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COLLATERAL IMBALANCES IN INTRA-EUROPEAN TRADE?

ACCOUNTING FOR THE DIFFERENCES BETWEEN GROSS AND VALUE ADDED TRADE BALANCES

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Abstract

One of the main stylised facts that has emerged from the recent literature on global value chains is that bilateral trade imbalances in gross terms can differ substantially from those measured in value added terms. However, the factors underlying the extent and sign of the differences between the two measures have so far not been investigated. Here, we propose a novel decomposition of bilateral gross trade balances that accounts for the differences between gross and value added concepts. The bilateral analysis contributes conceptually to the literature on double counting in trade by identifying the trade flow in which value added is actually recorded for the first time in international trade statistics. We apply our decomposition framework to the development of intra-EU27 trade balances from 1995-2011 and show that a growing share of intra-EU bilateral trade balances is due to demand in countries other than the two direct trading partners.

Keywords: Trade balances; Global value chains; Vertical specialisation; Value added; Input-output tables

JEL classification: F1, F2, C67, R15

Non-technical summary

The availability of global input-output tables has sparked a growing literature on global value chains and has rendered it possible to compute the value added content of trade (Dietzenbacher et al., 2013; Johnson and Noguera, 2012; Koopman et al., 2014). Value added trade between two countries describes in which country the particular parts of a good consumed in one country are produced. Value added trade flows can differ from gross trade flows due to trade in intermediate goods which are used as inputs to produce final goods. This is because an intermediate good, such as a subcomponent of a car engine, might cross several international borders until the final good, in this case the car, is purchased by a client abroad. Intermediate goods trade also leads to considerable discrepancies between bilateral value added and gross trade balances, which are a sensitive topic in the economic policy debate. For example, Johnson and Noguera (2012) find that in 2004 the trade deficit of the US with Japan was approximately 33% larger when measured on a value added basis, whereas the US trade deficit with China was approximately 30-40% smaller. Value added measures of bilateral trade arguably better reflect which countries benefit from trade in terms of income and employment (Foster-McGregor and Stehrer, 2013; Timmer et al., 2013). In contrast, measures of bilateral trade flows based on gross concepts can lead to misjudging the influence of domestic demand and relative price adjustments on bilateral trade balances (Bems and Johnson, 2012).

Despite the importance of bilateral trade balances in policy debates, the proximate factors that explain why value added and gross trade balance are different have so far not been investigated. Therefore, in this article we first identify to what extent gross and value added concepts overlap, which is a conceptual contribution to the trade in value added literature in general. Second, we describe the factors that account for the differences between value added and gross trade balances. We provide a novel distinction between value added that is due to demand of the direct trading partner, and value added that is due to demand in third countries. While both are ultimately the results of trade in intermediates, the difference between these two categories proves to be important from a policy perspective as demand in third countries is by definition unaffected by domestic demand in the two trading partners, which is often one of the main targets for current account adjustments. We apply our decomposition framework to the development of intra-EU27 trade balances from 1995 to 2011. A major determinant of the difference that emerges – along with foreign value added – is demand in countries other than the two trading partners. The latter accounted for 25% of the total variance of intra-EU gross bilateral trade balances in 2011, which marks a considerable rise from 3% in 1995. We find that the extent of intra-European imbalances has been overestimated, while trade imbalances with countries outside of the European have been underestimated using data of gross trade flows. A similar result holds for the euro area.

Our results matter for policy as, particularly in a currency union, it is important to establish with whom trade imbalances exist since the burden of adjustment may differ between trade deficits vis-á-vis member countries and third parties (di Mauro and Pappada, 2014). Even though their limitations are widely acknowledged gross bilateral trade balances still figure widely in the literature and policy debates (Davis and Weinstein, 2002; Bahmani-Oskooee and Brooks, 1999). Our paper therefore provides a strong case for considering value added instead of gross bilateral trade balances since a sizable portion of gross bilateral trade balances cannot be influenced by the two countries between which the imbalance exists.

1 Introduction

One of the main stylised facts that has emerged from the recent literature on global value chains is that bilateral trade balances in gross terms can differ substantially from those measured in value added terms, while aggregate trade balances are the same in both cases (Johnson and Noguera, 2012). Value added balances capture the difference between any two countries' domestically produced value added that is absorbed in final demand by their respective trading partner. In contrast to gross trade balances, they discount the part of trade flows that is double counted in official trade statistics (Koopman et al., 2014). Trade in intermediate inputs can lead to considerable discrepancies between the two concepts. For example, consider the stylised global production network depicted in Figure 1. The gross trade balance between country 3 and country 2 is 4 USD. However, the gross trade flow between the two countries also contains value added worth 1 USD of country 1 due to the presence of trade in intermediate goods. The value added balance between country 3 and country 2 is only 3 USD once foreign value added is discounted. In practice, value added and gross trade balances often differ substantially from each other. For example, Johnson and Noguera (2012) find that in 2004 the trade deficit of the US with Japan was approximately 33% larger when measured on a value added basis, whereas the US trade deficit with China was approximately 30-40% smaller. Value added measures of bilateral trade arguably better reflect which countries benefit from trade in terms of income and employment (Foster-McGregor and Stehrer, 2013; Timmer et al., 2013). In contrast, measures of bilateral trade flows based on gross concepts can lead to misjudging the influence of domestic demand and relative price adjustments on bilateral trade balances (Bems and Johnson, 2012).

While the importance of value added trade balances in economic theory and policy is beginning to gain recognition, the underlying factors that determine the sign and magnitude of the differences between the two concepts have so far not been investigated. The previous literature has generally ascribed discrepancies between gross and value added balances to different characteristics of international production networks such as triangular production sharing and the relative position of countries in global value chains (Johnson and Noguera, 2012; OECD, 2013; Antras et al., 2012). However, a mathematical framework that clarifies the relation between the two concepts at the bilateral level[1] is still lacking and hampers the interpretation of bilateral gross balances, which are currently the only data available in a timely manner. In this paper, we compute and analyse the proximate factors that account for the differences between gross and value added bilateral trade balances. In order to do so, we propose a novel decomposition of bilateral gross trade balances. This requires a conceptual framework that relates bilateral value added and gross trade flows to each other. A caveat which turns out to be inherent in the subject matter is that the relation between value added and gross trade flows is theoretically ill-defined, and therefore necessarily a question of what value added flows are defined to be in relation to physical trade flows. A first conceptual contribution is that we are, to the best of our knowledge, the first to identify this problem and discuss the two most parsimonious solutions to it, which determine the intersection between value added and gross trade flows at the bilateral level. The two solutions anchor value added flows either to the country of production or to the country of final absorption, which we call the source-based and

[1] Koopman et al. (2014) provide a decomposition of the *total* gross exports of a country into value added exports and six other components that are double counted in international trade statistics.

the sink-based approach, respectively (discussed in detail in the Methodology section and in Figure 2). This paper contributes conceptually to the literature on double counting by identifying the trade flow in which value added is actually recorded for the first time in international trade statistics. In comparison to other decompositions suggested in the literature, we show that Koopman et al. (2014) implicitly use a variation of the source-based view when identifying value added exports in gross trade flows, but that their subdivision of value added exports is to some extent arbitrary and not based on the number of international border crossings.

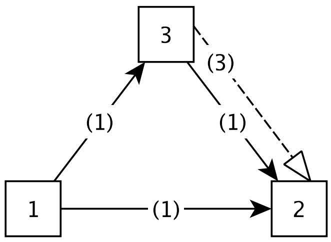

Figure 1: **Illustration of the difference between value added and gross trade balances.** Stylised global production network. Country 1 ships domestic value added worth 1 USD embedded in final goods to country 2 and domestic value added worth 1 USD in intermediate goods to country 3. Country 3 then adds 3 USD of domestic value added and ships final goods worth 4 USD (containing 1 USD of value added of country 1 and 3 USD of country 3) to country 2 for final consumption. In this case, the bilateral gross trade balances between the countries are $\mathrm{NX}^{12} = 1$, $\mathrm{NX}^{13} = 1$ and $\mathrm{NX}^{32} = 4$. The value added balances differ from the gross trade balances due to intermediates trade: $\mathrm{NVAX}^{12} = 2$, $\mathrm{NVAX}^{13} = 0$ and $\mathrm{NVAX}^{32} = 3$.

A second conceptual contribution is that we identify a range of categorically different components of value added that constitute the difference between bilateral value added and gross trade balances, which are independent of the methodological choice above. We find two quantities that explain the lion's share of the difference between gross and value added balances (other quantities like double counting and reflection are less important as shown below): (a) foreign value added absorbed by the two trading partners and (b) final demand in countries other than the two trading partners. The latter means that in a world of international production sharing the bilateral trade balance between two countries is to some degree a function of demand in the rest of the world.[2] The differences between the two quantities are of considerable importance, since - due to data availability - a majority of analyses currently still focuses on gross trade flows. For example, while both quantities in our decomposition may or may not be affected by bilateral exchange rate movements[3], domestic demand adjustments and trade shocks in third countries impact each of them differently. A decrease in domestic demand leads to an adjustment of the portion of the trade balance capturing foreign value added absorbed by the two trade partners, whereas by definition it will have no effect on the part of the trade balance which is due to demand

[2]For example, consider the bilateral trade balance between Germany and France. To satisfy demand in China, Germany exports final goods directly to Chinese firms and costumers. In order to produce these goods Germany needs to import a part of the required intermediates from France thereby pushing Germany more towards a trade deficit with France. Similarly, France also imports some intermediates from Germany in order to produce goods which are meant for final consumption and investment in China, which pushes the German trade balance with France more towards a trade surplus. The magnitude and the sign of the net effect depend on the level and sectoral composition of external demand and the relative position in the value chain of the two countries.

[3]Depending on whether it is more appropriate to measure price competitiveness in terms of goods or in tasks (Bems and Johnson, 2012; Bayoumi et al., 2013).

in third countries. Therefore, a completely balanced bilateral gross trade position is unlikely to be a good benchmark for assessing demand or price adjustments. Furthermore, this part of the trade balance may also occasionally be subject to volatility deriving from demand shocks in third countries that are independent of developments in the economies of the two trade partners.

Prior to the financial crisis there has been a substantial build-up in European trade imbalances. Particular attention has been paid to imbalances with other EU countries and especially with those countries with large current account deficits that were heavily affected by the financial crisis and the ensuing sovereign debt crisis (Berger and Nitsch, 2010; European Commission, 2010). We therefore apply our decomposition framework to the development of intra-EU27 trade balances from 1995 to 2011. We show that a major part of the difference, an average of 32% over the sample period, between intra-EU value added and gross bilateral balances is due to foreign value added consumed[4] by the respective trading partner. However, a sizable share of intra-EU bilateral trade balances is due to demand in countries other than the two trading partners. The latter accounted for 25% of the total variance of intra-EU gross bilateral trade balances in 2011, which is a considerable rise from 3% in 1995. A structural decomposition analysis indicates that this evolution was especially due to the rising importance of production fragmentation in the European Union while the change in global demand was the second most important factor.

The rest of the paper is structured as follows. Section 2 describes our decomposition framework and defines the two approaches that assign value added flows to gross trade balances and discusses the relation of our decomposition framework to the previous literature. Section 3 presents our empirical results and Section 4 concludes.

2 Methodology

In this section we first provide a short reminder of how bilateral gross and value added trade balances are calculated and, second, we present our decomposition framework, which can account for the differences between these two concepts. In the interest of space and readability, this section focuses on the case of three countries without loss of generality. The generalisation to the N-country case and technical details of the derivation are presented in the appendix.

2.1 Bilateral balances in gross and value added terms

A key finding of the literature on global value chains is that bilateral trade balances in gross terms differ from those in value added terms, while aggregate trade balances are the same in both cases. A global input-output table allows for the calculation of bilateral value added exports (Johnson and Noguera, 2012). Value added exports between country 1 and country 2 (VAX^{12})[5] are defined as the value added of country 1, which is ultimately absorbed in final demand by country 2. In the case of three countries,

[4] We use the terms *consumption* and *absorption in final demand* interchangeably. When using the term *consumption* we refer to all final demand categories specified in WIOD including final consumption expenditure by households, final consumption expenditure by non-profit organisations serving households, final consumption expenditure by the government, gross fixed capital formation and changes in inventories and valuables.

[5] Note that the acronym VAX in this article, which stands for value added exports, should not be confused with the VAX-ratio (Johnson and Noguera, 2012), which denotes the ratio of value added to gross exports of a country.

this is computed in the following way:

$$\text{VAX}^{12} = \begin{pmatrix} v^1 & 0 & 0 \end{pmatrix} \begin{pmatrix} l^{11} & l^{12} & l^{13} \\ l^{21} & l^{22} & l^{23} \\ l^{31} & l^{32} & l^{33} \end{pmatrix} \begin{pmatrix} f^{12} \\ f^{22} \\ f^{32} \end{pmatrix} = v^1 l^{11} f^{12} + v^1 l^{12} f^{22} + v^1 l^{13} f^{32} \tag{1}$$

where v^i is the value added coefficient of country i of the value added vector $\mathbf{v}$, l^{ij} refers to the ith row, jth column element[6] of the Leontief inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$, $\mathbf{A}$ is the global input-output coefficient matrix and f^{ij} denotes final goods flows from country i to country j. Value added imports of country 1 from 2 are equal to the value added exports of country 2 to 1. Hence, the value added trade balance of country 1 with 2 (NVAX^{12}) is the difference between their respective bilateral value added exports:

$$\begin{aligned} \text{NVAX}^{12} &= \text{VAX}^{12} - \text{VAX}^{21} \\ &= \left(v^1 l^{11} f^{12} + v^1 l^{12} f^{22} + v^1 l^{13} f^{32}\right) - \left(v^2 l^{21} f^{11} + v^2 l^{22} f^{21} + v^2 l^{23} f^{31}\right) \end{aligned}$$

Similarly, the gross trade balance between country 1 and 2 (NX^{12}) can be expressed as the difference between their respective bilateral gross exports:

$$\text{NX}^{12} = e^{12} - e^{21} = f^{12} + z^{12} - f^{21} - z^{21} = f^{12} + a^{12} x^2 - f^{21} - a^{21} x^1 \tag{2}$$

where e^{ij} denotes gross exports from country i to j, z^{ij} is the flow of intermediates between i and j, which equals the share of intermediates of country i in production of country j (a^{ij} of the global input-output matrix $\mathbf{A}$) multiplied by the level of gross output in country j (x^j).

At this stage, it is also worth highlighting the specific assumptions and main limitations of our study that are associated with its reliance on global input-output tables and which are shared by the literature on value added trade as a whole. These limitations include a proportionality assumption used to assign intermediate input flows to the production of different domestic sectors (Puzzello, 2012). In addition, in the process of preparing global input-output tables changes have to be made in order to balance the global coefficient matrix and in some cases methodological discrepancies exist in the underlying national accounts concepts between different countries (Dietzenbacher et al., 2013). Furthermore, input-output tables in general assume a fixed input structure and a constant returns to scale production function (Miller and Blair, 2009). However, it should be noted that some of these limitations are less relevant for an ex-post analysis of trade flows and when data is available, as in our case, for more than a single year.

2.2 Decomposition framework

To guide the reader through this section, we provide a brief outline of our decomposition framework and a summary of the main ideas. The purpose of the decomposition proposed here is to elucidate the differences between gross and value added trade balances. To identify meaningful categories in bilateral gross trade flows we use two simple criteria: (a) the origin of the value added embedded in gross trade flows and (b) the country, which ultimately absorbs the value added in its final demand.

[6] l^{ij} is an $S \times S$ matrix if the number of sector S is greater than one.

Accordingly, in a first step all gross trade flows are expressed as a function of final demand in accordance with a demand driven Leontief model and are broken down by the origin of their value added content. Second, a crucial step is to identify the intersection between bilateral gross and value added exports. The key question that arises in this regard is to which bilateral gross trade flow value added exports should be assigned if the embedded value added crosses international borders multiple times. It turns out that the solution to this problem is a matter of definition and therefore to some extent arbitrary. We consider two extreme cases taking the perspective of (1) the country, in which the value added originates (source-based approach) and (2) the country, which ultimately absorbs the value added in final demand (sink-based approach). While infinitely many other allocations are possible in theory, the two cases considered are arguably the most parsimonious and they allow us to assess the sensitivity of our results to this methodological choice. Our preferred solution is the source-based approach since it entails an intuitive definition of double counted value added based on the number of border crossings. In the source-based approach a value added export is assigned to the gross trade flow, in which it leaves the producing country for the very first time, whereas it is labeled as *double counted* in case it has crossed international borders (and hence has been counted as value added exports) previously. In accordance with the aforementioned definitions, bilateral gross trade balances are decomposed into six components explained in more detail below: a) the intersection with the respective value added trade balance, value added of one of the two trading partners that is b) double counted or c) reflected back via third countries for consumption in the country of origin, d) foreign value added consumed by the respective trading partner, e) domestic and f) foreign value added ultimately absorbed in final demand of third countries.

2.3 Bilateral exports and value added components

Bilateral gross exports from country 1 to country 2, as shown in equation (2), are a function of both demand in country 2 for final goods of country 1 and gross output of country 2, x^2:

$$e^{12} = f^{12} + a^{12}x^2 \tag{3}$$

As outlined above, in a first step, gross exports are expressed as a function of final demand of the country that ultimately absorbs the final goods and services in its final demand. In order to do so, we note that the gross output of country 2, x^2, is endogenous in a demand-driven Leontief system, i.e. gross output can be expressed as a function of final demand in all countries in the world

$$x^2 = l^{21}(f^{11} + f^{12} + f^{13}) + l^{22}(f^{21} + f^{22} + f^{23}) + l^{23}(f^{31} + f^{32} + f^{33}) \tag{4}$$

In a second step, gross exports are broken down by the origin of their value added content. We note that the identity $\mathbf{v}'(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{v}'\mathbf{L} = \iota'$ holds which follows from first principles[7] and consequently in

[7] The output x^j of a one sector economy j is simply the sum of its inputs z^{ij} from all countries C and the value added w^j generated in country j.

$$x^j = \sum_{i=1}^{C} z^{ij} + w^j$$

or in vector notation

$$\mathbf{x}' = \iota'\mathbf{Z} + \mathbf{w}' \tag{5}$$

the three-country case

$$\begin{pmatrix} v^1l^{11} + v^2l^{21} + v^3l^{31} \\ v^1l^{12} + v^2l^{22} + v^3l^{32} \\ v^1l^{13} + v^2l^{23} + v^3l^{33} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{6}$$

Substituting x^2 in equation (3) by (4) and multiplying the resulting expression by the first line of equation (6)[8] yields:

$$\begin{aligned} e^{12} &= v^1l^{11}f^{12} + v^2l^{21}f^{12} + v^3l^{31}f^{12} \\ &+ v^1l^{11}a^{12}\big[l^{21}(f^{11}+f^{12}+f^{13}) + l^{22}(f^{21}+f^{22}+f^{23}) + l^{23}(f^{31}+f^{32}+f^{33})\big] \\ &+ v^2l^{21}a^{12}\big[l^{21}(f^{11}+f^{12}+f^{13}) + l^{22}(f^{21}+f^{22}+f^{23}) + l^{23}(f^{31}+f^{32}+f^{33})\big] \\ &+ v^3l^{31}a^{12}\big[l^{21}(f^{11}+f^{12}+f^{13}) + l^{22}(f^{21}+f^{22}+f^{23}) + l^{23}(f^{31}+f^{32}+f^{33})\big] \end{aligned}$$

In order to identify meaningful categories in bilateral gross trade flows we use (a) the origin of the value added embedded in gross trade flows and (b) the country, which ultimately absorbs the value added in its final demand. Accordingly, we apply the label $v^r..f^{\cdot s}$ to the terms in e^{12}, where r indicates the country of origin of the value added and s denotes the country, in which the value added is eventually consumed, independent of the path the value added takes in between which can cross borders multiple times:

$$\begin{aligned} e^{12} = &\underbrace{v^1l^{11}a^{12}\sum_i^3 l^{2i}f^{i1}}_{v^1..f^{\cdot 1}} + \underbrace{v^1l^{11}f^{12} + v^1l^{11}a^{12}\sum_i^3 l^{2i}f^{i2}}_{v^1..f^{\cdot 2}} + \underbrace{v^1l^{11}a^{12}\sum_i^3 l^{2i}f^{i3}}_{v^1..f^{\cdot 3}} \\ &+ \underbrace{v^2l^{21}a^{12}\sum_i^3 l^{2i}f^{i1}}_{v^2..f^{\cdot 1}} + \underbrace{v^2l^{21}f^{12} + v^2l^{21}a^{12}\sum_i^3 l^{2i}f^{i2}}_{v^2..f^{\cdot 2}} + \underbrace{v^2l^{21}a^{12}\sum_i^3 l^{2i}f^{i3}}_{v^2..f^{\cdot 3}} \\ &+ \underbrace{v^3l^{31}a^{12}\sum_i^3 l^{2i}f^{i1}}_{v^3..f^{\cdot 1}} + \underbrace{v^3l^{31}f^{12} + v^1l^{11}a^{12}\sum_i^3 l^{2i}f^{i2}}_{v^3..f^{\cdot 2}} + \underbrace{v^3l^{31}a^{12}\sum_i^3 l^{2i}f^{i3}}_{v^3..f^{\cdot 3}} \end{aligned}$$

Before presenting our trade balance decomposition, we need to clarify the relation between domestic value added absorbed in final demand by country 2 ($v^1..f^{\cdot 2}$) and value added exports from country 1 to 2 (VAX^{12}). Phrasing the question differently, we need to determine how to distribute VAX^{12} to bilateral gross trade flows between countries. To illustrate the problem, let's consider the first term, $v^1l^{11}f^{12}$, in the expression for VAX^{12} in equation (1). It is important to note that the coefficient l^{11} describes

The global input-output coefficient matrix can be obtained by scaling the row entries of the input matrix $\mathbf{Z}$ by the total output of the respective country: $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$, where $\hat{\mathbf{x}}$ denotes the diagonal matrix created from vector $\mathbf{x}$. Similarly, the value added coefficient vector can be obtained by scaling the value added generated in a particular country by its total output: $\mathbf{v} = \mathbf{w}\hat{\mathbf{x}}^{-1}$. Substituting $\mathbf{Z} = \mathbf{A}\hat{\mathbf{x}}$ and postmultiplying by $\hat{\mathbf{x}}^{-1}$ in equation (5) yields

$$\mathbf{x}'\hat{\mathbf{x}}^{-1} = \iota'\mathbf{A}\hat{\mathbf{x}}\hat{\mathbf{x}}^{-1} + \mathbf{w}'\hat{\mathbf{x}}^{-1}$$

or

$$\iota' = \iota'\mathbf{A} + \mathbf{v}'$$

which can be re-written as $\mathbf{v}'(\mathbf{I} - \mathbf{A})^{-1} = \iota'$ to yield the expression used in the main text. Note that this derivation was adopted from Miller and Blair (2009) with minor modifications in notation.

[8]The left-hand side of the expression is unchanged since all entries of the vector in equation (6) are equal to one.

all possible ways that value added from country 1 can take to travel embedded in intermediate goods through international production networks back to country 1 (see Appendix A.1). A similar argument holds for the other two terms in Equation (1). This means that in the presence of international production sharing a certain proportion of value added in VAX^{12} passes - embedded in intermediates - through gross trade flows between any two countries in the world. This implies that it is recorded several times in international trade statistics. At least in theory, it is conceivable to assign these portions of VAX^{12} to the respective bilateral flows.[9] In the following, we consider two extreme cases. First, we take the perspective of the country, in which the value added originates (source-based approach) and, second, we consider the country which ultimately absorbs the value added in its final demand as a reference point (sink-based approach). While it is conceivable to attribute value added exports of country 1 to 2 to gross trade flows between third countries, this would be theoretically unappealing and the two cases considered in this text are arguably the most parsimonious.

In the source-based approach (for a numerical example see Figure 2a and Section 2.6), we assign only that portion of $v^1..f^{\cdot 2}$ to VAX^{12} which leaves the country for the first time in final goods or in the form of intermediate goods for further processing abroad, which we denote by VAX^{*12}. The remainder of $v^1..f^{\cdot 2}$ has by definition been re-imported by 1 after processing abroad and hence has crossed international borders at least twice, which we denote by DBC for double counting. To implement this mathematically the entry of the Leontief inverse l^{11} is split into a portion that represents intra-country processing, $I + a^{11} + a^{11}a^{11} + a^{11}a^{11}a^{11} + \ldots = (1 - a^{11})^{-1}$ (i.e. every possible way of going from country 1 to country 1 without leaving the country) captured by the domestic Leontief inverse, and the remainder $l^{11} - (1 - a^{11})^{-1}$, which has crossed international borders at least twice.

$$\begin{aligned}
v^1..f^{\cdot 2} &= v^1 l^{11} f^{12} + v^1 l^{11} a^{12} \sum_i^3 l^{2i} f^{i2} \\
&= \underbrace{v^1 (1 - a^{11})^{-1} f^{12} + v^1 (1 - a^{11})^{-1} a^{12} \sum_i^3 l^{2i} f^{i2}}_{\text{VAX}^{*12}} \\
&+ \underbrace{v^1 (l^{11} - (1 - a^{11})^{-1}) f^{12} + v^1 (l^{11} - (1 - a^{11})^{-1}) a^{12} \sum_i^3 l^{2i} f^{i2}}_{\text{DBC}}
\end{aligned}$$

Similarly, we can consider gross exports between country 1 and 3, e^{13}, which also include a term $v^1..f^{\cdot 2}$ that can be split into VAX^{*12} and DBC. Note that in the source-based approach VAX^{*ij} terms appear only in bilateral gross exports of country i, i.e. the value added producing country. Therefore, in the source-based approach, summing VAX^{*ij} terms across all bilateral imports of country j, i.e. the value added absorbing country, in general does not yield value added imports from country i. In Appendix A.5 we prove that the sum of all VAX^{*12} terms in e^{12} and e^{13} equals the value added exports from country 1 to 2, VAX^{12}.

[9] For example, in the stylised production chain depicted in Figure 3a domestic value added of country 1 travels embedded in intermediates via country 2 and 3 to country 4 where it is absorbed in final demand, and hence counts as value added exports from country 1 to 4, VAX^{14}. At least in theory, the value added trade worth 1 USD of country 1 could be assigned to the gross trade flow between country 2 and 3, even though this involves neither the country that generated nor consumes the value added.

In the sink-based approach (for a numerical example see Figure 2b and Section 2.6), instead of considering the producing country we take the absorbing country as a reference point. In this case, we assign value added exports to the gross trade flow, in which value added last enters the country of final demand. Hence, we allocate only that portion of $v^1..f^{\cdot 2}$ to VAX^{12} which never leaves country 2 again before being absorbed in final demand. The remainder of $v^1..f^{\cdot 2}$ will once again be flagged as having been double counted. Domestic value added in final goods exports between country 1 and 2 is directly consumed in country 2 so it is unambiguously assigned to VAX*^{12}. For intermediate goods the situation is slightly more complicated. The portion of country 1's value added which is directly used in the production of final goods immediately consumed in country 2 is allocated to VAX*^{12}. The remainder leaves country 2 again embedded in intermediates even though it eventually re-enters 2 again for final absorption. The part of country 1's value added that never leaves country 2 again before being consumed, can be expressed mathematically as $1 + a^{22} + a^{22}a^{22} + a^{22}a^{22}a^{22} + \ldots = (1 - a^{22})^{-1}$ instead of using l^{22}.

$$
\begin{aligned}
v^1..f^{\cdot 2} &= v^1 l^{11} f^{12} + v^1 l^{11} a^{12} l^{22} f^{22} + v^1 l^{11} a^{12} \sum_{i \neq 2}^{3} l^{2i} f^{i2} \\
&= \underbrace{v^1 l^{11} f^{12} + v^1 l^{11} a^{12} (1 - a^{22})^{-1} f^{22}}_{\text{VAX*}^{12}} \\
&\quad \underbrace{+ v^1 l^{11} a^{12} (l^{22} - (1 - a^{22})^{-1}) f^{22} + v^1 l^{11} a^{12} \sum_{i \neq 2}^{3} l^{2i} f^{i2}}_{\text{DBC}}
\end{aligned}
$$

Similarly, we can consider the gross exports between country 3 and 2, e^{32}, which also include a term $v^1..f^{\cdot 2}$ that can be split into VAX*^{12} and DBC. Note that in contrast to the source-based approach VAX*^{ij} terms appear only in bilateral gross imports of country j, i.e. the value added absorbing country. Therefore, in the sink-based approach, summing VAX*^{ij} terms across all bilateral exports of country i, i.e. the value added producing country, in general does not yield value added exports of country i. Again, in Appendix A.5 we prove that the sum of the VAX*^{12} terms in e^{12} and e^{32} equals the value added imports of country 2 from 1, VAM^{21}, and hence the value added exports from country 1 to 2, VAX^{12}.

Arguments can be found against and in favour of either one of the source- and the sink-based approach. We lean slightly towards the source-based approach since it entails an intuitive definition of value added that is double counted based on the number of border crossings. In the interest of space, we focus on the source-based approach in the main text and present the sink-based decomposition and its results in the appendix.

2.4 Relation to alternative decomposition approaches

Several decompositions of gross trade flows have recently been suggested in the literature (Daudin et al., 2011; Johnson and Noguera, 2012; Koopman et al., 2014; Foster-McGregor and Stehrer, 2013). However, none of these investigates the relation between bilateral gross and value added trade in general and bilateral gross and value added trade balances in particular. While Johnson and Noguera (2012) relate value added balances to gross trade balances in terms of differences in bilateral value added to export ratios, their analysis leaves unexplained why these ratios differ between countries in the first place. The decomposition that is most similar to our contribution is the work by Koopman et al. (2014) (henceforth KWW), who decompose gross exports - albeit at the aggregate level - into value added exports and terms that are double counted in international trade statistics. The implicit definition of value added exports in the decomposition by KWW is worth clarifying in the light of our bilateral decomposition framework introduced here. In KWW all value added trade flows are assigned to the exporting country and the remainder of the gross trade flows between countries is labeled as double counted. Hence, KWW in principle endorse the source-based approach. However, value-added exports are then further subdivided into (1) domestic value added in direct final goods exports ($v^s \sum_{r \neq s}^{C} l^{ss} f^{sr}$), (2) domestic value added in intermediates exports absorbed by direct importers ($v^s \sum_{r \neq s}^{C} l^{sr} f^{rr}$) and (3) domestic value added in intermediates re-exported to third countries in intermediate goods ($v^s \sum_{r \neq s}^{C} \sum_{t \neq s,r}^{C} l^{sr} f^{rt}$). Note that this subdivision is not based on how often value added crosses international borders, but on the distinction between final and intermediate goods, and a somewhat ambiguous partition of intermediate exports.

Our decomposition framework suggests two refinements to the subdivision of value added exports. First, the division into final and intermediate goods exports is to a certain degree arbitrary since the first term involves both intermediate and final goods flows. In (1) l^{ss} describes all possible ways that domestic value added can travel through international production networks back to country s (see discussion above and Appendix A.1). Hence, a certain share of the domestic value added in (1) is initially exported in intermediates before it is included in final goods exports of s and shipped off to country r for absorption in final demand. Only $v^s(I - a^{ss})^{-1} f^{sr}$ leaves country s for the very first time in final goods that are absorbed in country r and therefore should be considered to be part of (1). The remainder $v^s(l^{ss} - (I - a^{ss})^{-1}) f^{sr}$ crosses international borders at least twice in intermediates before being absorbed in final demand and hence is more appropriately subsumed in category (3) which captures domestic value added in intermediates that is re-exported to third countries in intermediate goods. Second, not all intermediates in (2) are absorbed by their *direct* importer since the Leontief inverse l^{sr} describes all possible ways that value added can flow between country s and country r involving third countries, which is not in general equivalent to a^{sr}, i.e. direct sourcing of intermediates by country r from country s. In the following l^{sr} will be substituted by three terms. As a first step only value added should be considered that has not left country s for processing abroad previously. In order to do so, once again, the term $(I - a^{ss})^{-1}$ should be used. In a second step, we are only interested in that part of domestic value added, which is *directly* shipped from country s to r for which we use the international sourcing coefficient a^{sr} denoting the share of total inputs of country r that comes from country s. Finally, we exclusively want to consider value added that does not leave country r again (since this would imply that value added crosses international borders at least twice in intermediates before being absorbed in final demand and hence this part would be more appropriately allocated to category (3)). In order to do so we choose $(I - a^{rr})^{-1}$ which allows for intra-country processing. In summary, the term l^{sr} is substituted by $(I - a^{ss})^{-1} a^{sr} (I - a^{rr})^{-1}$.[10] Once again, the remainder $v^s(l^{sr} - (I - a^{ss})^{-1} a^{sr} (I - a^{rr})^{-1}) f^{rr}$ crosses international borders multiple

[10] Another way to derive this expression is from the definition of the matrix inverse (see Appendix A.4). Equation (12) states that $l^{12} = (1 - a^{11})^{-1}(a^{12} l^{22} + a^{13} l^{32})$ for the three-country case. The generalisation to the N-country case of this expression is $l^{sr} = (1 - a^{ss})^{-1}(\sum_{t \neq s} a^{st} l^{tr})$. Note that in this expression only $(1 - a^{ss})^{-1} a^{sr} l^{rr}$ describes direct imports of intermediates of country r from s. In order to exclude value added leaving country r for additional processing abroad, once again, l^{rr} needs to be substituted by $(1 - a^{rr})^{-1}$. Collecting terms and substituting l^{sr} in $v^s l^{sr} f^{rr}$ one arrives at $v^s(I - a^{ss})^{-1} a^{sr} (I - a^{rr})^{-1} f^{rr}$ as before.

times and is assigned to category (3). In summary, for a consistent decomposition with the source-based approach as defined above (i.e. based on the number of international border crossings) and the labels suggested by KWW, it would be appropriate to decompose value added exports as follows: (1) $v^s \sum_{r\neq s}^{C}(I-a^{ss})^{-1}f^{sr}$, (2) $v^s \sum_{r\neq s}^{C}(I-a^{ss})^{-1}a^{sr}(I-a^{rr})^{-1}f^{rr}$ and (3) $v^s \sum_{r\neq s}^{C}(l^{ss}-(I-a^{ss})^{-1})f^{sr} + v^s \sum_{r\neq s}^{C}(l^{sr}-(I-a^{ss})^{-1}a^{sr}(I-a^{rr})^{-1})f^{rr} + v^s \sum_{r\neq s}^{C}\sum_{t\neq s,r}^{C} l^{sr}f^{rt}$. Hence, (1) describes value added that leaves country s for the very first time in final goods exports, (2) captures domestic value added in intermediates that is absorbed immediately by direct importers without being re-exported for further processing and (3) collects all the domestic value added terms that cross international borders several times before being incorporated in domestic final goods or final goods of other countries and before being absorbed in final demand abroad. Note that value added in terms (1) and (2) crosses international borders only once before being absorbed in final demand, whereas value added in term (3) crosses international borders multiple times and therefore leads to double counting in international trade statistics.

While the two decompositions differ conceptually from each other, how important the differences are quantitatively is an empirical question. Table 1 shows the subdivision of value added exports according to KWW and our alternative subdivision based on the first international border crossing denoted by KWW* for the top ten value added exporters in 2011 in billion USD. The KWW* subdivision assigns the share of (1) domestic value added in direct final goods exports and (2) domestic value added in intermediates exports absorbed by direct importers that crosses international borders more than once to the third category (3*). Therefore (1) and (2) in the KWW decomposition are always larger than (1*) and (2*) in the KWW* decomposition, and (3) is always smaller than (3*). For example, China exported domestic value added worth 744 billion USD in final goods exports, 8 billion USD of which were originally exported as intermediates by China. Since this part was re-imported by China after processing abroad, it crossed international borders several times and was hence added to the third category (3*). China exported 676 billion USD worth of domestic value added in intermediates which was included and consumed in final goods in the same country. Domestic value added worth 128 billion USD of this amount crossed international borders more than once and was added to the third category (3*). In general, for domestic value added in direct final goods exports the difference between the two decompositions (cf. (1) vs. (1*)) is quantitatively relatively minor since only a small portion of domestic value added is re-imported by the country of production. For domestic value added in intermediates exports absorbed by direct importers, however, the difference between the two decompositions (cf. (2) vs. (2*)) is quantitatively non-negligible. This is because domestic value added in intermediate goods often crosses international borders more than once before being absorbed in final demand. The revised decomposition KWW* takes multiple border crossings into account and therefore yields a considerably larger value of domestic value added in intermediates re-exported to third countries in intermediate goods. In general, whether to choose the KWW or the KWW* decomposition depends on the particular research or policy question under consideration. If the interest lies in categorising value added flows according to differences in final absorption, KWW could be used since the KWW decomposition splits value added exports into three categories according to which country is involved in producing the final good before consumption.[11] In

[11] f^{sr}, f^{rr} and f^{rt}, where s is the country that produces the value added, r is a direct trading partner of s and t is a third country.

our case, KWW* is preferable since it accurately identifies the part of the value added that leads to double counting in international trade flows.

	KWW			KWW*			
	(1)	(2)	(3)	(1*)	(2*)	(3*)	
countries	DV in direct final goods exports	DV in intermediates exports absorbed by direct importers	DV in intermediates re-exported to third countries in intermediate goods	DV in direct final goods exports	DV in intermediates exports absorbed by direct importers	DV in intermediates re-exported to third countries in intermediate goods	VAX
CHN	744	676	154	736	548	290	1, 574
USA	463	852	140	458	694	303	1, 455
DEU	451	539	122	442	415	255	1, 112
JPN	258	379	93	257	301	172	730
GBR	169	303	67	168	223	148	539
FRA	201	227	55	200	176	107	483
RUS	39	331	80	39	229	182	451
ITA	195	185	49	194	141	93	429
CAN	109	264	34	108	223	75	406
KOR	119	197	46	119	155	88	362
world	4, 414	7, 400	1, 510	4,370	5, 839	3, 116	13, 325

Table 1: **Comparison between the KWW and KWW* decomposition.**
Subdivision of value added exports (column 8) according to KWW (column 2-4) and our adjusted subdivision denoted by KWW* (column 4-7) for the top ten value added exporters in 2011 in billion USD. (Deviations between the totals of the two subdivisions are due to rounding.)

2.5 Decomposition of bilateral gross trade balances (source-based approach)

This brings us back to apply the source-based decomposition approach above to bilateral trade balances (see Appendix A.2 for the sink-based approach). Using the definition of a bilateral trade balance in gross terms and inserting the respective expressions yields:

$$
\begin{aligned}
NX^{12} &= e^{12} - e^{21} \\
&= \underbrace{v^1(I-a^{11})^{-1}f^{12} + v^1(I-a^{11})^{-1}a^{12}\sum_i^3 l^{2i}f^{i2}}_{\text{NVAX*}^{12}\ (\text{e}^{12})} - \underbrace{v^2(I-a^{22})^{-1}f^{21} - v^2(I-a^{22})^{-1}a^{21}\sum_i^3 l^{1i}f^{i1}}_{\text{NVAX*}^{12}\ (\text{e}^{21})} \\
&+ \underbrace{v^1(l^{11}-(I-a^{11})^{-1})f^{12} + v^1(l^{11}-(I-a^{11})^{-1})a^{12}\sum_i^3 l^{2i}f^{i2}}_{\text{DBC}\ (\text{e}^{12})} + \underbrace{v^2 l^{21}a^{12}\sum_i^3 l^{2i}f^{i1}}_{\text{DBC}\ (\text{e}^{12})} \\
&- \underbrace{v^2(l^{22}-(I-a^{22})^{-1})f^{21} - v^2(l^{22}-(I-a^{22})^{-1})a^{21}\sum_i^3 l^{1i}f^{i1}}_{\text{DBC}\ (\text{e}^{21})} - \underbrace{v^1 l^{12}a^{21}\sum_i^3 l^{1i}f^{i2}}_{\text{DBC}\ (\text{e}^{21})} \\
&+ \underbrace{v^2 l^{21}f^{12} + v^1 l^{11}a^{12}\sum_i^3 l^{2i}f^{i1} + v^2 l^{21}a^{12}\sum_i^3 l^{2i}f^{i2}}_{\text{REFL}\ (\text{e}^{12})} - \underbrace{v^1 l^{12}f^{21} - v^1 l^{12}a^{21}\sum_i^3 l^{1i}f^{i1} - v^2 l^{22}a^{21}\sum_i^3 l^{1i}f^{i2}}_{\text{REFL}\ (\text{e}^{21})} \\
&+ \underbrace{v^3 l^{31}f^{12} + v^3 l^{31}a^{12}\sum_i^3 l^{2i}(f^{i1}+f^{i2})}_{\text{FVA}^{tp}\ (\text{e}^{12})} - \underbrace{v^3 l^{32}f^{21} - v^3 l^{32}a^{21}\sum_i^3 l^{1i}(f^{i1}+f^{i2})}_{\text{FVA}^{tp}\ (\text{e}^{21})} \\
&+ \underbrace{v^1 l^{11}a^{12}\sum_i^3 l^{2i}f^{i3} + v^2 l^{21}a^{12}\sum_i^3 l^{2i}f^{i3}}_{\text{DVA}^{3rd}\ (\text{e}^{12})} - \underbrace{v^2 l^{22}a^{21}\sum_i^3 l^{1i}f^{i3} - v^1 l^{12}a^{21}\sum_i^3 l^{1i}f^{i3}}_{\text{DVA}^{3rd}\ (\text{e}^{21})} \\
&+ \underbrace{v^3 l^{31}a^{12}\sum_i^3 l^{2i}f^{i3}}_{\text{FVA}^{3rd}\ (\text{e}^{12})} - \underbrace{v^3 l^{32}a^{21}\sum_i^3 l^{1i}f^{i3}}_{\text{FVA}^{3rd}\ (\text{e}^{21})} \qquad (7)
\end{aligned}
$$

where individual terms come with a label indicating whether they belong to e^{12} or e^{21} and have been grouped into the following categories. NVAX*^{12} is the subset of NVAX^{12} that is part of the bilateral gross trade balance between country 1 and 2 as described above. DBC refers to the double counting terms identified previously and, in addition, value added of the respective trading partner that is double counted, i.e. $\sum_i \sum_j v^j..f^{\cdot i}(e^{ij})$, where $i \neq j \in \{1,2\}$ and (e^{ij}) denotes the appropriate bilateral gross trade flow. REFL is value added of one of the two trade partners that is reflected back via third countries and eventually consumed in the country of production, i.e. $\sum_i v^i..f^{\cdot i}$, where $i \in \{1,2\}$. FVA^{tp} is foreign value added consumed by one of the two trading partners, i.e. $\sum_k \sum_i v^k..f^{\cdot i}$, where $i \in \{1,2\}$ and $k \in \{3\}$ in the three country case. DVA^{3rd} is value added of one of the two trading partners ultimately absorbed in a third country, i.e. $\sum_i \sum_k v^i..f^{\cdot k}$, where $i \in \{1,2\}$ and $k \in \{3\}$ in the three country case. FVA^{3rd} is foreign value added ultimately absorbed in a a third country, i.e. $\sum_k \sum_l v^k..f^{\cdot l}$, where $k \in \{3\}$ and $l \in \{3\}$ in the three country case. Please refer to the appendix A.3 for the generalisation of the source-based decomposition to the N-country case.

Note that in the results section we group DBC and REFL together as a residual term, and we combine DVA^{3rd} and FVA^{3rd} to capture any imbalance between the two direct trading partners that is due to demand in third countries. Since we are interested in the differences between gross and value added trade balances, we also introduce the additional term CORR (for value added correction). CORR denotes value added produced by one of the two trade partners that is absorbed in final demand by the respective trade partner, which is part of the value added trade balance, but does not figure in the balance in gross terms:

$$\text{CORR} = \text{NVAX}^{*12} - \text{NVAX}^{12} \tag{8}$$

This term reflects value added that is part of other bilateral gross trade balances (of the exporting country in the source-based approach and of the importing country in the sink-based approach).

2.6 Examples

This section provides some stylised examples of the different categories in the decomposition of bilateral gross trade balances and the value added correction factor, CORR, described in the previous section in order to help build intuition. First, consider the three country example already referred to in the introduction (Figure 1), in which country 1 ships domestic value added worth 1 USD embedded in final goods to country 2 and domestic value added worth 1 USD in intermediate goods to country 3. Country 3 then adds 3 USD of domestic value added and ships final goods worth 4 USD (containing 1 USD of value added of country 1 and and 3 USD of country 3) to country 2 for final consumption. In this case, the bilateral gross trade balances between the countries are $\text{NX}^{12} = 1$, $\text{NX}^{13} = 1$ and $\text{NX}^{32} = 4$. The value added balances differ from the gross trade balances due to intermediates trade: $\text{NVAX}^{12} = 2$, $\text{NVAX}^{13} = 0$ and $\text{NVAX}^{32} = 3$. Our decomposition framework allows the allocation of bilateral value added trade to gross trade flows. Consider the value added exports of country 1, which produces a total of 2 USD of value added that are ultimately absorbed in final demand in country 2. To which bilateral gross trade flow in this three country world should the value added exports of country 1 be assigned? The answer differs in the source-based and the sink-based approach.

In the source-based approach, value added trade is assigned to the very first trade flow in which it leaves the country of production (Figure 2a). In our example, domestic value added of country 1 worth 1 USD leaves country 1 for the first time in intermediate goods exports to country 3, and 1 USD of value added exports is accordingly assigned to the gross trade balance between country 1 and 3, NX^{13}. Note that we do so even though the value added will eventually be absorbed in final demand in country 2 (and hence is part of the value added exports of country 1 to country 2, VAX^{12}, and also the value added trade balance between country 1 and country 2, NVAX^{12}). Similarly, domestic value added of country 1 worth 1 USD leaves country 1 for the first time embedded in final goods exports to country 2, and 1 USD of value added exports is accordingly assigned to the gross trade balance between country 1 and 2, NX^{12}. Note that this portion of value added of country 1 is also consumed by country 2, and therefore it is part of the value added exports of country 1 to country 2, VAX^{12}, and the their value added trade balance, NVAX^{12}. Since this fraction of value added of country 1 is part of both the value added and the gross trade balance between country 1 and 2, it represents the intersection between the two concepts and accordingly we label it NVAX^{*12}. As mentioned above not all of the value added trade balance between country 1 and country 2 ($\text{NVAX}^{12} = 2$) is contained in their respective gross trade balance ($\text{NVAX}^{*12} = 1$). 1 USD was exported in intermediates to country 3 for further processing

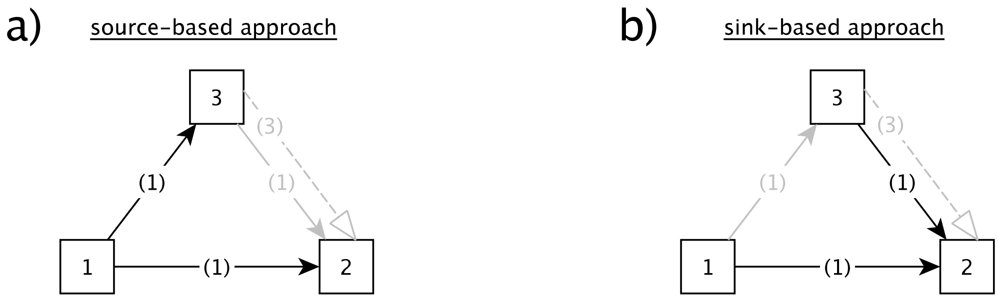

Figure 2: **Illustration of the difference between source-based and sink-based allocations of value added.**
a) Source-based approach. The source-based approach assigns the value added exports of country 1 to country 2 to the gross trade flow, in which the value added leaves the country of production, country 1, for the first time. In this case, 1 USD of value added is assigned to the gross trade flow between country 1 and 2, and 1 USD of value added is assigned to the gross trade flow between country 1 and 3. b) Sink-based approach. The sink-based approach assigns the value added imports of country 2 from country 1 to the gross trade flow, in which the value added last enters the country of consumption, country 2. In this case, 1 USD of value added is assigned to the gross trade flow between country 1 and 2, and 1 USD of value added is assigned to the gross trade flow between country 2 and 3.

before final absorption in country 2. In order to keep track of this part of the value added balance that enters other bilateral gross trade balances, we compute the difference between the intersection of value added and gross trade balance, and the value added balance itself, which we label CORR for value added correction factor (CORR=NVAX^{*12} - $\text{NVAX}^{12} = 1 - 2 = -1$).

In the sink-based approach, value added trade is assigned to the very last trade flow in which it enters the country of final absorption (Figure 2b). In our example, domestic value added of country 1 worth 1 USD enters country 2 in final goods, that are immediately consumed, from country 3, and 1 USD of the value added imports of country 3 from country 1 is accordingly assigned to the gross trade balance between country 2 and 3, NX^{23}. Note that we do so even though the value added does not originate in country 3 (and is part of the value added imports of country 2 from country 1, VAM^{21}, and also the value added trade balance between country 1 and country 2, NVAX^{12}). Similarly, domestic value added of country 1 worth 1 USD enters country 2 in final goods, which are immediately absorbed in final demand, from country 1. Note that this portion of value added of country 1 is assigned to the same gross trade balance in both the source-based and in the sink-based approach. Therefore, as in the source-based approach, this fraction of value added of country 1 is part of both the value added and the gross trade balance between country 1 and 2, it represents the intersection between the two concepts and accordingly we label it NVAX^{*12}. Again, the value added trade balance between country 1 and country 2 ($\text{NVAX}^{12} = 2$) is not a subset of their respective gross trade balance ($\text{NVAX}^{*12} = 1$). 1 USD of value added of country 1 is assigned to the gross trade balance between country 2 and 3. As before, we compute the difference between the intersection of value added and gross trade balance, and the value added balance itself (CORR=NVAX^{*12} - $\text{NVAX}^{12} = 1 - 2 = -1$). Although in this simple example the correction factor turns out to be the same in both the source-based and in the sink-based approach, this is not true in general. The following examples are independent of whether the source- or the sink-based approach is chosen (with the exception of the example for the double counting term).

Foreign value added consumed by the respective trading partner (FVA^{tp}): Country 3 has a gross trade surplus of 4 USD with country 2. 3 USD of the gross trade balance is value added of country 3 that is absorbed in country 2, i.e. NVAX^{32} (Figure 1). The remainder of 1 USD of the gross trade balance is value added of country 1 that country 3 imported in intermediates, which is ultimately absorbed by the direct trading partner, country 2. This counts as foreign value added absorbed by one of the two trading partners, i.e. FVA^{tp}. In summary, the gross trade balance between country 3 and 2 can be decomposed into $\text{NX}^{32} = \text{NVAX}^{32} + \text{FVA}^{tp} = 3 + 1 = 4$.

Domestic value added ultimately absorbed in final demand of third countries (DVA^{3rd}): Country 1 has a gross trade surplus of 1 USD with country 3 (Figure 1). In value added terms, however, trade between the two countries is completely balanced since neither country consumes any value added of the other. The difference between gross and value added balances arises because country 1 ships 1 USD worth of intermediate goods to country 3, which are ultimately deemed for consumption in a third country (in this case country 2). Since the value added in intermediate goods originates in one of the two trading partners, it is labeled domestic value added ultimately absorbed in final demand in a third country. In summary, $\text{NX}^{13} = \text{DVA}^{3rd} = 1$.

Foreign value added ultimately absorbed in final demand of third countries (FVA^{3rd}): For this category an example with a longer production chain is necessary. Consider a world with four countries forming a sequential production chain (Figure 3a), in which country 1 ships intermediate goods of 1 USD to country 2. Country 2 then adds 2 USD of domestic value added to those intermediate goods and exports them to country 3 for further processing. Country 3 in turn adds another 3 USD of domestic value added and ships everything of as final goods worth 6 USD (containing 1 USD of value added of country 1, 2 USD of value added of country 2 and 3 USD of value added of country 3) to country 4 for final consumption. Country 2 has a gross trade surplus of 3 USD with country 3, but balanced trade in value added terms since neither country consumes any value added of the other. The difference between gross and value added balances arises, on the one hand, due to domestic value added of country 2 worth 2 USD that is shipped off to country 3 for final absorption in country 4 (DVA^{3rd}, see (d) above). In addition, country 2 exports 1 USD of foreign value added (of country 1) to country 3 that is also deemed for final absorption in country 4 and therefore should be adequately labeled FVA^{3rd}. In summary, $\text{NX}^{23} = \text{DVA}^{3rd} + \text{FVA}^{3rd} = 2 + 1 = 3$.

Value added of one of the two trading partners that is reflected back via third countries for consumption in the country of origin (REFL): In order to illustrate the reflection term[12] consider a three country example (Figure 3b), in which country 1 exports 1 USD of domestic value added in intermediates to country 2. Country 2 adds 2 USD of domestic value added and ships 3 USD worth of intermediates to country 3. Country 3 then adds another 3 USD of value added and ships 6 USD of final goods (containing 1 USD of value added of country 1, 2 USD of value added of country 2 and 3 USD of value added of country 3) to country 1. Country 1 has a gross trade surplus of 1 USD with country 2, but balanced trade in value added terms. The difference arises from domestic value added worth 1 USD that is exported to country 2, which re-enters country 1 for final consumption after processing in country 3. Essentially,

[12]For ease of exposition only the second term of the reflection expression in equation (7) is considered.

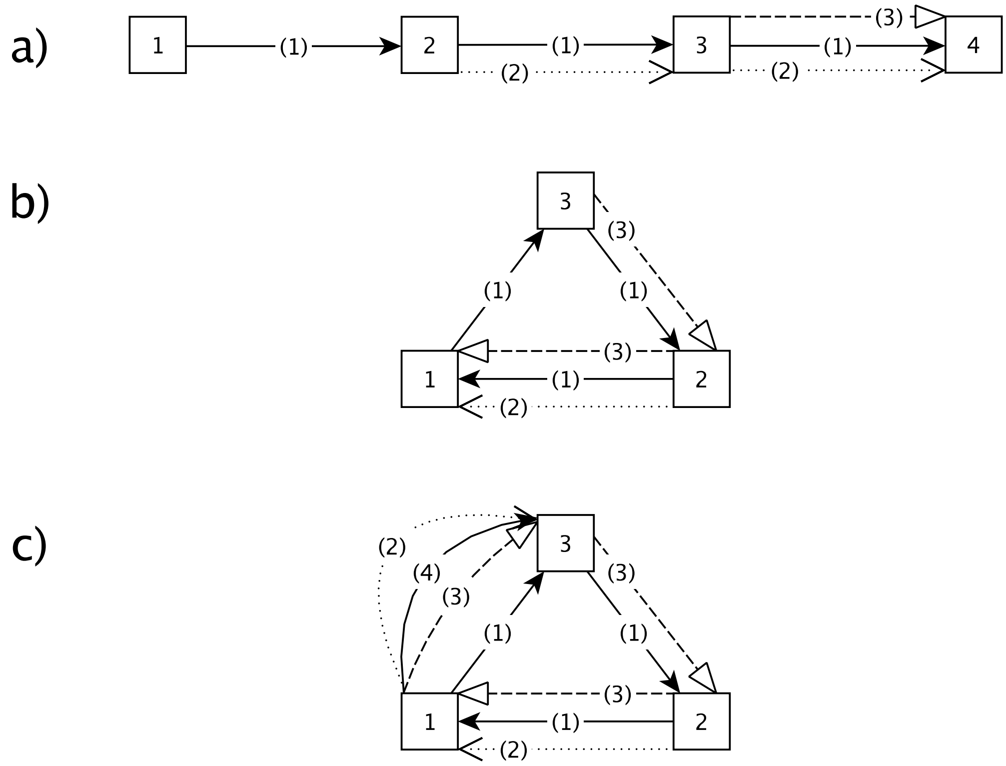

Figure 3: **Illustration of different categories in the decomposition of bilateral gross trade balances.**
a) Example for FVA^{3rd}. Country 1 ships intermediate goods of 1 USD to country 2. Country 2 then adds 2 USD of domestic value added to those intermediate goods and exports them to country 3 for further processing. Country 3 in turn adds another 3 USD of domestic value added and ships everything of as final goods worth 6 USD (containing 1 USD of value added of country 1, 2 USD of value added of country 2 and 3 USD of value added of country 3) to country 4 for final consumption. b) Example for the reflection term, REFL. Country 1 exports 1 USD of domestic value added in intermediates to country 2. Country 2 adds 2 USD of domestic value added and ships 3 USD worth of intermediates to country 3. Country 3 then adds another 3 USD of value added and ships 6 USD of final goods (containing 1 USD of value added of country 1, 2 USD of value added of country 2 and 3 USD of value added of country 3) to country 1. c) Example for the double counting term, DBC. As b) but instead of country 3 exporting the final good worth 6 USD to country 1, suppose that country 3 exports an intermediate good worth 6 USD (containing 1 USD of value added of country 1, 2 USD of value added of country 2 and 3 USD of value added of country 3) to country 1 for further processing. Country 1 then adds 3 USD of value added and ships a final good worth 9 USD to country 2 for final consumption (containing a total of 4 USD of value added of country 1).

the value added is reflected back to the country of initial production via a third country. In summary, $NX^{12} = REFL = 1$.

Value added of one of the two trading partners that is double counted: Consider the same three country example as before with one small modification that illustrates the double counting term for the source-based approach[13] (Figure 3c). Instead of country 3 exporting the final good worth 6 USD to country 1, suppose that country 3 exports an intermediate good worth 6 USD (containing 1 USD of value added of country 1, 2 USD of value added of country 2 and 3 USD of value added of country 3) to country 1 for further processing. Country 1 then adds 3 USD of value added and ships a final good worth

[13] For ease of exposition only the third term of the double counting expression in equation (7) is considered.

9 USD to country 2 for final consumption (containing a total of 4 USD of value added of country 1). The gross trade balance of country 1 with country 2 is now equal to 10 USD (1 USD in intermediate goods and 9 USD in final goods). The value added surplus, however, is only equal to 4 USD. The first factor that accounts for the discrepancy is foreign value added worth 5 USD (2 USD of country 2 and 3 USD of country 3) deemed for final consumption by the direct trading partner, country 2 (labeled FVA^{tp}). The remainder, 1 USD, is domestic value added that has been counted twice – once in intermediate goods and then again in final goods exports. In summary, $\text{NX}^{12} = \text{NVAX}^{12} + \text{FVA}^{tp} + \text{REFL} = 4 + 5 + 1 = 10$.

3 Decomposing bilateral trade balances

In the following sections we focus on the most important results using the source-based approach while the differences between the source- and the sink-based approach are presented in the appendix (see Appendix A.7). None of our main findings is sensitive to the particular decomposition approach employed.[14] Results are based on the World Input Output Database (WIOD).[15]

3.1 Selected results for individual country pairs

	(1) gross trade balance	(2) value added trade balance	(3) value added in gross trade balance	(4) value added correction: (3)-(2)	(5) foreign value added (trade partner demand)	(6) domestic and foreign value added (3rd country demand)	(7) residual
in million USD							
NLD-DEU	43,969	14,374	11,131	−3,243	20,098	12,514	226
DEU-FRA	39,098	27,961	22,461	−5,500	9,946	5,599	1,092
NLD-BEL	23,990	4,230	3,917	−312	2523	17,498	52
GBR-IRL	22,504	3,472	3,927	455	−2,878	21,272	183
NLD-ITA	22,134	13,166	11,118	−2,048	6,864	4,000	152
in % of gross trade balance							
NLD-DEU	100	33	25	−7	46	28	1
DEU-FRA	100	72	57	−14	25	14	3
NLD-BEL	100	18	16	−1	11	73	0
GBR-IRL	100	15	17	2	−13	95	1
NLD-ITA	100	59	50	−9	31	18	1

Table 2: Decomposition of the five largest bilateral trade balances between EU27 countries in 2011. (Deviations from totals and 100 % are due to rounding.)

Table 2 shows the five largest gross bilateral trade balances between EU27 countries in 2011 and the components derived using the source-based decomposition framework outlined above. In general, the composition of gross trade balances is fairly heterogeneous across country pairs, although some first patterns become apparent. As documented previously, value added balances differ markedly from

[14] By definition the only difference that can arise are shifts between the contributions of the part of the value added balance that overlaps with the bilateral gross trade balance and double counting terms. In the appendix (see Appendix A.7) we show that for these two categories the quantitative differences between the two approaches are relatively minor.

[15] See www.wiod.org

balances in gross terms. For example, while the Netherlands reported a trade surplus of 44 billion USD with Germany, the value added balance between the two countries was 67% lower and stood at 14.4 billion USD. Our decomposition framework disentangles the two concepts and quantifies the factors that contribute to their differences. For most country pairs[16] the value added contained in the gross bilateral balance (column 3) is smaller than the value added balance (column 2), while the remainder appears in the gross bilateral balances of other countries (column 4). Foreign value added consumed by the respective trading partner makes up a substantial share of the difference between gross and value added concepts for most of the trade balances considered. Another important factor that emerges is demand in countries other than the two trade partners, which, for example, makes up a sizable portion of 28% of the gross bilateral trade balance between the Netherlands and Germany. The residual, which is composed of trade that is double counted and domestic value added that is reflected back via third countries for domestic consumption, does not play a significant role in any of the five bilateral trade balances under consideration.

NLD-DEU			
top 5 demand countries	million USD	% of 3rd country demand	% of gross trade balance
ROW	4,352	35	10
USA	1,438	11	3
CHN	1,237	10	3
FRA	913	7	2
AUT	896	7	2

DEU-FRA			
top 5 demand countries	million USD	% of 3rd country demand	% of gross trade balance
ROW	3,842	69	10
ESP	1,319	24	3
AUT	−665	−12	−2
ITA	640	11	2
BEL	579	10	1

Table 3: Third country demand decomposition of the NLD-DEU and DEU-FRA gross trade balance in 2011 by demand country.

Given the importance of demand in third countries in explaining the difference between value added and gross trade balances, Table 3 provides a further breakdown of which countries matter for the third country demand effect in the gross trade balance between the Netherlands and Germany as well as Germany and France. Demand in countries not further specified in WIOD explain about 10% of both gross trade balances. Both final demand in the United States and China are responsible for 3% of the trade surplus of the Netherlands vis-à-vis Germany while France and Austria each contribute another 2%. Germany's trade surplus with France is partially due to demand in Spain (3%), Italy (2%) and Belgium (1%), while demand in Austria actually leads to a small reduction (2%) of its surplus. Demand in other countries not listed in Table 3 account for the remainder of the third country effect.

A point that is worth highlighting is that bilateral value added balances do not necessarily have to be (i) smaller, but can also be (ii) larger in absolute terms than gross trade balances. Furthermore, the trade balance can also reverse its sign, which means that a country could go, for example, from (iii) a trade surplus in gross terms to a trade deficit in value added terms, and (iv) in some rarer cases this value added trade deficit could also be larger in absolute terms than its gross trade surplus. Which particular

[16]Note that the value added in the gross trade balance between the UK and Ireland is greater than the value added trade balance. While the value added in gross exports to a particular country is by definition equal to or smaller than the value added exports to that country, it is evident that the same does not hold for a bilateral trade balance, which is the difference between the bilateral exports of the two countries.

	(1) gross trade balance	(2) value added trade balance	(3) value added in gross trade balance	(4) value added correction: (3)-(2)	(5) foreign value added (trade partner demand)	(6) domestic and foreign value added (3rd country demand)	(7) residual
in million USD							
(i) NLD-DEU	43,969	14,374	11,131	−3,243	20,098	12,514	226
(ii) NLD-GBR	6,774	10,771	8,135	−2,636	10,026	−11,421	34
(iii) GBR-LUX	19,650	−83	58	141	−193	19,664	120
(iv) SWE-DNK	223	−774	−785	−11	836	168	4
in % of gross trade balance							
(i) NLD-DEU	100	33	25	−7	46	28	1
(ii) NLD-GBR	100	159	120	−39	148	−169	1
(iii) GBR-LUX	100	0	0	1	−1	100	1
(iv) SWE-DNK	100	−348	−352	−5	375	76	2

Table 4: Decomposition of greatest differences between gross and value added bilateral balances in 2011. (Deviations from totals and 100% are due to rounding.)

case applies to a given country pair depends on the sign and magnitude of the value added trade balance and the remaining components of the decomposition. Table 4 shows the decomposition of the greatest differences between gross and value added bilateral balances between EU27 countries in 2011 for the four different cases described above.[17] (i) The value added balance between the Netherlands and Germany shows the largest reduction relative to the gross trade balance without changing its sign, and the factors that contribute to the difference were already discussed above. (ii) The value added trade surplus between the Netherlands and the UK is actually larger than the surplus measured in gross terms. This is due to considerable Dutch value added flows entering the UK via third countries (column 4) and because the UK provides more intermediates to the Netherlands destined for consumption in third countries (column 6), which is not offset by the large foreign value added surplus of the Netherlands with the UK (column 5). (iii) The UK has a large trade surplus with Luxembourg in gross terms, yet a small trade deficit in value added terms. This is mainly due to demand in countries other than the UK and Luxembourg (column 6), i.e. because the UK provides a large volume of domestic and foreign intermediates to Luxembourg that are eventually consumed in other countries. (iv) Finally, the Swedish value added balance with Denmark changes its sign and is larger in absolute terms than its gross trade balance. In this case, a combination of foreign value added (column 5) and demand in third countries (column 6) explains the difference.

3.2 Decomposition of intra-EU27 trade imbalances

Although these country-specific results highlight the importance of the various factors driving the differences between gross and value added trade balances, we aim at a broader view of these patterns across EU27 countries (the results for EMU17 countries are presented in Appendix A.6). In gross terms intra-EU27 trade imbalances increased substantially between 1995 and 2008 (Figure 4a) as measured by

[17](i) value added balance *smaller* in absolute terms than and *same* sign as gross trade balance; (ii) value added balance *larger* in absolute terms than and *same* sign as gross trade balance; (iii) value added balance *smaller* in absolute terms than and *different* sign as gross trade balance; (iv) value added balance *larger* in absolute terms than and *different* sign as gross trade balance. Note that for (i) and (iii) the trade imbalance in gross terms is exacerbated, i.e. appears larger than it actually is, while for (ii) and (iv) it is attenuated, i.e. appears smaller than it actually is.

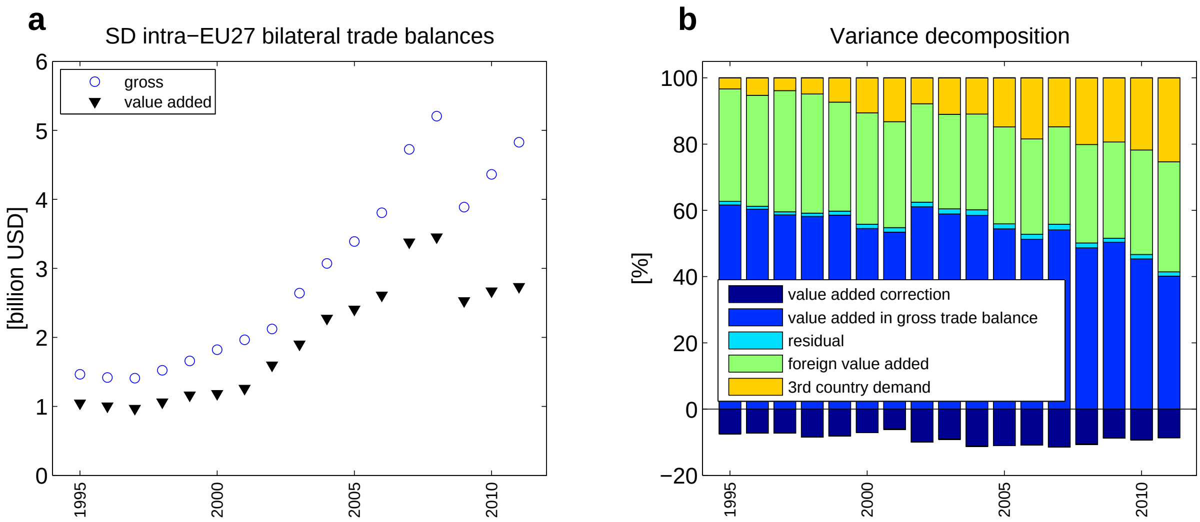

Figure 4: **Development of intra-EU27 bilateral trade balances and their components.** a) Standard deviation of intra-EU27 bilateral trade balances in gross and value added terms. b) Variance decomposition of intra-EU27 bilateral gross trade balances.

the standard deviation of the bilateral trade balances between all EU27 countries[18]. The great trade collapse led to a substantial reduction in the imbalance measure in 2009, while bilateral imbalances have rebounded since and almost reached their pre-crisis level in 2011. A similar trend is observed when considering imbalances in value added terms. However, in the past decade there has been a growing divergence between the measure in gross and value added terms with the increase in the latter being much weaker. We apply a variance decomposition of intra-EU bilateral gross trade balances in order to assess the importance of individual components of the trade balance and to account for the divergent development of gross and value added balances over time.[19] Figure 4b shows that intra-EU gross trade balances have become less representative of value added trade balances over time. In 1995 the trade balance in value added terms (*value added in gross trade balance* + *value added correction*) accounted for 69% of the gross trade balance, while in 2011 its share was down to only 49%. This trend was in particular due to a rising importance of demand in third countries, which increased from 3% in 1995 to 25% in 2011. Foreign value added directly consumed by one of the two trading partners determined the major part of the difference between gross and value added concepts, although its importance has remained relatively constant accounting for an average of 32% of the variance in the period under consideration. Similarly, the part of the value added trade balance not included in the gross trade balance (*value added correction*) has not changed appreciably since 1995 and on average was equivalent to about 9% of the gross trade balance. The overall conclusions remain unchanged when considering the sample of euro area countries,

[18] A total of $27^2 - 27 = 702$ bilateral trade balances or 351 country pairs were considered, i.e. two trade balances - one deficit and one surplus - for each pair of countries.

[19] The overall variance of gross trade balances is decomposed into

$$\mathrm{var}(\sum_i x_i) = \sum_i \mathrm{var}(x_i) + \sum_i \sum_{j \neq i} \mathrm{cov}(x_i, x_j)$$

where x_i refers to the i-th component that constitutes the trade balance. The contribution of component x_i is then computed as

$$\phi(x_i) = \frac{\mathrm{var}(x_i) + \sum_{j \neq i} \mathrm{cov}(x_i, x_j)}{var(\sum_i x_i)}$$

i.e. the contribution of the covariance term of x_i and x_j is equally split between components i and j.

i.e. EMU17 instead of EU27 countries (see Appendix A.6).

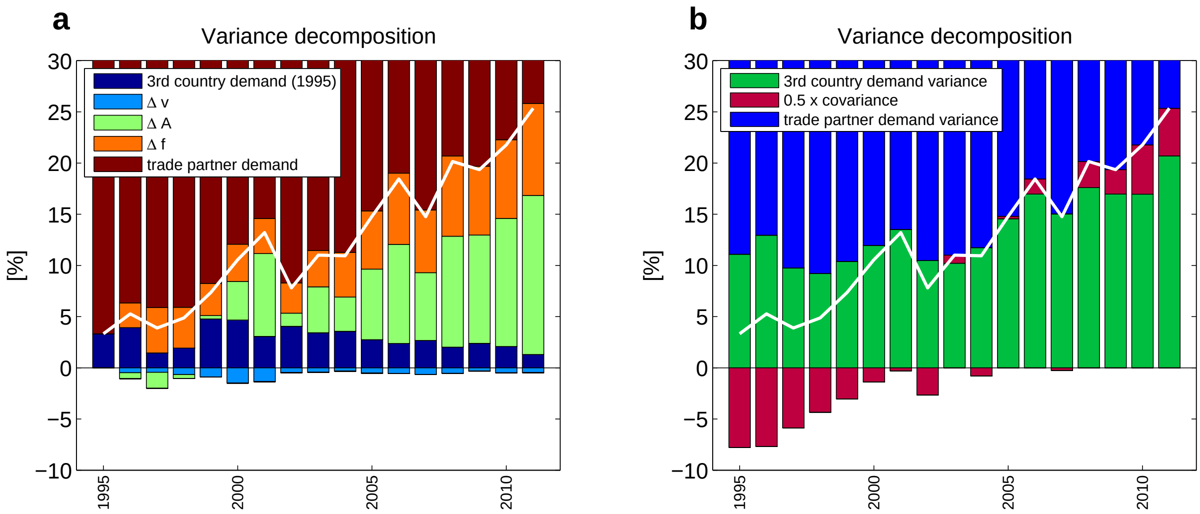

Figure 5: **Accounting for changes in the third country demand effect.** a) Structural decomposition analysis of intra-EU27 bilateral gross trade balances decomposing the shift in the third country demand effect relative to the reference year 1995 into changes of final demand ($\Delta\mathbf{f}$), international production sharing ($\Delta\mathbf{A}$) and value added content ($\Delta\mathbf{v}$). b) Variance decomposition of intra-EU27 bilateral gross trade balances into variance and covariance components of third country and trade partner demand. (White line - overall third country demand contribution.)

To shed light on the driving forces behind the increase in the third country demand effect over time two additional analyses are performed. First, we used a structural decomposition analysis (Dietzenbacher and Los, 1998; Miller and Blair, 2009)[20] of the third country demand effect in order to gain further insights into what determined its change over time. The structural decomposition analysis provides a breakdown of the shift in the third country demand effect into changes of final demand (Δf), international production sharing (Δa and Δl) and value added content (Δv). Figure 5a indicates that relative to the reference year 1995 intensified international production sharing contributed roughly two thirds (+15.5pp) to the larger prominence of the third country demand effect, while changes in final demand were responsible for most of the remainder (+9pp).[21] The sectoral value added content of exports - which, for example, is reduced when firms outsource the generation of value added to other sectors - had a negligible impact (-0.5pp).

Second, we split the gross trade balance into trade partner and third country demand shares and

[20] Structural decomposition analysis provides an additive decomposition of a matrix product y of n-terms into contributions of its individual factors x_i

$$\begin{aligned}\Delta y = &\frac{1}{2}(\Delta x_1)[(x_2^0 \ldots x_n^0) + (x_2^1 \ldots x_n^1)] \\ &+ \frac{1}{2}[x_1^0(\Delta x_2)(x_3^1 \ldots x_n^1) + x_1^1(\Delta x_2)(x_3^0 \ldots x_n^0)] + \ldots \\ &+ \frac{1}{2}[(x_1^0 \ldots x_{n-2}^0)(\Delta x_{n-1})x_n^1 + (x_1^1 \ldots x_{n-2}^1)(\Delta x_{n-1})x_n^0] \\ &+ \frac{1}{2}[(x_1^0 \ldots x_{n-1}^0) + (x_1^1 \ldots x_{n-1}^1)](\Delta x_n)\end{aligned}$$

where superscripts indicate data for different years ($t = 0, 1$).

[21] Note that the changing contribution of *3rd country demand in 1995* is due to two effects. First, while the variance of the third country demand effect in 1995 remained constant over time, the overall variance of intra-EU bilateral balances increased and hence one would expect the former's relative contribution to decline. Second, the covariance terms between *3rd country demand in 1995* and all the other components does change over time. This generates a second source of variation in *3rd country demand in 1995*, since half of the covariance terms are assigned to third country demand effect in 1995.

repeated the variance decomposition exercise from before considering both the variance and covariance terms that contribute to the overall magnitude of the third country demand effect. Figure 5b suggests that the rising importance of demand in third countries for intra-EU bilateral balances was due to both an increase in the magnitude - the variance - of the third country demand effect (from 11.1% in 1995 to 20.7% in 2011) and a stronger alignment - a positive covariance - between the third country demand effect and the remainder of the trade balance (from -7.7% in 1995 to 4.6% in 2011). The increase in the covariance term means that countries running a *classic* trade deficit (i.e. importing more from their trade partner than they are exporting to them) were also more likely to move relatively more downstream in international production chains (i.e. importing more intermediates from their trade partners than they were exporting to them in order to satisfy final demand in third countries).

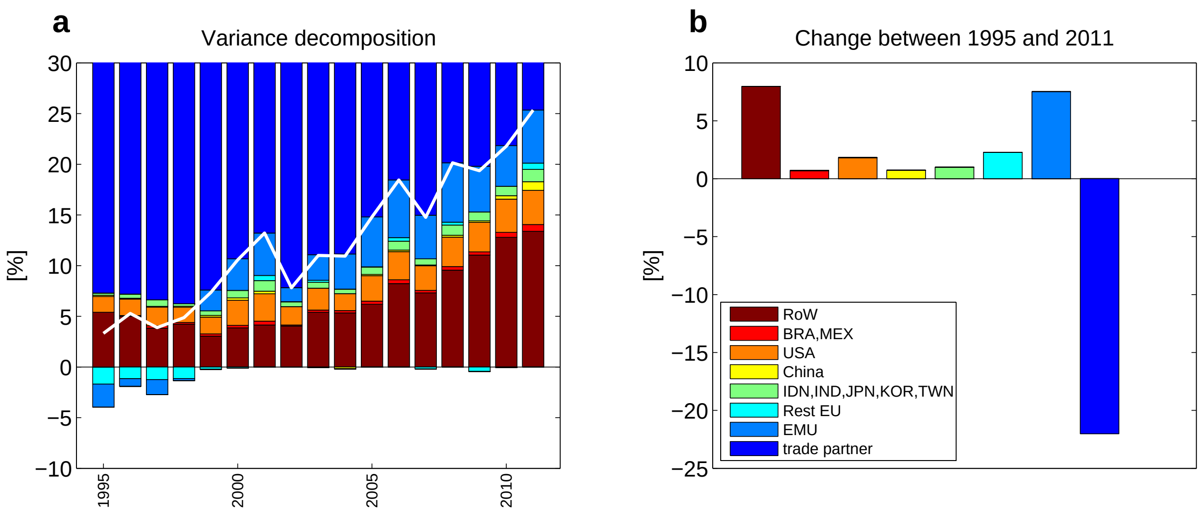

Figure 6: **Contributions of individual countries to intra-European imbalances.**
a) Variance decomposition of intra-EU27 gross trade balances between 1995 and 2011 by country of final demand. b) Difference of individual country and regional contributions between 2011 and 1995.
(White line - overall third country demand contribution.)

To further characterise the third country demand effect we consider an additional variance decomposition which identifies the contributions of individual countries to intra-European imbalances. Figure 6a provides a breakdown of intra-EU27 bilateral trade balances by demand for both domestic and foreign value added in different countries and regions. EU27 and in particular EMU countries other than the trading partners were on average responsible for 5% of bilateral imbalances. The US with 3% was the single most important country in 2011, while China accounted for 1% of the variance. Demand in Indonesia, India, Japan, Korea and Taiwan together, as well as Brazil and Mexico combined contributed about 1% each. The remainder of 13% was due to demand in the rest of the world. The large dispersion of contributions across countries shows that - while the cumulative third country demand effect was a non-negligible 25% - any single country had only a relatively minor impact on the average bilateral trade balance in the EU. Figure 6b shows that particularly demand in other EU countries, the US and the rest of the world had a larger impact on intra-EU imbalances in 2011 than in 1995, whereas the significance of direct trade partners declined sharply. Demand in other countries only marginally increased in importance. It is worth highlighting that due to the changes that occurred in the past decade about one fifth of intra-EU27 trade balances in 2011 was due to demand in non-EU countries. This suggests that intra-European trade imbalances were overstated while those with countries outside the European Union were slightly underestimated. Indeed, the value added trade balance vis-á-vis the European Union as a whole was smaller (and therefore the one with non-European countries larger) than the gross trade balance for 21 of the 27 countries. A similar result holds for the euro area for which the value added trade balance of individual countries with the EMU aggregate was smaller than the gross trade balance for 15 out of 17 countries. This finding makes it undesirable to assess intra-EU (intra-EMU) imbalances in terms of gross trade flows since a sizeable share of these are in fact trade imbalances with countries outside of the European Union (euro area) and they will by definition be unaffected by adjustment of domestic demand within the European Union (euro area).

4 Concluding remarks

The intensification of international production sharing has been a defining feature of the international economy in the recent decade. The availability of global input-output tables (Dietzenbacher et al., 2013) has sparked a growing literature on global value chains that has increasingly allowed to frame trade in value added terms (Foster-McGregor and Stehrer, 2013; Timmer et al., 2013; Johnson and Noguera, 2012). Elucidating the relation between the gross and value added concepts, and particularly what accounts for their differences, is a crucial step for interpreting and fully comprehending value added measures of trade. The previous literature has often highlighted the numerical differences between gross and value added flows, but – in the absence of a framework that relates the two concepts – has left unexplained what constitutes their disparities.[22] This paper contributes to this literature by providing a novel decomposition of *bilateral* trade balances that accounts for the differences between gross and value added concepts. A caveat which turns out to be inherent in the subject matter is that the question regarding the relation between value added and gross trade flows is theoretically ill-defined, and that the answer necessarily has to be a matter of what we define value added flows to be. A first conceptual contribution is that we are, to the best of our knowledge, the first to describe this problem and to discuss the two most parsimonious solutions to it, which determine the intersection between value added and gross trade flows at the bilateral level.

A second conceptual contribution is that we identify a range of categorically different components of value added that constitute the difference between bilateral value added and gross trade balances, which are independent of the methodological choice above. We provide a novel distinction between (foreign) value added that is due to demand of the direct trading partner, and (domestic and foreign) value added that is due to demand in third countries. While both are ultimately the results of trade in intermediates, the difference between these two categories proves to be important from a policy perspective as demand in third countries is by definition unaffected by domestic demand in the two trading partners, which is often one of the main targets for current account adjustments. In an application of our decomposition framework to intra-European trade imbalances we show that gross trade balances have become increas-

[22]With the exception of (Koopman et al., 2014), who decompose gross exports into value added exports and double counted terms at the *aggregate* level.

ingly less representative of value added balances. We demonstrate that the major determinant behind this increasing discrepancy is the rising importance of the part of the trade balance which is due to demand in countries other than the two direct trading partners. We use a structural decomposition analysis that allows us to quantify the relative contributions made by changes in international production sharing and final demand respectively, and we find that changes in the European division of labour account for roughly two thirds of the increase. Focusing on the countries behind the imbalances, we find that a sizeable share of about one fifth of the variance of intra-European imbalances in 2011 was due to demand in countries outside the European Union. This means that the problem of intra-European imbalances has been overstated in the past, while trade imbalances with countries outside of Europe were larger than inferred from gross trade balances. A similar result holds for the euro area. Particularly in a currency union, it is important to establish with whom trade imbalances exist since the burden of adjustment may differ between trade deficits vis-á-vis member countries and third parties (di Mauro and Pappada, 2014). Even though their limitations are widely acknowledged gross bilateral trade balances still figure widely in the economic literature and policy debates (Davis and Weinstein, 2002; Bahmani-Oskooee and Brooks, 1999). Despite some general limitations of input-output models, our paper provides a strong case for considering value added instead of gross bilateral trade balances since a sizable portion of gross bilateral trade balances is no longer under the influence of the direct trading partners themselves.

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A Appendix

A.1 Leontief inverse

In the main text it was argued that a coefficient of the Leontief inverse describes all possible ways that value added from a country r travels embedded in intermediate goods through international production networks to country s. In this section, we illustrate this statement mathematically. Denote the global input-output matrix by $\mathbf{A}$ and the matrix with the domestic coefficients by $\tilde{\mathbf{A}}$. For the three-country case:

$$\mathbf{A} = \begin{pmatrix} a^{11} & a^{12} & a^{13} \\ a^{21} & a^{22} & a^{23} \\ a^{31} & a^{32} & a^{33} \end{pmatrix} \qquad \tilde{\mathbf{A}} = \begin{pmatrix} a^{11} & 0 & 0 \\ 0 & a^{22} & 0 \\ 0 & 0 & a^{33} \end{pmatrix}$$

The Leontief inverses of these two matrices can be written as a geometric series, i.e.

$$\mathbf{L} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \mathbf{A}^3 + \cdots = (\mathbf{I} - \mathbf{A})^{-1}$$

and

$$\tilde{\mathbf{L}} = \mathbf{I} + \tilde{\mathbf{A}} + \tilde{\mathbf{A}}^2 + \tilde{\mathbf{A}}^3 + \cdots = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}$$

where

$$\mathbf{A}^2 = \begin{pmatrix} a^{11} & a^{12} & a^{13} \\ a^{21} & a^{22} & a^{23} \\ a^{31} & a^{32} & a^{33} \end{pmatrix} \begin{pmatrix} a^{11} & a^{12} & a^{13} \\ a^{21} & a^{22} & a^{23} \\ a^{31} & a^{32} & a^{33} \end{pmatrix} = \begin{pmatrix} \sum_i a^{1i}a^{i1} & \sum_i a^{1i}a^{i2} & \sum_i a^{1i}a^{i3} \\ \sum_i a^{2i}a^{i1} & \sum_i a^{2i}a^{i2} & \sum_i a^{2i}a^{i3} \\ \sum_i a^{3i}a^{i1} & \sum_i a^{3i}a^{i2} & \sum_i a^{3i}a^{i3} \end{pmatrix}$$

and

$$\begin{aligned} \mathbf{A}^3 &= \begin{pmatrix} \sum_i a^{1i}a^{i1} & \sum_i a^{1i}a^{i2} & \sum_i a^{1i}a^{i3} \\ \sum_i a^{2i}a^{i1} & \sum_i a^{2i}a^{i2} & \sum_i a^{2i}a^{i3} \\ \sum_i a^{3i}a^{i1} & \sum_i a^{3i}a^{i2} & \sum_i a^{3i}a^{i3} \end{pmatrix} \begin{pmatrix} a^{11} & a^{12} & a^{13} \\ a^{21} & a^{22} & a^{23} \\ a^{31} & a^{32} & a^{33} \end{pmatrix} \\ &= \begin{pmatrix} \sum_j(\sum_i a^{1i}a^{ij})a^{j1} & \sum_j(\sum_i a^{1i}a^{ij})a^{j2} & \sum_j(\sum_i a^{1i}a^{ij})a^{j3} \\ \sum_j(\sum_i a^{2i}a^{ij})a^{j1} & \sum_j(\sum_i a^{2i}a^{ij})a^{j2} & \sum_j(\sum_i a^{2i}a^{ij})a^{j3} \\ \sum_j(\sum_i a^{3i}a^{ij})a^{j1} & \sum_j(\sum_i a^{3i}a^{ij})a^{j2} & \sum_j(\sum_i a^{3i}a^{ij})a^{j3} \end{pmatrix} \end{aligned}$$

A typical element of the matrix $\mathbf{A}^3$ is

$$a^{(3)kl} = \sum_{i_3}(\sum_{i_2} a^{ki_2}a^{i_2i_3})a^{i_3l}$$

More generally for the matrix $\mathbf{A}^n$ a typical element is

$$a^{(n)kl} = \sum_{i_n}\Big(\sum_{i_{n-1}}\cdots\sum_{i_3}\big(\sum_{i_2} a^{ki_2}a^{i_2i_3}\big)a^{i_3i_4}\dots a^{i_{n-1}i_n}\Big)a^{i_nl}$$

Therefore, a typical element of the Leontief inverse can be written using the expressions derived from the geometric series above

$$l^{(n)kl} = \delta^{kl} + a^{kl} + \sum_{m=2}^{n} a^{(m)kl}$$

with $\delta^{kl} = 1$ for $k = l$ and $\delta^{kl} = 0$ for $k \neq l$ (Kronecker delta). Thus, the entries of the Leontief inverse describe *all* possible ways to go from country k to country l. Note that naturally this also holds true for the diagonal elements in the matrix with the domestic coefficients $\tilde{\mathbf{A}}$. For calculating the part of the value added which does not leave a country at all one therefore has to take account only of the diagonal elements of the $\tilde{\mathbf{A}}$ matrix

$$\tilde{l}^{kk} = \sum_{m=0}^{\infty} (a^{kk})^m = (1 - a^{kk})^{-1}$$

Thus in matrix notation

$$\tilde{\mathbf{L}} = (\mathbf{I} - \tilde{\mathbf{A}})^{-1}$$

In the text the Leontief is therefore split into

$$\mathbf{L} = \tilde{\mathbf{L}} + (\mathbf{L} - \tilde{\mathbf{L}})$$

or

$$\mathbf{L} = \begin{pmatrix} l^{11} & l^{12} & l^{13} \\ l^{21} & l^{22} & l^{23} \\ l^{31} & l^{32} & l^{33} \end{pmatrix} = \begin{pmatrix} \tilde{l}^{11} & 0 & 0 \\ 0 & \tilde{l}^{22} & 0 \\ 0 & 0 & \tilde{l}^{33} \end{pmatrix} + \begin{pmatrix} l^{11} - \tilde{l}^{11} & l^{12} & l^{13} \\ l^{21} & l^{22} - \tilde{l}^{22} & l^{23} \\ l^{31} & l^{32} & l^{33} - \tilde{l}^{33} \end{pmatrix}$$

It is important to note that $(l^{kk} - \tilde{l}^{kk}) \geq 0$ (or a non-negative matrix in the multi-sectoral case) which holds by definition (see also Stone (1985)).

A.2 Decomposition of bilateral gross trade balances (sink-based approach)

As mentioned in the text, the corresponding decomposition for the sink-based approach is given by

$$
\begin{aligned}
NX^{12} &= e^{12} - e^{21} \\
&= \underbrace{v^1 l^{11} f^{12} + v^1 l^{11} a^{12}(1-a^{22})^{-1} f^{22}}_{\text{NVAX*}^{12}\ (\text{e}^{12})} - \underbrace{v^2 l^{22} f^{21} - v^2 l^{22} a^{21}(1-a^{11})^{-1} f^{11}}_{\text{NVAX*}^{12}\ (\text{e}^{21})} \\
&+ \underbrace{v^1 l^{11} a^{12}(l^{22} - (1-a^{22})^{-1}) f^{22} + v^1 l^{11} a^{12} \sum_{i\neq 2}^{3} l^{2i} f^{i2}}_{\text{DBC}\ (\text{e}^{12})} + \underbrace{v^2 l^{21} a^{12} \sum_{i}^{3} l^{2i} f^{i1}}_{\text{DBC}\ (\text{e}^{12})} \\
&- \underbrace{v^2 l^{22} a^{21}(l^{11} - (1-a^{11})^{-1}) f^{11} + v^2 l^{22} a^{21} \sum_{i\neq 1}^{3} l^{1i} f^{i1}}_{\text{DBC}\ (\text{e}^{21})} - \underbrace{v^1 l^{12} a^{21} \sum_{i}^{3} l^{1i} f^{i2}}_{\text{DBC}\ (\text{e}^{21})} \\
&+ \underbrace{v^2 l^{21} f^{12} + v^1 l^{11} a^{12} \sum_{i}^{3} l^{2i} f^{i1} + v^2 l^{21} a^{12} \sum_{i}^{3} l^{2i} f^{i2}}_{\text{REFL}\ (\text{e}^{12})} - \underbrace{v^1 l^{12} f^{21} - v^1 l^{12} a^{21} \sum_{i}^{3} l^{1i} f^{i1} - v^2 l^{22} a^{21} \sum_{i}^{3} l^{1i} f^{i2}}_{\text{REFL}\ (\text{e}^{21})} \\
&+ \underbrace{v^3 l^{31} f^{12} + v^3 l^{31} a^{12} \sum_{i}^{3} l^{2i}(f^{i1} + f^{i2})}_{\text{FVA}^{tp}\ (\text{e}^{12})} - \underbrace{v^3 l^{32} f^{21} - v^3 l^{32} a^{21} \sum_{i}^{3} l^{1i}(f^{i1} + f^{i2})}_{\text{FVA}^{tp}\ (\text{e}^{21})} \\
&+ \underbrace{v^1 l^{11} a^{12} \sum_{i}^{3} l^{2i} f^{i3} + v^2 l^{21} a^{12} \sum_{i}^{3} l^{2i} f^{i3}}_{\text{DVA}^{3rd}\ (\text{e}^{12})} - \underbrace{v^2 l^{22} a^{21} \sum_{i}^{3} l^{1i} f^{i3} - v^1 l^{12} a^{21} \sum_{i}^{3} l^{1i} f^{i3}}_{\text{DVA}^{3rd}\ (\text{e}^{21})} \\
&+ \underbrace{v^3 l^{31} a^{12} \sum_{i}^{3} l^{2i} f^{i3}}_{\text{FVA}^{3rd}\ (\text{e}^{12})} - \underbrace{v^3 l^{32} a^{21} \sum_{i}^{3} l^{1i} f^{i3}}_{\text{FVA}^{3rd}\ (\text{e}^{21})}
\end{aligned}
$$

where individual terms were labeled as in the source-based approach. Note that NVAX*^{12} and DBC are the only terms that are different from those in the source-based approach.

A.3 Generalisation to N-country case

The generalisation of the source- and sink-based approach to the N-country case is achieved by summing the corresponding terms over all trade partners.

A.3.1 Source-based approach

$$
\begin{aligned}
NX^{ij} &= e^{ij} - e^{ji} \\
&= \underbrace{v^i(I-a^{ii})^{-1}f^{ij} + v^i(I-a^{ii})^{-1}a^{ij}\sum_k^N l^{jk}f^{kj}}_{\text{NVAX*}^{ij}\ (\text{e}^{ij})} - \underbrace{v^j(I-a^{jj})^{-1}f^{ji} - v^j(I-a^{jj})^{-1}a^{ji}\sum_k^N l^{ik}f^{ki}}_{\text{NVAX*}^{ij}\ (\text{e}^{ji})} \\
&+ \underbrace{v^i(l^{ii}-(I-a^{ii})^{-1})f^{ij} + v^i(l^{ii}-(I-a^{ii})^{-1})a^{ij}\sum_k^N l^{jk}f^{kj}}_{\text{DBC}\ (\text{e}^{ij})} + \underbrace{v^j l^{ji}a^{ij}\sum_k^N l^{jk}f^{ki}}_{\text{DBC}\ (\text{e}^{ij})} \\
&- \underbrace{v^j(l^{jj}-(I-a^{jj})^{-1})f^{ji} - v^j(l^{jj}-(I-a^{jj})^{-1})a^{ji}\sum_k^N l^{ik}f^{ki}}_{\text{DBC}\ (\text{e}^{ji})} - \underbrace{v^i l^{ij}a^{ji}\sum_k^N l^{ik}f^{kj}}_{\text{DBC}\ (\text{e}^{ji})} \\
&+ \underbrace{v^j l^{ji}f^{ij} + v^i l^{ii}a^{ij}\sum_k^N l^{jk}f^{ki} + v^j l^{ji}a^{ij}\sum_k^N l^{jk}f^{kj}}_{\text{REFL}\ (\text{e}^{ij})} - \underbrace{v^i l^{ij}f^{ji} - v^i l^{ij}a^{ji}\sum_k^N l^{ik}f^{ki} - v^j l^{jj}a^{ji}\sum_k^N l^{ik}f^{jk}}_{\text{REFL}\ (\text{e}^{ji})} \\
&+ \underbrace{\sum_{k\neq i,j}^N \left[v^k l^{ki}f^{ij} + v^k l^{ki}a^{ij}\sum_m^N l^{jm}(f^{mi}+f^{mj})\right]}_{\text{FVA}^{tp}\ (\text{e}^{ij})} - \underbrace{\sum_{k\neq i,j}^N \left[v^k l^{kj}f^{ji} - v^k l^{kj}a^{ji}\sum_m^N l^{im}(f^{mi}+f^{mj})\right]}_{\text{FVA}^{tp}\ (\text{e}^{ji})} \\
&+ \underbrace{\left[v^i l^{ii}a^{ij} + v^j l^{ji}a^{ij}\right]\sum_k^N l^{jk}\sum_{m\neq i,j}^N f^{km}}_{\text{DVA}^{3rd}\ (\text{e}^{ij})} - \underbrace{\left[v^j l^{jj}a^{ji} - v^i l^{ij}a^{ji}\right]\sum_k^N l^{ik}\sum_{m\neq i,j}^N f^{km}}_{\text{DVA}^{3rd}\ (\text{e}^{ji})} \\
&+ \underbrace{\sum_{k\neq i,j}^N v^k l^{ki}a^{ij}\sum_m^N l^{jm}\sum_{n\neq i,j}^N f^{mn}}_{\text{FVA}^{3rd}\ (\text{e}^{ij})} - \underbrace{\sum_{k\neq i,j}^N v^k l^{kj}a^{ji}\sum_m^N l^{im}\sum_{n\neq i,j}^N f^{mn}}_{\text{FVA}^{3rd}\ (\text{e}^{ji})}
\end{aligned}
$$

A.3.2 Sink-based approach

$$
\begin{aligned}
NX^{ij} &= e^{ij} - e^{ji} \\
&= \underbrace{v^i l^{ii} f^{ij} + v^i l^{ii} a^{ij} (1-a^{jj})^{-1} f^{jj}}_{\text{NVAX*}^{ij}\ (\text{e}^{ij})} - \underbrace{v^j l^{jj} f^{ji} - v^j l^{jj} a^{ji} (1-a^{ii})^{-1} f^{ii}}_{\text{NVAX*}^{ij}\ (\text{e}^{ji})} \\
&+ \underbrace{v^i l^{ii} a^{ij} (l^{jj} - (1-a^{jj})^{-1}) f^{jj} + v^i l^{ii} a^{ij} \sum_{k \neq j}^{N} l^{jk} f^{kj}}_{\text{DBC}\ (\text{e}^{ij})} + \underbrace{v^j l^{ji} a^{ij} \sum_{k}^{N} l^{jk} f^{ki}}_{\text{DBC}\ (\text{e}^{ij})} \\
&- \underbrace{v^j l^{jj} a^{ji} (l^{ii} - (1-a^{ii})^{-1}) f^{ii} - v^j l^{jj} a^{ji} \sum_{k \neq i}^{N} l^{ik} f^{ki}}_{\text{DBC}\ (\text{e}^{ji})} - \underbrace{v^i l^{ij} a^{ji} \sum_{k}^{N} l^{ik} f^{kj}}_{\text{DBC}\ (\text{e}^{ji})} \\
&+ \underbrace{v^j l^{ji} f^{ij} + v^i l^{ii} a^{ij} \sum_{k}^{N} l^{jk} f^{ki} + v^j l^{ji} a^{ij} \sum_{k}^{N} l^{jk} f^{kj}}_{\text{REFL}\ (\text{e}^{ij})} - \underbrace{v^i l^{ij} f^{ji} - v^i l^{ij} a^{ji} \sum_{k}^{N} l^{ik} f^{ki} - v^j l^{jj} a^{ji} \sum_{k}^{N} l^{ik} f^{jk}}_{\text{REFL}\ (\text{e}^{ji})} \\
&+ \underbrace{\sum_{k \neq i,j}^{N} \left[v^k l^{ki} f^{ij} + v^k l^{ki} a^{ij} \sum_{m}^{N} l^{jm} (f^{mi} + f^{mj}) \right]}_{\text{FVA}^{tp}\ (\text{e}^{ij})} - \underbrace{\sum_{k \neq i,j}^{N} \left[v^k l^{kj} f^{ji} - v^k l^{kj} a^{ji} \sum_{m}^{N} l^{im} (f^{mi} + f^{mj}) \right]}_{\text{FVA}^{tp}\ (\text{e}^{ji})} \\
&+ \underbrace{\left[v^i l^{ii} a^{ij} + v^j l^{ji} a^{ij} \right] \sum_{k}^{N} l^{jk} \sum_{m \neq i,j}^{N} f^{km}}_{\text{DVA}^{3rd}\ (\text{e}^{ij})} - \underbrace{\left[v^j l^{jj} a^{ji} - v^i l^{ij} a^{ji} \right] \sum_{k}^{N} l^{ik} \sum_{m \neq i,j}^{N} f^{km}}_{\text{DVA}^{3rd}\ (\text{e}^{ji})} \\
&+ \underbrace{\sum_{k \neq i,j}^{N} v^k l^{ki} a^{ij} \sum_{m}^{N} l^{jm} \sum_{n \neq i,j}^{N} f^{mn}}_{\text{FVA}^{3rd}\ (\text{e}^{ij})} - \underbrace{\sum_{k \neq i,j}^{N} v^k l^{kj} a^{ji} \sum_{m}^{N} l^{im} \sum_{n \neq i,j}^{N} f^{mn}}_{\text{FVA}^{3rd}\ (\text{e}^{ji})}
\end{aligned}
$$

A.4 Matrix inverse

If $\mathbf{A}$ is an n-by-n invertible matrix, there exists an n-by-n matrix $\mathbf{B}$ such that

$$\mathbf{AB} = \mathbf{BA} = \mathbf{I_n} \tag{9}$$

and the matrix $\mathbf{B}$ is called the matrix inverse of $\mathbf{A}$, denoted by $\mathbf{A^{-1}}$. This equation is sometimes referred to as the "property of inverse matrices". From the definition of the matrix inverse, several identities can be derived that will be useful for the proofs in the next section. Note that, technically speaking, the source- and sink based approach differ with respect to the corresponding identities which are applied.

A.4.1 Useful identities (source-based approach)

$$(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A})^{\mathbf{-1}} = (\mathbf{I} - \mathbf{A})\mathbf{L} = \mathbf{I} = \begin{bmatrix} I - a^{11} & -a^{12} & -a^{13} \\ -a^{21} & I - a^{22} & -a^{23} \\ -a^{31} & -a^{32} & I - a^{33} \end{bmatrix} \begin{bmatrix} l^{11} & l^{12} & l^{13} \\ l^{21} & l^{22} & l^{23} \\ l^{31} & l^{32} & l^{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

For the source-based approach three of the resulting nine identities will be relevant:

$$(1 - a^{11})l^{11} - a^{12}l^{21} - a^{13}l^{31} = 1$$

$$(1 - a^{11})l^{12} - a^{12}l^{22} - a^{13}l^{32} = 0$$

$$(1 - a^{11})l^{13} - a^{12}l^{23} - a^{13}l^{33} = 0$$

or solving for the entry of the Leontief inverse related to output of country 1:

$$l^{11} = (1 - a^{11})^{-1}(1 + a^{12}l^{21} + a^{13}l^{31}) \tag{11}$$

$$l^{12} = (1 - a^{11})^{-1}(a^{12}l^{22} + a^{13}l^{32}) \tag{12}$$

$$l^{13} = (1 - a^{11})^{-1}(a^{12}l^{23} + a^{13}l^{33}) \tag{13}$$

A.4.2 Useful identities (sink-based approach)

$$(\mathbf{I} - \mathbf{A})^{\mathbf{-1}}(\mathbf{I} - \mathbf{A}) = \mathbf{L}(\mathbf{I} - \mathbf{A}) = \mathbf{I} = \begin{bmatrix} l^{11} & l^{12} & l^{13} \\ l^{21} & l^{22} & l^{23} \\ l^{31} & l^{32} & l^{33} \end{bmatrix} \begin{bmatrix} I - a^{11} & -a^{12} & -a^{13} \\ -a^{21} & I - a^{22} & -a^{23} \\ -a^{31} & -a^{32} & I - a^{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For the sink-based approach one of the resulting nine identities will be relevant:

$$-l^{11}a^{12} + l^{12}(I - a^{22}) - l^{13}a^{32} = 0$$

or solving for l^{12}:

$$l^{12} = (l^{11}a^{12} + l^{13}a^{32})(1 - a^{22})^{-1} \tag{14}$$

A.5 Proofs

A.5.1 Source-based approach

In this section, we show that the sum of all VAX^{*12} terms in e^{12} and e^{13} equals the value added exports from country 1 to 2, VAX^{12}. As discussed in main text, in order to arrive at the VAX^{*12} terms we substitute $(1-a^{11})^{-1}$ for l^{11} in the expression for $v^1..f^{\cdot 2}$ in the bilateral gross exports to country 2

$$VAX^{*12}(e^{12}) = v^1(1-a^{11})^{-1}f^{12} + v^1(1-a^{11})^{-1}a^{12}\sum_i^3 l^{2i}f^{i2}$$

and country 3

$$VAX^{*12}(e^{13}) = v^1(1-a^{11})^{-1}a^{13}\sum_i^3 l^{3i}f^{i2}$$

In the following, we need to show that

$$VAX^{12} = VAX^{*12}(e^{12}) + VAX^{*12}(e^{13})$$

In order to do so, l^{11}, l^{12} and l^{13} is substituted by (11)-(13) in the expression for VAX^{12}:

$$\begin{aligned}
VAX^{12} &= v^1l^{11}f^{12} + v^1l^{12}f^{22} + v^1l^{13}f^{32}\\
&= v^1(1-a^{11})^{-1}(1+a^{12}l^{21}+a^{13}l^{31})f^{12}\\
&+ v^1(1-a^{11})^{-1}(a^{12}l^{22}+a^{13}l^{32})f^{22}\\
&+ v^1(1-a^{11})^{-1}(a^{12}l^{23}+a^{13}l^{33})f^{32}\\
&= v^1(1-a^{11})^{-1}f^{12} + v^1(1-a^{11})^{-1}a^{12}\sum_i^3 l^{2i}f^{i2}\\
&+ v^1(1-a^{11})^{-1}a^{13}\sum_i^3 l^{3i}f^{i2}\\
&= VAX^{*12}(e^{12}) + VAX^{*12}(e^{13})
\end{aligned}$$

A.5.2 Sink-based approach

Here, we show that the sum of all VAX^{*12} terms in e^{12} and e^{32} equals the value added exports from country 1 to 2, VAX^{12} and hence the value added imports of country 2 from 1, VAM^{21}. As discussed in the description of the sink-based approach, to arrive at the VAX^{*12} terms we substitute $(1-a^{22})^{-1}$ for l^{22} in the expression for $v^1..f^{\cdot 2}$ in the bilateral gross imports of country 2 from country 1

$$VAX*^{12}(e^{12}) = v^1l^{11}f^{12} + v^1l^{11}a^{12}(1-a^{22})^{-1}f^{22}$$

and from country 3

$$VAX*^{12}(e^{32}) = v^1l^{13}a^{32}(1-a^{22})^{-1}f^{22} + v^1l^{13}f^{32}$$

In the following, we need to show that

$$VAX^{12} = VAX *^{12} (e^{12}) + VAX *^{12} (e^{32})$$

In order to do so, in the expression for VAX^{12} l^{12} is substituted by (14):

$$\begin{aligned}
VAX^{12} &= v^1 l^{11} f^{12} + v^1 l^{12} f^{22} + v^1 l^{13} f^{32} \\
&= v^1 l^{11} f^{12} \\
&+ v^1 (l^{11} a^{12} + l^{13} a^{32})(1 - a^{22})^{-1} f^{22} \\
&+ v^1 l^{13} f^{32} \\
&= v^1 l^{11} f^{12} + v^1 l^{11} a^{12} (1 - a^{22})^{-1} f^{22} \\
&+ v^1 l^{13} a^{32} (1 - a^{22})^{-1} f^{22} + v^1 l^{13} f^{32} \\
&= VAX^{*12}(e^{12}) + VAX^{*12}(e^{32})
\end{aligned}$$

A.6 Euro area results

All decomposition analyses presented in the main text were also performed for the sample of EMU17 countries. The results are qualitatively very similar for the euro area subset of EU27 countries and the main conclusions remain unchanged.

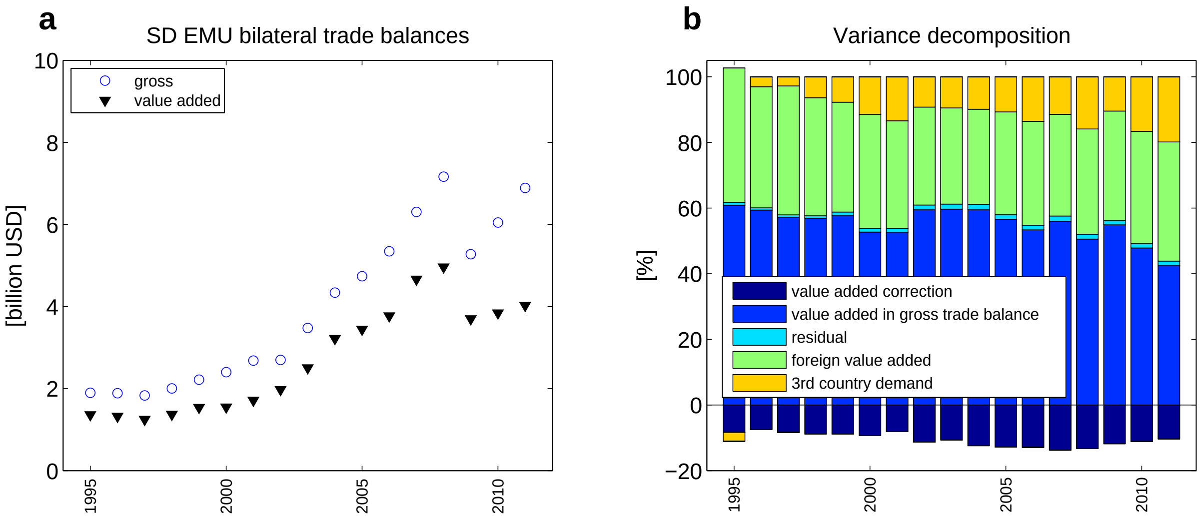

Figure 7: **Development of intra-EMU17 bilateral trade balances and their components.** a) Standard deviation of intra-EMU17 bilateral trade balances in gross and value added terms. b) Variance decomposition of intra-EMU17 bilateral gross trade balances.

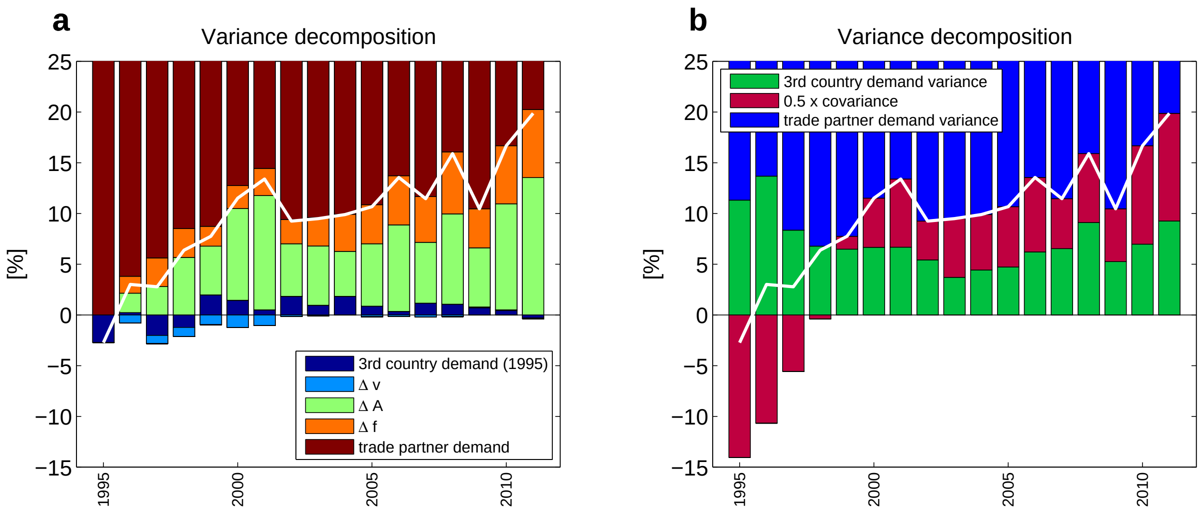

Figure 8: **Accounting for changes in the third country demand effect.** a) Structural decomposition analysis of intra-EMU17 bilateral gross trade balances decomposing the shift in the third country demand effect relative to the reference year 1995 into changes of final demand ($\Delta\mathbf{f}$), international production sharing ($\Delta\mathbf{A}$) and value added content ($\Delta\mathbf{v}$). b) Variance decomposition of intra-EMU17 bilateral gross trade balances into variance and covariance components of third country and trade partner demand. (White line - overall third country demand contribution.)

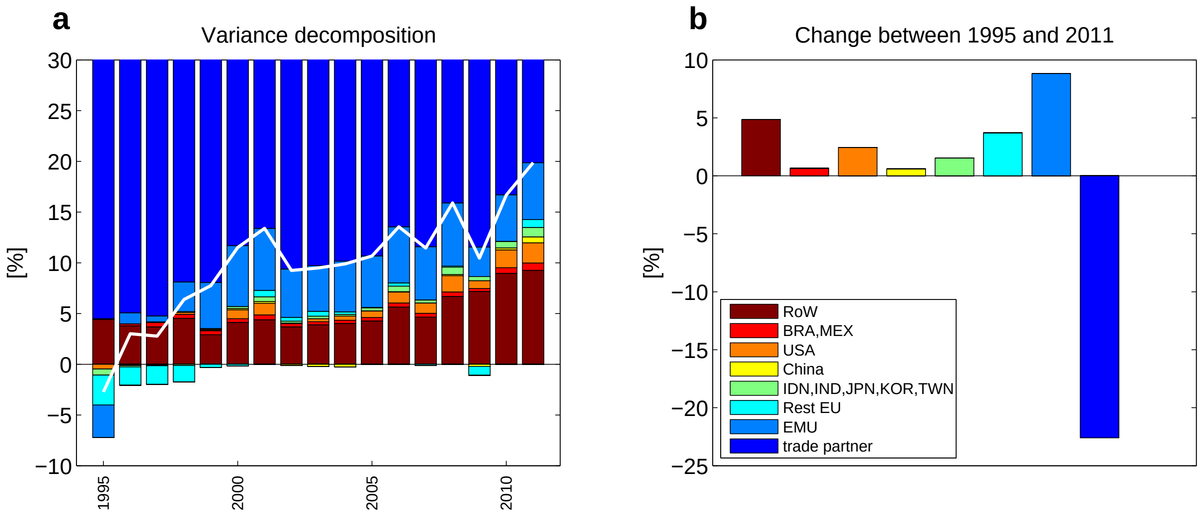

Figure 9: **Contributions of individual countries to intra-EMU imbalances.**
a) Variance decomposition of intra-EMU17 gross trade balances between 1995 and 2011 by country of final demand. b) Difference of individual country and regional contributions between 2011 and 1995.
(White line - overall third country demand contribution.)

A.7 Comparison sink- and source-based approach

This section presents a short comparison of individual country and variance decomposition results for the sink- and source-based approach. By definition, the two approaches only differ with regards to the contributions of double counting terms and the part of the value added balance that coincides with the bilateral gross trade balance. Hence, the results pertaining to foreign value added and demand in third countries are unaffected by this methodological choice. Table 5 shows how domestic value added of the two trade partners which is absorbed by the respective trade partner (and hence in theory qualifies to be part of the value added balance) is split into being part of the value added balance and a double counting term. In general the double counting term differs appreciably in the two decompositions. However, in both cases the double counting term is quantitatively relatively small in comparison with the value added in the gross trade balance. Table 6 shows the results of the variance decomposition of intra-EU27 trade balances for both source- and sink-based decompositions. As before, the contribution of the double counting terms is discernibly different in the two approaches, but the absolute contribution is very small in both cases. In the source-based approach, the relatively small contributions of the double counting term reflects the fact that the share of domestic value added that is re-imported for further processing in the country of production and eventually re-exported for absorption in final demand by the direct trade partner is comparatively minor. Similarly, for the sink-based approach the double counting term is marginal since the value added of the direct trade partner that is re-exported and later re-imported for absorption in final demand is relatively small in comparison with the direct trade partner's value added that is directly consumed. In conclusion, although the two approaches are conceptually very different from each other, the quantitative differences are in general relatively minor and do not affect the conclusions drawn in the main text.

		source-based approach		sink-based approach	
	domestic value added absorbed by respective trade partner	value added in gross trade balance	double counting	value added in gross trade balance	double counting
in million USD					
NLD-DEU	11,240	11,131	109	10,809	431
DEU-FRA	23,499	22,461	1,038	23,678	−179
NLD-BEL	4,004	3,917	87	3,848	156
GBR-IRL	3,962	3,927	35	3,963	−1
NLD-ITA	11,232	11,118	114	11,195	37

Table 5: **Individual country results according to source- and sink-based decompositions.** Domestic value added (of one of the two trading partners) absorbed by the respective trade partner is subdivided into the part of the value added trade balance that coincides with the bilateral gross trade balance and a double counting term according to the source- and the sink-based approach.
(Deviations from totals are due to rounding.)

	domestic value added absorbed by respective trade partner	source-based approach		sink-based approach	
		value added in gross trade balance	double counting	value added in gross trade balance	double counting
in %					
1995	62.53	61.62	0.92	62.63	−0.10
1996	61.11	60.31	0.79	61.22	−0.11
1997	59.43	58.63	0.80	59.49	−0.06
1998	59.01	58.15	0.86	59.11	−0.10
1999	59.58	58.57	1.01	59.77	−0.19
2000	55.55	54.50	1.04	55.67	−0.12
2001	54.43	53.44	0.99	54.49	−0.07
2002	62.29	61.08	1.21	62.49	−0.20
2003	60.21	58.93	1.28	60.41	−0.20
2004	59.99	58.49	1.50	60.28	−0.29
2005	55.73	54.42	1.32	55.91	−0.17
2006	52.60	51.26	1.34	52.75	−0.16
2007	55.65	54.09	1.56	55.94	−0.29
2008	50.00	48.69	1.31	50.14	−0.14
2009	51.39	50.36	1.03	51.51	−0.12
2010	46.52	45.35	1.17	46.59	−0.07
2011	41.27	40.12	1.15	41.23	0.04

Table 6: **Variance decomposition of intra-EU27 imbalances according to source- and sink-based decompositions.**
(Deviations from totals are due to rounding.)