Constructing Multi-country Rational Expectations Models*

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Abstract

This paper considers some of the technical issues involved in using the GVAR approach to construct a multi-country rational expectations, RE, model and illustrates them with a new Keynesian model for 33 countries estimated with quarterly data over the period 1980-2011. The issues considered are: the measurement of steady states; the determination of exchange rates and the specification of the short-run country-specific models; the identification and estimation of the model subject to the theoretical constraints required for a determinate rational expectations solution; the treatment of the nominal anchor and the solution of a large RE model; the structure and estimation of the covariance matrix; and the simulation of shocks. The model used as an illustration shows that global demand and supply shocks are the most important drivers of output, inflation and interest rates in the long run. By contrast, monetary or exchange rate shocks have only a short-run impact in the evolution of the world economy. The paper also shows the importance of international connections, directly as well as indirectly through spillover effects. Overall, ignoring global inter-connections as country-specific models do, could give rise to misleading conclusions.

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1 Introduction

The multi-country, cointegrating global vector autoregression, GVAR, has proved an effective tool for a range of purposes, including forecasting, counterfactual analysis and investigating the transmission of shocks across economies or markets. The Handbook edited by di Mauro and Pesaran (2013) provides a review of many applications of the GVAR approach. Smith and Galesi (2013) provide a toolbox for constructing GVARs. The GVAR has the advantage that country-specific models can be easily estimated with the current values of foreign variables being treated as weakly exogenous, an assumption that is found to be generally acceptable after testing. The structure allows for both between country and within country cointegration, dealing with the issue originally raised by Banerjee et al. (2004). The country specific models can be combined to form a system which treats all variables as endogenous. Even though the presence of the current foreign variables give the GVAR a simultaneous equations type structure, it is subject to the critique that its shocks cannot be given simple economic interpretations as demand, supply or monetary policy shocks. Dynamic stochastic general equilibrium, DSGE, models do generate shocks that can be given such a simple interpretation, but extending DSGE models to a multi-country framework is not straightforward. Existing open-economy DSGE models tend to consider either two countries of comparable size, such as the euro area and the US, or small open economy models where the rest of the world is treated as exogenous. But it is the interactions between many countries that is often crucial for questions about the global economy. Carabenciov et al. (2008, p.6) who consider developing multi-country models, state that: "Large scale DSGE models show promise in this regard, but we are years away from developing empirically based multi-country versions of these models".

This paper considers some of the technical issues involved in using a GVAR approach to construct a multi-country rational expectations (RE) model, similar to those used within the DSGE literature. The approach involves estimating a set of country-specific rational expectations models and then combining them to solve the system as a whole. Such a model can be used to identify the contribution of domestic and international demand, supply, and monetary policy shocks to business cycle fluctuations. We illustrate the various issues involved in the analysis of multi-country RE models with a multi-country new Keynesian (MCNK) model encompassing 33 countries. The country specific models are kept simple intentionally and have the familiar standard three equation structure comprising a Phillips curve, an IS curve and a Taylor rule. To capture the multi-country nature of the analysis each country model also includes an exchange rate equation with the exception of the model for the US whose currency serves as the numeraire. The country specific models are also augmented with an equation for oil prices included in the US model as an endogenous variable. The countries are indexed by $i = 0, 1, 2, ..., N$, with the US denoted as country 0. The MCNK model thus explains output, inflation, short interest rates and exchange rates, but not long interest rates and equity prices which are included in the GVAR of Dees et al. (2007). This is because there are no widely accepted standard structural models for the term structure or equity premium. The MCNK model is estimated over 1980Q3-2011Q2 and has 131 variables.[1]

[1] This paper was previously circulated under the title: "Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian Model", where a shorter sample 1980Q1-2006Q4 and a slightly different specification were

Developing a multi-country model faces a number of important challenges and we discuss these in turn. The issues considered are: the measurement of steady states in Section 2; the specification of the short-run individual country models and the determination of exchange rates in Section 3; the identification and estimation of the model subject to the theoretical constraints required for a determinate solution in Section 4; the treatment of the nominal anchor and the solution of a large rational expectations (RE) model in Section 5; the structure and estimation of the covariance matrix, in Section 6 ; the simulation of shocks in Section 7; the robustness to alternative assumptions in Section 8. Section 9 provides some concluding comments.

2 Steady states

Most DSGE models are highly non-linear and for empirical analysis they are typically log-linearised around the model's steady states. Here we follow this approach and treat all variables as being measured as deviations from their steady states. In cases where the variables under consideration are either stationary or trend-stationary, the steady state values are either fixed constants or can be approximated by linear trends, and the deviations in the DSGE model can be replaced by realised values with constant terms or linear trends added to the equations (as appropriate) to take account of the non-zero deterministic means of the stationary or trend stationary processes. But there exists ample evidence that most macroeconomic variables, including inflation and interest rates, real exchange rates and real output, are likely to contain stochastic trends and could be cointegrated. Common stochastic trends at national and global levels can lead to within country as well as between country cointegration. The presence of such stochastic trends must be appropriately taken into account in the identification and estimation of steady state values (and hence the deviations), otherwise the estimates of the structural parameters and the associated impulse responses that are based on such deviations can be badly biased, even in large samples.

There are a variety of methods that can be used to handle permanent components, some of which are discussed by Fukac and Pagan (2010). While it is common to use purely statistical univariate de-trending procedures like the Hodrick-Prescott (HP) filter, these type of methods may neglect important multivariate characteristics of the underlying processes such as unit roots and cointegration, and need not be consistent with the underlying economic model. Here we follow Dees et al. (2009 p.1490-1492), and measure the steady states as long-horizon forecasts from an underlying GVAR.

Suppose that we have $N+1$ countries $i = 0, 1, 2, ..., N$, with country 0 denoting the US, and the GVAR is specified in terms of the realised values denoted by $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt})'$, with the deviations given by

$$\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}_t^P,$$

where $\mathbf{x}_t^P$ denotes the permanent component of $\mathbf{x}_t$. $\mathbf{x}_t^P$ is further decomposed into deterministic

used. All results in this paper are obtained using the MCNK 1.0 Matlab program by LV Smith (2013). A short non-technical introduction to the MCNK approach is available in Smith (2013).

and stochastic components

$$\mathbf{x}_t^P = \mathbf{x}_{d,t}^P + \mathbf{x}_{s,t}^P, \text{ and } \mathbf{x}_{d,t}^P = \mathbf{g}_0 + \mathbf{g}_1 t,$$

where $\mathbf{g}_0$ and $\mathbf{g}_1$ are $k \times 1$ vectors of constants and t a deterministic time trend. The steady state (permanent-stochastic component) $\mathbf{x}_{st}^P$, is then defined as the 'long-horizon forecast' (net of the permanent-deterministic component)

$$\mathbf{x}_{s,t}^P = \lim_{h\to\infty} E_t\left(\mathbf{x}_{t+h} - \mathbf{x}_{d,t+h}^P\right) = \lim_{h\to\infty} E_t\left[\mathbf{x}_{t+h} - \mathbf{g}_0 - \mathbf{g}_1(t+h)\right].$$

In the case where $\mathbf{x}_t$ is trend stationary then $\mathbf{x}_{s,t}^P = \mathbf{0}$, and we revert back to the familiar case where deviations are formed as residuals from regressions on linear trends. However, in general, $\mathbf{x}_{s,t}^P$ is non-zero and must be estimated from a multivariate time series model of $\mathbf{x}_t$ that allows for stochastic trends and cointegration. Once a suitable multivariate model is specified, it is then relatively easy to show that $\mathbf{x}_{s,t}^P$ corresponds to a multivariate Beveridge-Nelson (1981, BN) decomposition as argued by Garratt *et al.* (2006).[2] The economic model used to provide the long-horizon forecasts is a global VAR (GVAR) which takes account of unit roots and cointegration in the global economy (within as well as across economies). Dees et al. (2009) provide more detail on the GVAR and explain how it relates to the solution of structural models such as the MCNK considered here.

For each country, $i = 0, 1, 2, ..., N$, the GVAR model consists of VARX* models of the form:

$$\mathbf{x}_{it} = \mathbf{h}_{i0} + \mathbf{h}_{i1} t + \mathbf{A}_{i1}\mathbf{x}_{i,t-1} + \mathbf{A}_{i2}\mathbf{x}_{i,t-2} + \mathbf{C}_{i0}\mathbf{x}_{it}^* + \mathbf{C}_{i1}\mathbf{x}_{i,t-1}^* + \mathbf{C}_{i2}\mathbf{x}_{i,t-2}^* + \mathbf{u}_{it},\ i = 0, 1, .., N,$$

and the corresponding VECMX* models, subject to r_i cointegrating restrictions:

$$\Delta\mathbf{x}_{it} = \mathbf{c}_{i0} - \boldsymbol{\alpha}_i\boldsymbol{\beta}_i'[\mathbf{z}_{i,t-1} - \boldsymbol{\gamma}_i(t-1)] + \mathbf{C}_{i0}\Delta\mathbf{x}_{it}^* + \mathbf{G}_i\Delta\mathbf{z}_{i,t-1} + \mathbf{u}_{it},$$

where $\mathbf{z}_{it} = (\mathbf{x}_{it}', \mathbf{x}_{it}^{*\prime})'$, $\boldsymbol{\alpha}_i$ is a $k_i \times r_i$ matrix of rank r_i, and $\boldsymbol{\beta}_i$ is a $(k_i + k_i^*) \times r_i$ matrix of rank r_i. This allows for cointegration within $\mathbf{x}_{it}$ and between $\mathbf{x}_{it}$ and $\mathbf{x}_{it}^*$. Then using the identity $\mathbf{z}_{it} = \mathbf{W}_i\mathbf{x}_t$, where $\mathbf{x}_t = (\mathbf{x}_{0t}', \mathbf{x}_{1t}', ..., \mathbf{x}_{Nt}')'$, and $\mathbf{W}_i$ is the 'link' matrix for country i defined in terms of a set of weights w_{ij}, for example trade shares, we can stack the $N+1$ individual country models and solve for the standard VAR specification in $\mathbf{x}_t$:

$$\mathbf{x}_t = \mathbf{a}_0 + \mathbf{a}_1 t + \mathbf{F}_1\mathbf{x}_{t-1} + \mathbf{F}_2\mathbf{x}_{t-2} + \mathbf{u}_t. \tag{1}$$

This VAR model can now be readily used to derive $\mathbf{x}_t^P$ as the long-horizon forecast of $\mathbf{x}_t$.

The long-horizon forecasts from this GVAR model provide estimates of the steady states $\mathbf{x}_t^P$, which match the economic concept of a steady state and are derived from a multivariate economic model rather than a univariate statistical model, so they will reflect the long-run cointegrating relationships and stochastic trends in the system. The deviations from steady states used as variables

[2] However, it is worth noting that the long run expectations can be derived with respect to other information sets, such information dated $t-1$ or earlier. In such cases the long run expectations will not coincide with the permanent component in the BN decomposition.

in the MCNK model, $\widetilde{\mathbf{x}}_t = \mathbf{x}_t - \mathbf{x}_t^P$, are uniquely identified and stationary by construction.

These measures of steady state depend on the underlying economic model, which seems a desirable property. However, they may be sensitive to misspecification and it is possible that intercept shifts, broken trends or other forms of structural instability not allowed for in the estimated economic model, will be reflected in the measured deviations from the steady states. For instance, Perron and Wada (2009) argue that the difference between the univariate BN decomposition and other methods of measuring trend US GDP are the artifacts created by neglect of the change in slope of the trend function in 1973. Although our estimation period is all post 1973, so this is not an issue, and various tests indicate that the estimated GVARs seem structurally stable, possible structural breaks could be dealt with using the average long-horizon forecasts from models estimated over different samples. The evidence in Pesaran et al. (2009) indicates that averaging over observation windows improves forecasts. The extension of this procedure from forecasting to the estimation of steady states is an area for further research.

The GVAR model used to estimate the steady state values are based on country-specific VARX*(p, q) models whose orders are selected by the Akaike Information Criterion with a maximum lag of $p_{\max} = 4$ for the domestic variables and $q_{\max} = 2$ for the foreign variables. These lag orders are higher than are usually assumed ($p_{\max} = 2$ and $q_{\max} = 1$) in the literature, and are chosen to deal with the unusually large fluctuations observed during the recent financial crisis that occurred at the end of the sample, 1980Q3-2011Q2, under consideration. Initially, we used the lower maximum lag orders, but encountered significant residual serial correlation in the case of many of the country-specific models. Otherwise, the parameters of the country-specific models remained reasonably stable despite the severity of the recent crisis. See the online supplement for further results and supporting evidence.

3 Exchange rate determination and specification of individual country models

The variables included in the illustrative MCNK model are inflation deviations, $\widetilde{\pi}_{it}$, output deviations, $\widetilde{y}_{it}$, the interest rate deviations, $\widetilde{r}_{it}$ (except for Saudi Arabia where an interest rate variable is not available) and the real effective exchange rate deviations, $\widetilde{re}_{it}$, except for the US. There are also country-specific foreign variables, which are trade weighted averages of the corresponding variables of other countries. For example, the foreign output variable of country i is defined by $\widetilde{y}_{it}^* = \Sigma_{j=0}^N w_{ij}\widetilde{y}_{jt}$, where w_{ij} is the trade weight of country j in the total trade (exports plus imports) of country i. By construction $\sum_{j=0}^N w_{ij} = 1$, $w_{ii} = 0$.

The treatment of exchange rates is central to the construction of a coherent multi-country model where the number of countries is more than 2. In such cases the modelling of exchange rates must also take account of cross-rate effects that are absent in two-country or small open economy models. With $N + 1$ countries there are $N(N - 1)/2$ cross rates to be considered and the choice of the numeraire could become important. To allow for all the relevant cross-rates, deviations from steady state of the real effective exchange rate appear in the MCNK model. They are also the dependent variable in the exchange rate equations, except for the US where there is no exchange rate equation, since the US dollar is used as the numeraire.

More specifically, denote the log nominal exchange rate of country i against the US dollar by e_{it}, and the bilateral log exchange rate of country i with respect to country j by e_{ijt}. Then $e_{ijt} = e_{it} - e_{jt}$, and the log real effective exchange rate of country i with respect to its trading partners is given by

$$re_{it} = \sum_{j=0}^{N} w_{ij}(e_{it} - e_{jt}) + \sum_{j=0}^{N} w_{ij}p_{jt} - p_{it},$$

where p_{it} is the log general price level in country i. Therefore (recalling that $\sum_{j=0}^{N} w_{ij} = 1$)

$$re_{it} = (e_{it} - p_{it}) - \sum_{j=0}^{N} w_{ij}(e_{jt} - p_{jt}) = ep_{it} - ep_{it}^{*}, \tag{2}$$

where $ep_{it} = e_{it} - p_{it}$, and $ep_{it}^{*} = \sum_{j=0}^{N} w_{ij}ep_{jt}$. Deviations from steady states are defined accordingly as $\widetilde{re}_{it} = \widetilde{ep}_{it} - \widetilde{ep}_{it}^{*}$.

While the MCNK model estimates equations for the log real effective exchange rate deviations for countries $i = 1, 2, ..., N$, it solves the model for the $N + 1$ deflated exchange rate variables, $\widetilde{ep}_{it}$, $i = 0, 1, 2, ..., N$. In the case of the US, where $e_{0t} = 0$, we have $\widetilde{ep}_{0t} = -\widetilde{p}_{0t}$. It is important that possible stochastic trends in the log US price level are appropriately taken into account when computing $\widetilde{p}_{0t}$. This is achieved by first estimating $\widetilde{\pi}_{0t}$ and then cumulating the values of $\widetilde{\pi}_{0t}$ to obtain $\widetilde{p}_{0t}$ up to an arbitrary constant.

3.1 Country-specific models

The equations in the country-specific models include a standard Phillips curve (PC), derived from the optimising behaviour of monopolistically competitive firms subject to nominal rigidities, which determines inflation deviations $\widetilde{\pi}_{it}$, where $\pi_{it} = p_{it} - p_{i,t-1}$. This takes the form

$$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{if}E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right) + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st},\ i = 0, 1, ..., N, \tag{3}$$

where $E_{t-1}\left(\widetilde{\pi}_{i,t+1}\right) = E\left(\widetilde{\pi}_{i,t+1} \mid \Im_{i,t-1}\right)$.[3] There are no intercepts included in the equations since deviations from steady state values have mean zero by construction. The error term, $\varepsilon_{i,st}$, is interpreted as a supply shock or a shock to the price-cost margin in country i. The parameters are non-linear functions of underlying structural parameters. For instance, suppose that there is staggered price setting, with a proportion of firms, $(1 - \theta_i)$, resetting prices in any period, and a proportion θ_i keeping prices unchanged. Of those firms able to adjust prices only a fraction $(1-\omega_i)$ set prices optimally on the basis of expected marginal costs. A fraction ω_i use a rule of thumb

[3] Here we condition expectations on information sets dated at time $t - 1$, since in the case of macroeconomic relations information is aggregated across heterogeneous agents and it is likely that aggregate information is only available to the 'representative agent' with a time delay. But the analysis can be readily modified to deal with dated t information sets, if needed.

based on lagged inflation. Then for a subjective discount factor, λ_i, we have

$$\begin{aligned} \beta_{if} &= \lambda_i \theta_i \phi_i^{-1}, \ \beta_{ib} = \omega_i \phi_i^{-1}, \\ \beta_{iy} &= (1-\omega_i)(1-\theta_i)(1-\lambda_i\theta_i)\phi_i^{-1}, \end{aligned}$$

where $\phi_i = \theta_i + \omega_i[1 - \theta_i(1 - \lambda_i)]$. Notice that there is no reason for these parameters to be the same across countries with very different market institutions and property rights (which will influence λ_i), so we allow them to be heterogeneous from the start. If $\omega_i = 0$, all those who adjust prices do so optimally, then $\beta_{fi} = \lambda_i$, and $\beta_{bi} = 0$. Since $\theta_i \geq 0$, $\omega_i \geq 0$, $\lambda_i \geq 0$ the theory implies $\beta_{ib} \geq 0$, $\beta_{if} \geq 0$, and $\beta_{iy} \geq 0$, which needs to be imposed at the estimation stage. The restriction $\beta_{ib} + \beta_{if} < 1$ ensures a unique rational expectations solution in the case where $\widetilde{y}_{it}$ is exogenously given and there are no feedbacks from lagged values of inflation to the output gap. The corresponding condition in a multi-country model is likely to be more complicated. Some versions of the PC use marginal cost rather than output, but data for this is not available for all the countries and even if data were available using it would add another 33 variables to the model.

The aggregate demand or IS curve is obtained by log-linearising the Euler equation in consumption and substituting the result in the economy's aggregate resource constraint. In the standard closed economy case, this yields an equation for the output gap, $\widetilde{y}_{it}$, which depends on the expected future output gap, $E_{t-1}(\widetilde{y}_{i,t+1})$, and the real interest rate deviations, $\widetilde{r}_{it} - E_{t-1}(\widetilde{\pi}_{i,t+1})$. Lagged output will enter the IS equation if the utility of consumption for country i at time t is $u(C_{it} - h_i C_{i,t-1})$ where h_i is a habit persistence parameter. For an open economy model, the aggregate resource constraint will also contain net exports, which in turn will be a function of the real effective exchange rate, $\widetilde{re}_{it}$, and the foreign output gap, $\widetilde{y}^*_{it}$. The open economy version of the standard IS equation is then

$$\widetilde{y}_{it} = \alpha_{ib}\widetilde{y}_{i,t-1} + \alpha_{if}E_{t-1}(\widetilde{y}_{i,t+1}) + \alpha_{ir}[\widetilde{r}_{it} - E_{t-1}(\widetilde{\pi}_{i,t+1})] + \alpha_{ie}\widetilde{re}_{it} + \alpha_{iy*}\widetilde{y}^*_{it} + \varepsilon_{i,dt}, \ i = 0, 1, ..., N. \quad (4)$$

The coefficient of the real interest rate, α_{ir}, is interpreted as the inter-temporal elasticity of consumption, see Clarida *et al.* (1999), while $\alpha_{if} = 1/(1 + h_i)$ and $\alpha_{ib} = h_i/(1 + h_i)$. The error, $\varepsilon_{i,dt}$, is interpreted as a demand shock. A number of authors note that unless technology follows a pure random walk process, $\varepsilon_{i,dt}$ may reflect technology shocks, though by conditioning on the foreign output variable the convolution of demand shocks with technology shocks might be somewhat obviated. We would expect $\alpha_{ir} \leq 0$, $\alpha_{iy*} \geq 0$. In estimation it became apparent that when $\alpha_{if} \neq 0$, there was a strong tendency for $\alpha_{ir} > 0$. Given the importance of a negative interest rate effect on output in what follows we set $\alpha_{if} = 0$.

We treat the US as a closed economy as is common in the literature, which corresponds to setting $\alpha_{0e} = 0$ and $\alpha_{0y*} = 0$. This is partly because the role of the US as a dominant unit in the world economy makes the assumption of the weak exogeneity of the real effective exchange rate and foreign output less plausible, and partly to ensure that feedbacks to the US do not preclude the existence of a global solution to the resultant multi-country rational expectations model. There are, however, some indirect linkages from the rest of the world to the US through the cross country error dependencies, and we discuss these below.

The interest rate deviations in country i, $\widetilde{r}_{it}$, are set according to a standard Taylor rule (TR) of the form:

$$\widetilde{r}_{it} = \gamma_{ib}\widetilde{r}_{i,t-1} + \gamma_{i\pi}\widetilde{\pi}_{it} + \gamma_{iy}\widetilde{y}_{it} + \varepsilon_{i,mt},\ i = 0, 1, ..., N. \tag{5}$$

The error $\varepsilon_{i,mt}$ is interpreted as a monetary policy shock.

In the MCNK model the log real effective exchange rate deviations, $\widetilde{re}_{it}$, are modelled as a stationary first order autoregression,

$$\widetilde{re}_{it} = \rho_i\widetilde{re}_{i,t-1} + \varepsilon_{i,et},\ \ |\rho_i| < 1,\ i = 1, 2, ..., N. \tag{6}$$

To the extent that long run theory restrictions such as purchasing power parity and uncovered interest parity (UIP) are supported by the data, they will be embodied in the cointegrating relations that underlie the global error-correcting model. These will then be reflected in the steady state and long-horizon expectations, to which the exchange rate adjusts. One could use a UIP equation which adheres more strictly to the short-run theory, though it is well known that UIP fails to hold in the short-run: high interest rates predict an appreciation of the currency rather than a depreciation as UIP would imply. This failure gives rise to a large UIP risk premium or carry trade return. Since the MCNK model explains the exchange rate and the forward rate (from domestic and foreign interest rates) this UIP risk premium is determined implicitly by the GVAR model.

Putting equations (3) to (6) together for all 33 countries, and adding a first order autoregression to explain the price of oil, the total number of variables in the MCNK is $k = \sum_{i=0}^{N} k_i + 1 = 131$, where k_i is the number of variables in country i. For the US, with no exchange rate equation, $k_0 = 3$, for Saudi Arabia, with no interest rate equation, $k_{SA} = 3$, for the other 31 countries $k_i = 4$.

4 Identification and estimation

There has been some concern in the literature that some parameters of DSGE models are not identified, e.g. Canova and Sala (2009), Koop et al. (2013). As explained in Dees et al. (2009), the large N framework can provide sources of identification, not available in closed economy models, through the use of cross-section averages of foreign variables as instruments. Individual country shocks, being relatively unimportant, will be uncorrelated with the cross section averages as N becomes large, whilst global factors make the cross section averages correlated with the included endogenous variables. This allows consistent estimation of the parameters of the model and thus the structural shocks.

DSGE models are usually estimated either by Generalised Method of Moments, GMM, or Bayesian methods. Bayesian estimation of a multi-country model, where N is large, faces considerable difficulties both in the specification of multivariate priors over a large number of parameters and the numerical issues that arise in maximum likelihood estimation of large systems. Thus in estimation of the MCNK model we follow the GMM route and use an inequality constrained instrumental variables (ICIV) estimator, where the inequality constraints reflect the theoretical restrictions required for a determinate rational expectations solution of the model.[4] In cases where the constraints are not satisfied, the parameters are set to their boundary values and the choice between any alternative estimates that satisfy the constraints is based on the in-sample prediction errors. Pesaran and Smith (1994, p. 708) discuss the relationship between this criterion and the IV minimand.

More specifically, ICIV estimation is a non-linear optimization problem. In general, if there are n inequality restrictions there are 2^n possible unconstrained and constrained models to consider. In our application the maximum number of constraints is 4, in the PC equation, and the constrained optimization problem can be carried out by searching over all the specifications and then selecting the specification that satisfies all the constraints and has the lowest in-sample mean square prediction errors. Note that since some of the regressors are endogenous, the in-sample prediction errors and the IV residuals would not be the same. See Pesaran and Smith (1994). Consider the regression equation

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

where $\mathbf{y}$ and $\boldsymbol{\varepsilon}$ are $T \times 1$ vectors for the dependent variable and unobserved disturbance, $\mathbf{X}$ a $T \times k$ matrix of potentially endogenous regressors and there is also a $T \times s$ matrix of instruments $\mathbf{Z}$, $s \geq k$. Define $\mathbf{P}_z = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$, $\mathbf{M}_z = \mathbf{I}_T - \mathbf{P}_z$ and $\hat{\mathbf{X}} = \mathbf{P}_z\mathbf{X}$. The IV estimator is $\hat{\boldsymbol{\beta}} = (\mathbf{X}'\mathbf{P}_z\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}_z\mathbf{y}$. The prediction errors are: $\hat{\mathbf{e}} = \mathbf{y} - \hat{\mathbf{X}}\hat{\boldsymbol{\beta}}$; the IV residuals $\mathbf{e} = \mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}$. Then as shown in Pesaran and Smith (1994), the sum of squared prediction errors equals the IV minimand plus a constant which depends only on the data, not the estimates:

$$\hat{\mathbf{e}}'\hat{\mathbf{e}} = \mathbf{e}'\mathbf{P}_z\mathbf{e} + \mathbf{y}'\mathbf{M}_\mathbf{z}\mathbf{y}.$$

Thus minimising the sum of squares of prediction errors, $\hat{\mathbf{e}}'\hat{\mathbf{e}}$, with respect to $\hat{\boldsymbol{\beta}}$ is equivalent to minimising the IV minimand, $\mathbf{e}'\mathbf{P}_z\mathbf{e}$. In some of the constrained cases, there are no endogenous variables so $\mathbf{X} = \mathbf{Z}$, $\hat{\mathbf{X}} = \mathbf{X}$ and IV reduces to least squares; in other cases $\hat{\boldsymbol{\beta}}$ is fully specified by the constraints, so no parameters are estimated. The sum of squared prediction errors remains well defined in all such cases.

For illustration consider the ICIV estimation of the Phillips curve, (3), subject to the four inequality restrictions $\beta_{ib} \geq 0$, $\beta_{if} \geq 0$, $\beta_{ib} + \beta_{if} \leq 0.99$, and $\beta_{iy} \geq 0$. The set of possible binding constraints gives 14 cases: two cases are redundant since imposing the boundary conditions $\beta_{ib} = 0$ and $\beta_{if} = 0$ implies $\beta_{ib} + \beta_{if} \leq 0.99$.

4.1 Parameter estimates

Table 1 presents information on the distribution of the ICIV estimates of the coefficients of the MCNK model for the sample 1980Q3-2011Q2. The table gives the mean of the constrained coefficients and the number of the coefficients constrained. In total there were 67 coefficients constrained, with 5 countries completely unconstrained. The coefficient of inflation expectations, β_{if}, turned out

[4] Inference in inequality constrained estimation is non-standard and will not be addressed here. Gouriéroux et al. (1982) consider the problem in the case of least squares estimation.

to be positive in all cases and is generally much larger than the coefficient of lagged inflation, β_{ib}. The mean value of β_{iy} at 0.05 is close to the standard prior in the literature, although this average hides a wide range of estimates obtained across countries. The parameters of the Phillips curve, (3), are estimated subject to the inequality restrictions $\beta_{ib} \geq 0$, $\beta_{if} \geq 0$, $\beta_{ib} + \beta_{if} \leq 0.99$, and $\beta_{iy} \geq 0$. Since under $\beta_{ib} = \beta_{if} = 0$, the third restriction, $\beta_{ib} + \beta_{if} \leq 0.99$, is satisfied, there are 14 possible specifications. All specifications are estimated and from those satisfying the restrictions the one with the lowest in-sample mean squared prediction error is selected. Application of this procedure to Argentina over the full sample resulted in the estimates, $\hat{\beta}_{ib} = \hat{\beta}_{if} = \hat{\beta}_{iy} = 0$, which does not seem plausible and could be due to structural breaks, so the PC for Argentina was estimated over the sub-sample, 1990Q1-2011Q2, which gave somewhat more plausible estimates.

For the IS equation (4) various restricted versions were considered. As noted above given the importance of having a negative interest rate effect in the IS curve for the monetary transmission mechanism, the IS specification without the future output variable was used. Thus the parameters of (4) are estimated subject to the constraints $\alpha_{if} = 0$, $\alpha_{ir} \leq 0$ and $\alpha_{iy*} \geq 0$. Including $\widetilde{y}_{it}^{*}$ tended to produce a more negative and significant estimate of the interest rate effect. The estimate of the coefficient of the real exchange rate variable averaged to about zero, but with quite a large range of variations across the different countries.

The Taylor Rule, (5) was estimated subject to the constraints $\gamma_{iy} \geq 0$ and $\gamma_{i\pi} \geq 0$. For the real effective exchange rate equation, (6), the OLS estimates of ρ_i were all less than unity, confirming that this is a stable process, as one would expect given that we are using deviations from the steady states. There is some evidence of misspecification in a number of equations, but because we want to keep close to the standard theoretical model, we do not conduct a specification search with the view to adding more lags or foreign variables.

Overall, these estimates are qualitatively similar to those obtained using the pre-crisis sample up to 2006Q4, as shown in the working paper version of this article (DPSS, 2010) where a slightly different model specification was employed. The main differences are that the shorter sample tended to show faster adjustment, with coefficients of output in the PC and coefficients of interest rates in the IS curve being further away from zero. The short-run response to inflation in the Taylor rule was larger, though the long-run response was smaller. For the IS equation, the mean of the exchange rate coefficients went from a small positive number in the shorter sample to a small negative number in the longer sample.

Table 1: Distribution of inequality-constrained IV estimates using GVAR estimates of deviations from steady states: 1980Q3-2011Q2

	Mean	# Constrained	Constraint
Phillips curve - Equation (3), N=33			
β_{ib}	0.13	8	$\beta_{ib} \geq 0$
β_{if}	0.82	0	$\beta_{if} \geq 0$
β_{iy}	0.05	7	$\beta_{iy} \geq 0$
$\beta_{ib}+\beta_{if}$	0.94	20	$\beta_{ib}+\beta_{if} \leq 0.99$
IS curve - Equation (4), N=33			
α_{ib}	0.33		
α_{ir}	-0.13	17	$\alpha_{ir} \leq 0$
κ_{ir}	-0.29		
α_{ie}	-0.05		
κ_{ie}	-0.07		
α_{iy*}	0.79	1	$\alpha_{iy*} \geq 0$
κ_{iy*}	1.18		
Taylor Rule - Equation (5), N=32			
γ_{ib}	0.65		
$\gamma_{i\pi}$	0.20	3	$\gamma_{i\pi} \geq 0$
$\mu_{i\pi}$	0.88		
γ_{iy}	0.04	10	$\gamma_{iy} \geq 0$
μ_{iy}	0.17		
Exchange rates - Equation (6), N=32			
ρ_i	0.62	0	$\lvert\rho_i\rvert < 1$

Note: The estimation period begins in t=1980Q3 and ends in 2011Q1 for PC and IS, and 2011Q2 for TR and ER. An exception is the Phillips curve in Argentina which is estimated over the sub-sample beginning in 1990Q1. N is the number of countries for which the equations are estimated. The column headed "Mean" gives the average over all estimates, constrained and unconstrained. The column headed "# Constrained" gives the number of estimates constrained at the boundary. The κ_i are the long-run coefficients in the IS curve, the μ_i the long-run coefficients in the Taylor rule. Individual country results are available in the supplement. For the IS of US, $\alpha_{ie} = \alpha_{iy*} = 0$.

The fact that the coefficients are relatively stable despite the estimation period covering the crisis period after 2007 should not be surprising. The model is estimated on deviations from long-run steady states calculated as long horizon forecasts. These will reflect the stochastic trends and respond immediately to the crisis innovations thus the deviations from these steady states will be less volatile.

Table 2 gives the ICIV estimates of all the coefficients for eight countries. In some simple closed economy models, a condition for a determinate solution is that the long run coefficient in the Taylor rule, $\mu_{i\pi}$, should be greater than one. This is clearly satisfied in the US which is treated as a closed economy and is also satisfied in twelve of the 31 other countries. However, it should be noted that this condition is not directly applicable in a complex multi-country model, partly because some of the monetary feedback to inflation can come from abroad. In fact, these estimates do produce a determinate solution. The estimate of ρ_i for Canada at 0.94 is the largest of the 32 estimates. More details about the country-specific estimates can be found in the supplement.

Table 2: Inequality-constrained IV estimates using GVAR estimates of deviations from steady states for eight major economies

	US	China	Japan	Germany	France	UK	Italy	Canada
				Phillips curve - Equation (3)				
β_{ib}	0.11	0.26	0.02	0.10	0.17	0.22	0.42	0.12
β_{if}	0.88	0.64	0.97	0.59	0.82	0.77	0.57	0.87
β_{iy}	0.06	0.12	0.01	0.07	0.04	0.04	0.03	0.03
				IS curve - Equation (4)				
α_{ib}	0.71	0.53	0.71	0.07	0.31	0.36	0.16	0.48
α_{ir}	-0.11	-0.49	-0.14	0.00	0.00	-0.40	0.00	0.00
κ_{ir}	-0.45	-1.05	-0.48	0.00	0.00	-0.62	0.00	0.00
α_{ie}		-0.05	-0.06	-0.03	-0.07	0.05	-0.20	-0.01
κ_{ie}		-0.19	-0.19	-0.03	-0.10	0.07	-0.24	-0.02
α_{iy*}		0.26	0.24	1.22	0.67	0.92	0.44	0.85
κ_{iy*}		0.55	0.83	1.31	0.98	1.44	0.52	1.62
				Taylor Rule - Equation (5)				
γ_{ib}	0.90	0.82	0.92	0.81	0.86	0.84	0.95	0.76
$\gamma_{i\pi}$	0.22	0.02	0.08	0.16	0.20	0.23	0.06	0.33
$\mu_{i\pi}$	2.23	0.12	0.98	0.81	1.46	1.44	1.20	1.40
γ_{iy}	0.00	0.03	0.00	0.03	0.02	0.00	0.04	0.00
μ_{iy}	0.00	0.15	0.00	0.17	0.17	0.00	0.82	0.00
				Exchange rates - Equation (6)				
ρ_i		0.79	0.74	0.57	0.57	0.62	0.51	0.94

Note: The estimation sample begins in t=1980Q3 and ends in 2011Q1 for the PC and IS equations, and 2011Q2 for the Taylor rule and exchange rate equations. Individual country results are available in the supplement.

5 System and solution

5.1 Nominal anchor

The model is solved for the deviation from steady state of the logarithm of the nominal exchange rate against the US dollar deflated by the US general price level. For the US this is just minus the log of the US price level relative to its steady state value, derived from the US Phillips Curve. This provides the nominal anchor for the multi-country model. While inflation is determined in each country through the Phillips Curve, the price levels enter the real effective exchange rate variables. Since the US is the numeraire country for exchange rates, we use the US price level to provide the nominal anchor. To do this we distinguish the vectors used in estimation and solution of the MCNK model. For all countries $i = 0, 1, ..., N$, let $\widetilde{\mathbf{x}}_{it} = (\widetilde{\pi}_{it}, \widetilde{y}_{it}, \widetilde{r}_{it}, \widetilde{ep}_{it})'$ with the associated world $(k+1) \times 1$ vector $\widetilde{\mathbf{x}}_t = (\widetilde{\mathbf{x}}'_{0t}, \widetilde{\mathbf{x}}'_{1t}, ..., \widetilde{\mathbf{x}}'_{Nt})'$, so that $\widetilde{\mathbf{x}}_{0t}$ includes both US inflation and the US price level, since $\widetilde{ep}_{0t} = -\widetilde{p}_{0t}$. Although $\widetilde{\pi}_{0t}$ and $\widetilde{p}_{0t}$ are related, $\widetilde{ep}_{0t}$ is still needed for the construction of $\widetilde{ep}^*_{it}$, $(i = 0, 1, ...N)$ that enter the IS equations. So when we link up the country models, the MCNK represents a system of k variables in $k+1$ RE equations, and therefore contains a redundant equation in the US model. To remove this redundancy we solve the model in terms of a new $k \times 1$ vector $\widetilde{\mathring{\mathbf{x}}}_t = (\widetilde{\mathring{\mathbf{x}}}'_{0t}, \widetilde{\mathring{\mathbf{x}}}'_{1t}, ..., \widetilde{\mathring{\mathbf{x}}}'_{Nt})'$, where $\widetilde{\mathring{\mathbf{x}}}_{0t} = (y_{0t}, r_{0t}, ep_{0t})'$ and $\widetilde{\mathring{\mathbf{x}}}_{it} = \widetilde{\mathbf{x}}_{it}$ for $i = 1, 2, ..., N$. In particular, for the US we can relate the 4×1 vector $\widetilde{\mathbf{x}}_{0t}$ to the 3×1 vector $\widetilde{\mathring{\mathbf{x}}}_{0t}$ by

$$\widetilde{\mathbf{x}}_{0t} = \mathbf{S}_{00}\widetilde{\mathring{\mathbf{x}}}_{0t} - \mathbf{S}_{01}\widetilde{\mathring{\mathbf{x}}}_{0,t-1},$$

where

$$\mathbf{S}_{00}=\begin{pmatrix}0&0&-1\\1&0&0\\0&1&0\\0&0&1\end{pmatrix},\ \mathbf{S}_{01}=\begin{pmatrix}0&0&-1\\0&0&0\\0&0&0\\0&0&0\end{pmatrix}.$$

Similarly, $\widetilde{\mathbf{x}}_t=(\widetilde{\mathbf{x}}'_{0t},\widetilde{\mathbf{x}}'_{1t},...,\widetilde{\mathbf{x}}'_{Nt})'$ can be related to the $k\times 1$ global vector $\widetilde{\mathring{\mathbf{x}}}_t=(\widetilde{\mathring{\mathbf{x}}}'_{0t},\widetilde{\mathring{\mathbf{x}}}'_{1t},...,\widetilde{\mathring{\mathbf{x}}}'_{Nt})'$ by

$$\widetilde{\mathbf{x}}_t=\mathbf{S}_0\widetilde{\mathring{\mathbf{x}}}_t-\mathbf{S}_1\widetilde{\mathring{\mathbf{x}}}_{t-1}, \tag{7}$$

where

$$\mathbf{S}_0=\begin{pmatrix}\mathbf{S}_{00}&\mathbf{0}_{4\times(k-3)}\\\mathbf{0}_{(k-3)\times 3}&\mathbf{I}_{k-3}\end{pmatrix},\mathbf{S}_1=\begin{pmatrix}\mathbf{S}_{01}&\mathbf{0}_{4\times(k-3)}\\\mathbf{0}_{(k-3)\times 3}&\mathbf{0}_{(k-3)\times(k-3)}\end{pmatrix}.$$

Thus we can convert a solution for $\widetilde{\mathring{\mathbf{x}}}_t$ into a solution for $\widetilde{\mathbf{x}}_t$.

5.2 Linking the country models

We now explain how the country models are linked. In terms of $\widetilde{\mathbf{x}}_{it}$ the country-specific models based on equations (3), (4), (5) and (6) can be written as

$$\mathbf{A}_{i0}\widetilde{\mathbf{x}}_{it}=\mathbf{A}_{i1}\widetilde{\mathbf{x}}_{i,t-1}+\mathbf{A}_{i2}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1})+\mathbf{A}_{i3}\widetilde{\mathbf{x}}^*_{it}+\mathbf{A}_{i4}\widetilde{\mathbf{x}}^*_{i,t-1}+\boldsymbol{\varepsilon}_{it},\ \text{for } i=0,1,...,N, \tag{8}$$

where $\widetilde{\mathbf{x}}^*_{it}=(\widetilde{y}^*_{it},\widetilde{ep}^*_{it})'$, and as before $\widetilde{y}^*_{it}=\sum_{j=0}^N w_{ij}\widetilde{y}_{jt}$, and $\widetilde{ep}^*_{it}=\sum_{j=0}^N w_{ij}\widetilde{ep}_{jt}$. The expectations are taken with respect to a common global information set formed as the union intersection of the individual country information sets, $\mathfrak{I}_{i,t-1}$,namely $\mathfrak{I}_{t-1}=\cup_{i=0}^N\mathfrak{I}_{i,t-1}$.

For US, $i=0$

$$\mathbf{A}_{00}=\begin{pmatrix}1&-\beta_{0y}&0&0\\0&1&-\alpha_{0r}&-\alpha_{0e}\\-\gamma_{0\pi}&-\gamma_{0y}&1&0\end{pmatrix},\quad \mathbf{A}_{01}=\begin{pmatrix}\beta_{0b}&0&0&0\\0&\alpha_{0b}&0&0\\0&0&\gamma_{0b}&0\end{pmatrix},$$

$$\mathbf{A}_{02}=\begin{pmatrix}\beta_{0f}&0&0&0\\-\alpha_{0r}&0&0&0\\0&0&0&0\end{pmatrix},\quad \mathbf{A}_{03}=\begin{pmatrix}0&0\\\alpha_{0y*}&-\alpha_{0e}\\0&0\end{pmatrix},$$

and $\boldsymbol{\varepsilon}_{0t}=(\varepsilon_{0,st},\varepsilon_{0,dt},\varepsilon_{0,mt})'$. Note that $\mathbf{A}_{04}=\mathbf{0}$, since there is no exchange rate equation for the US.

For the other countries, $i=1,2,...,N$, (except Saudi Arabia where there is no interest rate equation)

$$\mathbf{A}_{i0} = \begin{pmatrix} 1 & -\beta_{iy} & 0 & 0 \\ 0 & 1 & -\alpha_{ir} & -\alpha_{ie} \\ -\gamma_{i\pi} & -\gamma_{iy} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \mathbf{A}_{i1} = \begin{pmatrix} \beta_{ib} & 0 & 0 & 0 \\ 0 & \alpha_{ib} & 0 & 0 \\ 0 & 0 & \gamma_{ib} & 0 \\ 0 & 0 & 0 & \rho_i \end{pmatrix},$$

$$\mathbf{A}_{i2} = \begin{pmatrix} \beta_{if} & 0 & 0 & 0 \\ -\alpha_{ir} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{A}_{i3} = \begin{pmatrix} 0 & 0 \\ \alpha_{iy*} & -\alpha_{ie} \\ 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{A}_{i4} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & -\rho_i \end{pmatrix},$$

and $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{i,st}, \varepsilon_{i,dt}, \varepsilon_{i,mt}, \varepsilon_{i,et})'$.

Let $\widetilde{\mathbf{z}}_{it} = (\widetilde{\mathbf{x}}_{it}', \widetilde{\mathbf{x}}_{it}^{*\prime})'$ then the $N+1$ models specified by (8) can be written compactly as

$$\mathbf{A}_{iz0}\widetilde{\mathbf{z}}_{it} = \mathbf{A}_{iz1}\widetilde{\mathbf{z}}_{i,t-1} + \mathbf{A}_{iz2}E_{t-1}\left(\widetilde{\mathbf{z}}_{i,t+1}\right) + \boldsymbol{\varepsilon}_{it}, \text{ for } i = 0, 1, 2, ..., N, \tag{9}$$

where $E_{t-1}(\cdot) = E(\cdot \,|\mathfrak{I}_{t-1})$, and

$$\begin{aligned} \mathbf{A}_{0z0} &= (\mathbf{A}_{00}, \ -\mathbf{A}_{03}), \ \mathbf{A}_{0z1} = \left(\mathbf{A}_{01}, \ \mathbf{0}_{k_0\times(k_0+1+k_0^*)}\right), \ \mathbf{A}_{0z2} = \left(\mathbf{A}_{02}, \ \mathbf{0}_{k_0\times(k_0+1+k_0^*)}\right), \text{ for } i = 0, \\ \mathbf{A}_{iz0} &= (\mathbf{A}_{i0}, \ -\mathbf{A}_{i3}), \ \mathbf{A}_{iz1} = (\mathbf{A}_{i1}, \ \mathbf{A}_{i4}), \ \mathbf{A}_{iz2} = \left(\mathbf{A}_{i2}, \ \mathbf{0}_{k_i\times(k_i+k_i^*)}\right), \text{ for } i = 1, 2, ..., N. \end{aligned}$$

The variables $\widetilde{\mathbf{z}}_{it}$ are linked to the variables in the global model, $\widetilde{\mathbf{x}}_t$, through the identity

$$\widetilde{\mathbf{z}}_{it} = \mathbf{W}_i\widetilde{\mathbf{x}}_t, \tag{10}$$

where the 'link' matrices $\mathbf{W}_i$, $i = 0, 1, ..., N$ are defined in terms of the weights w_{ij}. For $i = 0$, $\mathbf{W}_0$ is $(k_0 + 1 + k_0^*) \times (k + 1)$ and for $i = 1, 2, ..., N$, $\mathbf{W}_i$ is $(k_i + k_i^*) \times (k + 1)$ dimensional.

Substituting (10) in (9) now yields

$$\mathbf{A}_{iz0}\mathbf{W}_i\widetilde{\mathbf{x}}_t = \mathbf{A}_{iz1}\mathbf{W}_i\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_{iz2}\mathbf{W}_iE_{t-1}\left(\widetilde{\mathbf{x}}_{t+1}\right) + \boldsymbol{\varepsilon}_{it}, \ i = 0, 1, ..., N,$$

and then stacking all the $N + 1$ country models we obtain the multi-country RE model for $\widetilde{\mathbf{x}}_t$ as

$$\mathbf{A}_0\widetilde{\mathbf{x}}_t = \mathbf{A}_1\widetilde{\mathbf{x}}_{t-1} + \mathbf{A}_2E_{t-1}\left(\widetilde{\mathbf{x}}_{t+1}\right) + \boldsymbol{\varepsilon}_t, \tag{11}$$

where the stacked $k \times (k + 1)$ matrices $\mathbf{A}_j$, $j = 0, 1, 2$ are defined by

$$\mathbf{A}_j = \begin{pmatrix} \mathbf{A}_{0zj}\mathbf{W}_0 \\ \mathbf{A}_{1zj}\mathbf{W}_1 \\ \vdots \\ \mathbf{A}_{Nzj}\mathbf{W}_N \end{pmatrix}, \ j = 0, 1, 2, \text{ and } \boldsymbol{\varepsilon}_t = \begin{pmatrix} \boldsymbol{\varepsilon}_{0t} \\ \boldsymbol{\varepsilon}_{1t} \\ \vdots \\ \boldsymbol{\varepsilon}_{Nt} \end{pmatrix}.$$

Using (7) in (11) we have

$$\mathbf{A}_0 \left(\mathbf{S}_0\tilde{\mathring{\mathbf{x}}}_t - \mathbf{S}_1\tilde{\mathring{\mathbf{x}}}_{t-1}\right) = \mathbf{A}_1 \left(\mathbf{S}_0\tilde{\mathring{\mathbf{x}}}_{t-1} - \mathbf{S}_1\tilde{\mathring{\mathbf{x}}}_{t-2}\right) + \mathbf{A}_2 E_{t-1}(\mathbf{S}_0\tilde{\mathring{\mathbf{x}}}_{t+1} - \mathbf{S}_1\tilde{\mathring{\mathbf{x}}}_t) + \boldsymbol{\varepsilon}_t,$$

or

$$\mathbf{H}_0\tilde{\mathring{\mathbf{x}}}_t = \mathbf{H}_1\tilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{H}_2\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_3 E_{t-1}(\tilde{\mathring{\mathbf{x}}}_{t+1}) + \mathbf{H}_4 E_{t-1}(\tilde{\mathring{\mathbf{x}}}_t) + \boldsymbol{\varepsilon}_t, \tag{12}$$

where

$$\mathbf{H}_0 = \mathbf{A}_0\mathbf{S}_0, \mathbf{H}_1 = \mathbf{A}_1\mathbf{S}_0 + \mathbf{A}_0\mathbf{S}_1, \mathbf{H}_2 = -\mathbf{A}_1\mathbf{S}_1, \mathbf{H}_3 = \mathbf{A}_2\mathbf{S}_0,\ \mathbf{H}_4 = -\mathbf{A}_2\mathbf{S}_1.$$

For a determinate solution the $k \times k$ matrix $\mathbf{H}_0$ must be non-singular. Pre-multiplying (12) by $\mathbf{H}_0^{-1}$

$$\tilde{\mathring{\mathbf{x}}}_t = \mathbf{F}_1\tilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{F}_2\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{F}_3 E_{t-1}(\tilde{\mathring{\mathbf{x}}}_{t+1}) + \mathbf{F}_4 E_{t-1}(\tilde{\mathring{\mathbf{x}}}_t) + \mathbf{u}_t, \tag{13}$$

where $\mathbf{F}_j = \mathbf{H}_0^{-1}\mathbf{H}_j$, for $j = 1, 2, 3, 4$, and $\mathbf{u}_t = \mathbf{H}_0^{-1}\boldsymbol{\varepsilon}_t$. Using a companion form representation (13) can be written as

$$\boldsymbol{\chi}_t = \mathbf{A}\boldsymbol{\chi}_{t-1} + \mathbf{B}E_{t-1}(\boldsymbol{\chi}_{t+1}) + \boldsymbol{\eta}_t, \tag{14}$$

where $\boldsymbol{\chi}_t = \left(\tilde{\mathring{\mathbf{x}}}'_t, \tilde{\mathring{\mathbf{x}}}'_{t-1}\right)'$, and

$$\mathbf{A} = \begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix},\ \mathbf{B} = \begin{pmatrix} \mathbf{F}_3 & \mathbf{F}_4 \\ \mathbf{0} & \mathbf{0} \end{pmatrix},\ \boldsymbol{\eta}_t = \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \end{pmatrix}.$$

5.3 Solution

The system of equations in (14) is the canonical rational expectations model and its solution has been considered in the literature e.g. Binder and Pesaran (1995, 1997), King and Watson (1998) and Sims (2002). Binder and Pesaran (1995, 1997) review the alternative solution strategies and show that the nature of the solution critically depends on the roots of the quadratic matrix equation

$$\mathbf{B}\boldsymbol{\Phi}^2 - \boldsymbol{\Phi} + \mathbf{A} = \mathbf{0}. \tag{15}$$

There will be a unique globally consistent stationary solution if (15) has a real matrix solution such that all the eigenvalues of $\boldsymbol{\Phi}$ and $(\mathbf{I} - \mathbf{B}\boldsymbol{\Phi})^{-1}\mathbf{B}$ lie strictly inside the unit circle. The solution is then given by

$$\boldsymbol{\chi}_t = \boldsymbol{\Phi}\boldsymbol{\chi}_{t-1} + \boldsymbol{\eta}_t. \tag{16}$$

Partitioning $\boldsymbol{\Phi}$ conformably to $\boldsymbol{\chi}_t$, (16) can be expressed as

$$\begin{pmatrix} \tilde{\mathring{\mathbf{x}}}_t \\ \tilde{\mathring{\mathbf{x}}}_{t-1} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Phi}_{11} & \boldsymbol{\Phi}_{12} \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix} \begin{pmatrix} \tilde{\mathring{\mathbf{x}}}_{t-1} \\ \tilde{\mathring{\mathbf{x}}}_{t-2} \end{pmatrix} + \begin{pmatrix} \mathbf{I}_k & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_k \end{pmatrix} \begin{pmatrix} \mathbf{u}_t \\ \mathbf{0} \end{pmatrix},$$

so that the solution in terms of $\tilde{\mathring{\mathbf{x}}}_t$, is given by

$$\tilde{\mathring{\mathbf{x}}}_t = \boldsymbol{\Phi}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1} + \boldsymbol{\Phi}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\boldsymbol{\varepsilon}_t, \tag{17}$$

where $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}'_{0t}, \boldsymbol{\varepsilon}'_{1t}, ..., \boldsymbol{\varepsilon}'_{Nt})'$.

The method used to solve (15) is based on an iterative back-substitution procedure which involves iterating on an initial arbitrary choice of $\mathbf{\Phi}$ and $\mathbf{\Psi}$, say $\mathbf{\Phi}_0$ and $\mathbf{\Psi}_0$, using the recursive relations

$$\mathbf{\Phi}_r = (\mathbf{I}_k - \mathbf{B}\mathbf{\Phi}_{r-1})^{-1}\mathbf{A}, \;\; \mathbf{\Psi}_r = (\mathbf{I}_k - \mathbf{B}\mathbf{\Phi}_{r-1})^{-1}\mathbf{B},$$

where $\mathbf{\Phi}_r$ and $\mathbf{\Psi}_r$ are the values of $\mathbf{\Phi}$ and $\mathbf{\Psi}$, respectively, at the r^{th} iteration ($r = 1, 2, ...$) and $\mathbf{\Psi}$ is the coefficient matrix in the forward equation

$$\mathbf{z}_t = \mathbf{\Psi} E_{t-1}(\mathbf{z}_{t+1}) + \mathbf{v}_t,$$

with

$$\mathbf{z}_t = \boldsymbol{\chi}_t - \mathbf{\Phi}\boldsymbol{\chi}_{t-1}, \;\; \mathbf{v}_t = (\mathbf{I}_k - \mathbf{B}\mathbf{\Phi})^{-1}\boldsymbol{\eta}_t.$$

See Binder and Pesaran (1995, 1997) for further details.

We set $\mathbf{\Phi}_0$ and $\mathbf{\Psi}_0$ to the identity matrix. As a check against multiple solutions, we also started the iterations with an initial value of $\mathbf{\Phi}$ and $\mathbf{\Psi}$ that had units along the diagonal and the off diagonal terms were drawn from a uniform distribution over the range -0.5 to +0.5. Both initial values resulted in the same solution. This iterative procedure is continued until one of the following convergence criteria is met

$$\|\mathbf{\Phi}_r - \mathbf{\Phi}_{r-1}\|_{\max} \leq 10^{-6} \text{ or } \|\mathbf{\Psi}_r - \mathbf{\Psi}_{r-1}\|_{\max} \leq 10^{-6},$$

where the max norm of a matrix $\mathbf{A} = \{a_{ij}\}$ is defined as $\|\mathbf{A}\|_{\max} = \max_{i,j}\{|a_{ij}|\}$.[5] In the numerical calculations all unknown parameters are replaced with the restricted IV estimates. Notice that given the size of the system we cannot provide an analytical expression for the conditions that ensure the RE model has a determinate solution. In the empirical application under consideration we did manage to obtain a determinate solution with the US model treated as a closed economy and once the theory-based inequality constraints were imposed.

In the MCNK model used as an illustration, the system is stable and in response to shocks the variables converge to their steady state values within 5 to 6 years in the vast majority of cases. Although there are only short lags in the system, no more than one period, and strongly forward looking behaviour in the Phillips curve, due to the inter-connections in the global model there is complicated dynamics and some slow adjustment to shocks. The largest eigenvalue of the system is 0.948. Many of the eigenvalues are complex, so adjustments often cycle back to zero. Inflation is a forward-looking variable in this model, so it jumps as expectations adjust to a shock, while interest rates respond strongly to inflation.

6 Covariance Matrix

When dealing with a large number (N) of countries, one needs to think differently about the nature of the shocks and their correlations. Following the closed economy literature, the Phillips curve

[5] Matlab and Gauss code for this procedure is available at http://ideas.repec.org/c/dge/qmrbcd/73.html.

error, $\varepsilon_{i,st}$, is interpreted as a supply shock, the IS curve error, $\varepsilon_{i,dt}$, as a demand shock, and the Taylor rule error, $\varepsilon_{i,mt}$, as a monetary policy shock. The estimates of the structural parameters can then be used to estimate $\varepsilon_{i,st}$, $\varepsilon_{i,dt}$ and $\varepsilon_{i,mt}$, respectively, for $i = 0, 1, ..., N$. As far as the cross correlations of the structural shocks are concerned we follow the literature and assume that these shocks are pair-wise orthogonal within each country, but allow for the shocks of the same type to be correlated across countries. In a multi-country context it does not seem plausible to assume that shocks of the same type are orthogonal across countries. Consider neighbouring economies with similar experiences of supply disruptions, or small economies that are affected by the same supply shocks originating from a dominant economy. As discussed in Chudik and Pesaran (2013), it is possible to deal with such effects explicitly by conditioning the individual country equations on the current and lagged variables of the dominant economy (if any), as well as on the variables of the neighbouring economies. This has been done partly in the specification of the IS equations. But following such a strategy more generally takes us away from the standard New Keynesian model and will not be pursued here. Instead we shall try to deal with such cross-country dependencies through suitably restricted error spillover effects.

We allow the exchange rate shocks, $\varepsilon_{i,et}$, defined by (6), to respond to all the other shocks through non-zero cross error correlations both within and across the countries. This provides a flexible way to allow the exchange rate to respond to all the shocks in the system. These assumptions yield the main case we consider: a block diagonal error covariance matrix which is bordered by non-zero covariances between $\varepsilon_{i,et}$ and $(\varepsilon_{i,st}, \varepsilon_{i,dt}, \varepsilon_{i,mt})$.

The structural shocks, $\boldsymbol{\varepsilon}_t$, can be recovered from (17) as:

$$\boldsymbol{\varepsilon}_t = \mathbf{H}_0(\widetilde{\tilde{\mathbf{x}}}_t - \boldsymbol{\Phi}_{11}\widetilde{\tilde{\mathbf{x}}}_{t-1} - \boldsymbol{\Phi}_{12}\widetilde{\tilde{\mathbf{x}}}_{t-2}). \tag{18}$$

To represent the cross-country correlations, we reorder the elements of $\boldsymbol{\varepsilon}_t$ in (18) to put shocks of the same type together as $\boldsymbol{\varepsilon}_t^0 = (\boldsymbol{\varepsilon}_{st}', \boldsymbol{\varepsilon}_{dt}', \boldsymbol{\varepsilon}_{mt}', \boldsymbol{\varepsilon}_{et}')'$, where $\boldsymbol{\varepsilon}_{st}$ and $\boldsymbol{\varepsilon}_{dt}$ are the $(N+1) \times 1$ vectors of supply and demand shocks, and $\boldsymbol{\varepsilon}_{mt}$ and $\boldsymbol{\varepsilon}_{et}$ are the $N \times 1$ vectors of monetary policy shocks (for all countries except Saudi Arabia) and shocks to the real effective exchange rates (for all countries except the US). We can then write

$$\boldsymbol{\varepsilon}_t^0 = \mathbf{G}\boldsymbol{\varepsilon}_t, \tag{19}$$

where $\mathbf{G}$ is a non-singular $k \times k$ matrix with elements 0 or 1. Also $E(\boldsymbol{\varepsilon}_t^0 \boldsymbol{\varepsilon}_t^{0\prime}) = \boldsymbol{\Sigma}_\varepsilon^0 = \mathbf{G}\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}\mathbf{G}'$, where $\boldsymbol{\Sigma}_\varepsilon = E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}_t')$.

The covariance of demand and supply shocks are given by the $(N+1) \times (N+1)$ dimensional matrices $\boldsymbol{\Sigma}_{ss}$ and $\boldsymbol{\Sigma}_{dd}$, and the covariance matrices of the monetary policy shocks and exchange rate shocks are given by the $N \times N$ matrices $\boldsymbol{\Sigma}_{mm}$ and $\boldsymbol{\Sigma}_{ee}$. The covariances between the exchange rate shocks and the structural shocks are given by $\boldsymbol{\Sigma}_{es}$, etc. These assumptions yield a block diagonal error covariance matrix which is bordered by non-zero covariances between the exchange rate shocks and the other shocks, so that $\mathbf{\Sigma}_{\varepsilon}^{0}$ has the form:

$$\mathbf{\Sigma}_{\varepsilon}^{0} = \begin{pmatrix} \mathbf{\Sigma}_{ss} & \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_{se} \\ \mathbf{0} & \mathbf{\Sigma}_{dd} & \mathbf{0} & \mathbf{\Sigma}_{de} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_{mm} & \mathbf{\Sigma}_{me} \\ \mathbf{\Sigma}_{es} & \mathbf{\Sigma}_{ed} & \mathbf{\Sigma}_{em} & \mathbf{\Sigma}_{ee} \end{pmatrix}. \tag{20}$$

The multi-country NK model is solved for all time periods in our estimation sample, and allows us to obtain estimates of all the structural shocks in the model. Altogether there are 130 different shocks; 98 structural and 32 reduced form, plus the oil price shock. Denote the shock of type $k = s, d, m, e$ in country $i = 1, 2, ..., 33$ at time t =1980Q3-2011Q2 by $\varepsilon_{i,k,t}$. It is now possible to compute pair-wise correlations of any pair of shocks both within and across countries. In Table 3 we provide averages of pair-wise correlations across the four types of shocks, computed by averaging, for instance, over the $(33 \times 32)/2 = 528$ pairs of correlation coefficients from the 33 supply shocks, and the $(33 \times 34)/2 = 561$ pairs of supply-demand shocks.

Table 3: Average pair-wise correlations of shocks using GVAR deviations.

	Supply	Demand	Mon. Pol.	Ex. Rate
Supply	0.444	0.158	0.056	0.018
Demand		0.079	0.073	-0.004
Mon. Pol.			0.126	-0.006
Ex. Rate				0.014

The largest average correlations are among the supply shocks, at 0.444; the other correlations are all less than 0.16. By comparison, the average pair-wise correlations of shocks of different types (given as the off-diagonal elements in Table 3) are small, with the largest figure given by the average correlation of demand and supply shocks given by 0.158. The other average correlations across the different types of shocks are small. This is in line with our maintained identifying assumption that supply, demand and monetary policy shocks are orthogonal. These correlations are very similar to those obtained using the pre-crisis sample up to 2006Q4. For instance the correlation of supply shocks for the shorter sample was 0.495; the correlation of demand and supply shocks was 0.166; and the correlation of monetary policy shocks 0.139. As with Table 3 all other correlations were less than 0.1 in absolute value.

Since the US is treated as a closed economy and the only influence of the rest of the world on the US is through the error covariances, it is interesting to consider the average correlations of the US with the other countries. These are shown in Table 4. Broadly the patterns are similar to those in Table 3, though the correlations tend to be a little higher, perhaps reflecting the omission of the foreign variables from the US IS curve.

Table 4: Average correlations of US with other countries shocks using GVAR deviations.

	Supply	Demand	Mon. Pol.	Ex. Rate
Supply	0.527	0.209	0.073	0.005
Demand		0.088	0.127	-0.065
Mon. Pol.			0.224	-0.045
Ex. Rate				-

Consider now the problem of consistent estimation of the covariance matrix of shocks defined by (20). One possibility would be to estimate the non-zero blocks $\mathbf{\Sigma}_{kl}$, $k, l = s, d, m, e$ with the sample covariance matrix using the estimates of $\boldsymbol{\varepsilon}_t$, defined by (18) and denoted by $\hat{\boldsymbol{\varepsilon}}_t$. For instance, $\mathbf{\Sigma}_{ss}$ can be estimated by $\sum_{t=1}^{T} \hat{\boldsymbol{\varepsilon}}_{st}\hat{\boldsymbol{\varepsilon}}'_{st}/T$. These estimates of the component matrices can then be inserted in (20) to provide an estimate of $\mathbf{\Sigma}^0_{\varepsilon}$, say $\hat{\mathbf{\Sigma}}^0_{\varepsilon}$. However, since the dimension of the variables, $k = 131$, is larger than the time series dimension, $T = 125$, $\hat{\mathbf{\Sigma}}^0_{\varepsilon}$ is not guaranteed to be a positive definite matrix. While the estimates of the individual correlations are consistent, the estimate of the whole matrix is not when $T < N$. This is an important consideration when we come to compute bootstrapped error bands for the impulse response functions. The same issue arises in other contexts including mean-variance portfolio optimisation where the number of assets is large.

A number of solutions have been suggested in the literature. Ledoit and Wolf (2004) consider an estimator which is a convex linear combination of the unrestricted sample covariance matrix and an identity matrix and provide an estimator for the weights. Friedman, Hastie and Tibshirani (2008) apply the lasso penalty to loadings in principal component analysis to achieve a sparse representation. Fan, Fan and Lv (2008) use a factor model to impose sparsity on the covariance matrix. Bickel and Levina (2008) propose thresholding the sample covariance matrix, where the threshold parameter is chosen using cross validation.

We use a simple shrinkage estimator of the covariance matrix given by

$$\hat{\mathbf{\Sigma}}^0_{\varepsilon}(\varrho) = (1-\varrho)diag(\hat{\mathbf{\Sigma}}^0_{\varepsilon}) + \varrho\hat{\mathbf{\Sigma}}^0_{\varepsilon}. \tag{21}$$

The diagonal matrix, $diag(\hat{\mathbf{\Sigma}}^0_{\varepsilon})$, which has $(\hat{\boldsymbol{\sigma}}^2_{1,ss}, ..., \hat{\boldsymbol{\sigma}}^2_{N+1,ss}, \hat{\boldsymbol{\sigma}}^2_{1,dd}, ..., \hat{\boldsymbol{\sigma}}^2_{N,ee})'$ on the diagonal and zeros elsewhere, is certainly positive definite. One can then use a convex combination of $\hat{\mathbf{\Sigma}}^0_{\varepsilon}$ and $diag(\hat{\mathbf{\Sigma}}^0_{\varepsilon})$, which shrinks the sample covariance matrix towards its diagonal, in order to obtain a positive definite matrix. We found that $\hat{\mathbf{\Sigma}}^0_{\varepsilon}(\varrho)$ is positive definite for all values of $\varrho \leq 0.6$. Accordingly, the initial estimates of the IRFs and FEVDs are based on the shrinkage covariance matrix, $\hat{\mathbf{\Sigma}}^0_{\varepsilon}(0.6)$. We then examine the sensitivity of the IRFs to the choice of covariance matrix. Since calculation of generalised IRFs does not require the covariance matrix to be positive definite we can compare the IRFs from the shrinkage covariance matrix, $\hat{\mathbf{\Sigma}}^0_{\varepsilon}(0.6)$ with the IRFs from $\hat{\mathbf{\Sigma}}^0_{\varepsilon}$, as well as the diagonal covariance matrix, $diag(\hat{\mathbf{\Sigma}}^0_{\varepsilon})$, and a block-diagonal covariance, $Bdiag(\hat{\mathbf{\Sigma}}^0_{\varepsilon})$ matrix, which sets the covariances between the exchange rate shocks and the structural shocks in $\hat{\mathbf{\Sigma}}^0_{\varepsilon}$, to zero.

7 Shocks

7.1 Impulse response functions

Impulse response functions provide counter-factual answers to questions concerning either the effects of a particular shock in a given economy, or the effects of a combined shock involving linear combinations of shocks across two or more economies. The effects of the shock can also be computed either on a particular variable in the global economy or on a combination of variables. Denote a composite shock, defined as a linear combination of the shocks, by $\xi_t = \mathbf{a}'\boldsymbol{\varepsilon}_t^0$, and consider the time profile of its effects on a composite variable $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$. The $k \times 1$ vector $\mathbf{a}$ and the $(k+1) \times 1$ vector $\mathbf{b}$ are either appropriate selection vectors picking out a particular error or variable or a suitable weighted average. The error weights, $\mathbf{a}$, can be chosen to define composite shocks, such as a global supply shock; the variable weights, $\mathbf{b}$, to define composite variables such as the real effective exchange rate or a PPP GDP weighted average of the countries in the euro area. The IRFs estimate the time profile of the response by $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$ to a unit shock (defined as one standard error shock of size $\sigma_\xi = \sqrt{\mathbf{a}'\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}$) to $\xi_t = \mathbf{a}'\boldsymbol{\varepsilon}_t^0$, and the FEVDs estimate the relative importance of different shocks in explaining the variations in output, inflation and interest rates from their steady states in a particular economy over time.

Using (17) and (19), we obtain

$$\tilde{\mathring{\mathbf{x}}}_t = \boldsymbol{\Phi}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1} + \boldsymbol{\Phi}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\mathbf{G}^{-1}\boldsymbol{\varepsilon}_t^0, \tag{22}$$

and the time profile of $\tilde{\mathring{\mathbf{x}}}_{t+n}$ in terms of current and lagged shocks can be written as

$$\tilde{\mathring{\mathbf{x}}}_{t+n} = \mathbf{D}_{n1}\tilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{D}_{n2}\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{C}_n\boldsymbol{\varepsilon}_t^0 + \mathbf{C}_{n-1}\boldsymbol{\varepsilon}_{t+1}^0 + ... + \mathbf{C}_1\boldsymbol{\varepsilon}_{t+n-1}^0 + \mathbf{C}_0\boldsymbol{\varepsilon}_{t+n}^0, \tag{23}$$

where $\mathbf{D}_{n1}$ and $\mathbf{D}_{n2}$ are functions of $\boldsymbol{\Phi}_{11}$ and $\boldsymbol{\Phi}_{12}$, $\mathbf{C}_j = \mathbf{P}_j\mathbf{H}_0^{-1}\mathbf{G}^{-1}$, and $\mathbf{P}_j$ can be derived recursively as

$$\mathbf{P}_j = \boldsymbol{\Phi}_{11}\mathbf{P}_{j-1} + \boldsymbol{\Phi}_{12}\mathbf{P}_{j-2},\ \mathbf{P}_0 = \mathbf{I}_k,\ \mathbf{P}_j = \mathbf{0}, \text{ for } j < 0.$$

Similarly, using (7) and (22), we have

$$\widetilde{\mathbf{x}}_{t+n} = \mathring{\mathbf{D}}_{n1}\tilde{\mathring{\mathbf{x}}}_{t-1} + \mathring{\mathbf{D}}_{n2}\tilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{B}_n\boldsymbol{\varepsilon}_t^0 + \mathbf{B}_{n-1}\boldsymbol{\varepsilon}_{t+1}^0 + ... + \mathbf{B}_1\boldsymbol{\varepsilon}_{t+n-1}^0 + \mathbf{B}_0\boldsymbol{\varepsilon}_{t+n}^0, \tag{24}$$

where

$$\mathring{\mathbf{D}}_{n1} = \mathbf{S}_0\mathbf{D}_{n1} - \mathbf{S}_1\mathbf{D}_{n-1,1},\quad \mathring{\mathbf{D}}_{n2} = \mathbf{S}_0\mathbf{D}_{n2} - \mathbf{S}_1\mathbf{D}_{n-1,2},$$

and

$$\mathbf{B}_0 = \mathbf{S}_0\mathbf{C}_0 \text{ and } \mathbf{B}_\ell = \mathbf{S}_0\mathbf{C}_\ell - \mathbf{S}_1\mathbf{C}_{\ell-1}, \text{ for } \ell = 1, 2, ..., n.$$

Notice that $\mathbf{B}_\ell$, $\ell = 0, 1, 2, ..., n$ are $(k+1) \times k$, dimensional matrices that transmit the effects of the k shocks in the system to the $(k+1)$ elements of $\widetilde{\mathbf{x}}_{t+n}$ that include both the US price level and the US inflation. Clearly, both representations (23) and (24) can be used to carry out the impulse response analysis. But it is more convenient to use (24) when considering the effects of shocks on US inflation.

The generalized impulse response function for the effect on $q_t = \mathbf{b}'\widetilde{\mathbf{x}}_t$ of a one standard error shock to $\xi_t = \mathbf{a}'\boldsymbol{\varepsilon}_t^0$ is then

$$\begin{aligned} g_q(n, \sigma_\xi) &= E(q_{t+n} \mid \xi_t = \sigma_\xi = \sqrt{\mathbf{a}'\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}, \Im_{t-1}) - E(\mathbf{b}'\widetilde{\mathbf{x}}_{t+n} \mid \Im_{t-1}) \\ &= \frac{\mathbf{b}'\mathbf{B}_n\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}{\sqrt{\mathbf{a}'\boldsymbol{\Sigma}_\varepsilon^0\mathbf{a}}},\ n = 0, 1, 2, \dots . \end{aligned} \tag{25}$$

While the IRFs of, say, supply shocks as a group can be identified because they are assumed to be orthogonal to demand and monetary policy shocks, the supply shock in any particular country cannot be identified, because they are correlated with the supply shocks in other countries. The issue of how to identify country-specific demand or supply shocks in a multi-country setting is beyond the scope of the present paper. Results are presented for a global supply shock, which uses $\mathbf{a}_s$, which has PPP GDP weights that add to one, corresponding to the supply shocks of each of the $N+1$ countries and zeros elsewhere, and a global demand shock which uses $\mathbf{a}_d$, which also has PPP GDP weights that add to one.

7.2 Demand and supply shocks

It is conventional to consider the effect of US monetary policy shocks. However, in the current economic environment, where the interest rates are at or approaching the zero lower bound, this focus could be of limited interest at best and might be even misleading.[6] However, this criticism does not apply to the analysis of the effects of global demand and supply shocks, to which we now turn.

We begin with the impulse response functions for a one standard error positive (global) supply shock to inflation. As noted above this uses $\mathbf{a}_s$, which has PPP GDP weights that add to one, corresponding to the supply shocks of each of the $N+1$ countries and zeros elsewhere. These are graphed for 26 countries, excluding the five Latin American countries (Argentina, Brazil, Chile, Mexico, Peru), Indonesia and Turkey. These countries tend to be outliers due to the much higher levels of inflation and nominal interest rates experienced in these economies over our estimation sample, though they show the same qualitative patterns in their impulse response functions. Also to focus on the differences across countries, the graphs only show the point estimates. Bootstrapped confidence bounds will be considered later. Figure 1a gives the impulse response function for a positive one standard error supply shock on inflation. Figure 1b gives the effects on output. Figure 1c gives the effects on interest rates. In response to the supply shock inflation increases sharply, but then falls back to zero very quickly, the pattern being almost identical across countries. Output falls, as one would expect with a (negative) supply shock, slowly returning to zero. Interest rates rise immediately to offset the increase in inflation, then fall below zero to offset the reduction in output. Although the responses are qualitatively similar across countries the effects on output and interest rates are more dispersed than on inflation reflecting the cross-country differences in Taylor rules and IS curves.

[6] We are grateful to a referee for making this point.

Figure 1a: Impulse response of a negative supply shock on inflation (per cent per quarter)

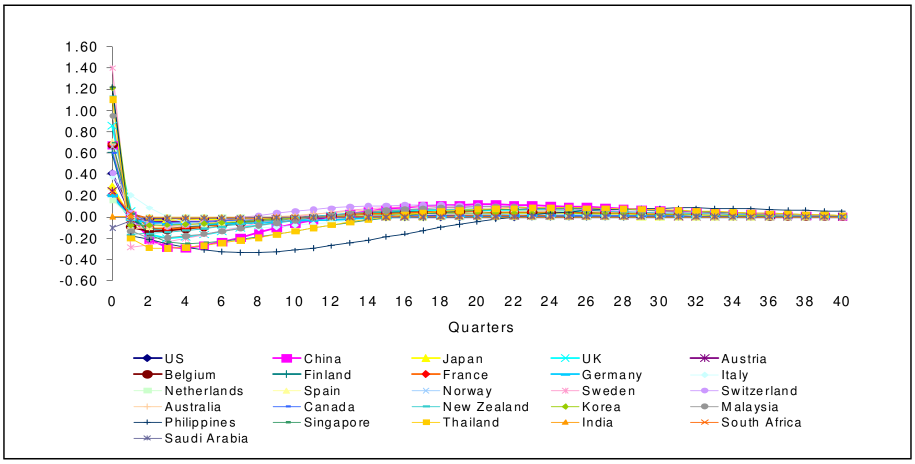


Figure 1b: Impulse responses of a one standard error supply shock on output (per cent per quarter)

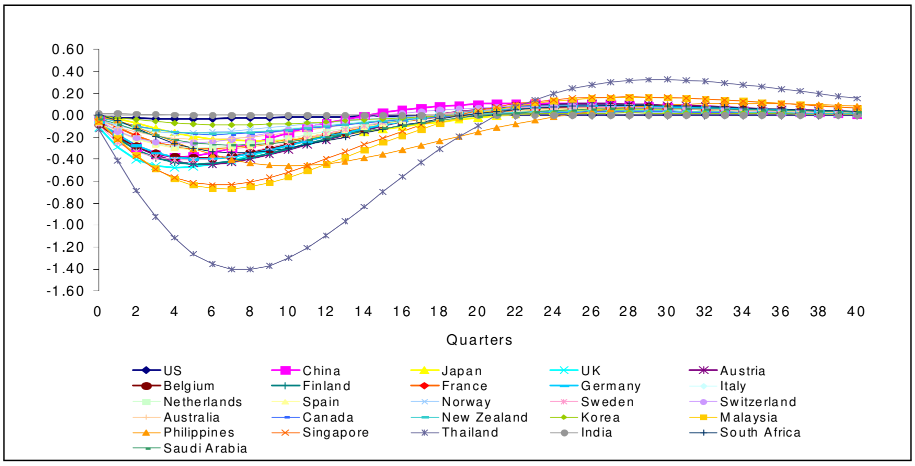
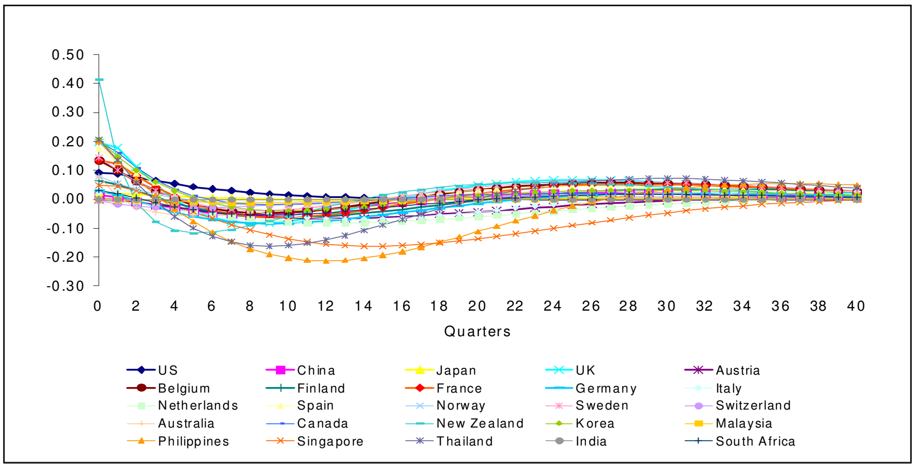


Figure 1c: Impulse responses of a one standard error supply shock on interest rates (per cent per quarter)



We now turn to a one standard error positive demand shock on output. These use $\mathbf{a}_d$, which has PPP GDP weights that add to one, corresponding to the demand shocks to output of each of the $N+1$ countries and zeros elsewhere. Figure 2a gives the impulse response function for a demand shock on output for 26 countries. Figure 2b gives the effect on inflation. Figure 2c gives the effect on interest rates. In response to the demand shock, output increases, as one would expect, returning to zero in the medium run. Inflation increases in the short run, goes negative in the medium run, returning to zero in the long run. Interest rates increase to offset the higher inflation and output.

Figure 2a: Impulse responses of a demand shock on output (per cent per quarter)

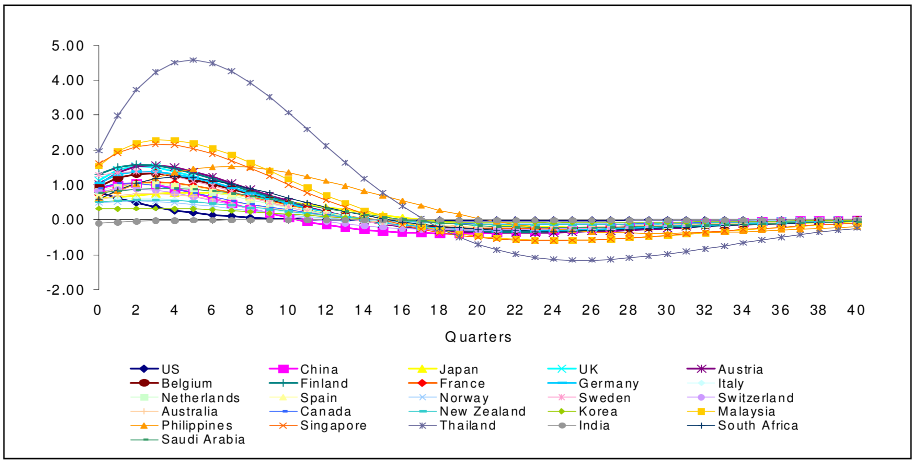


Figure 2b: Impulse responses of a demand shock on inflation (per cent per quarter)

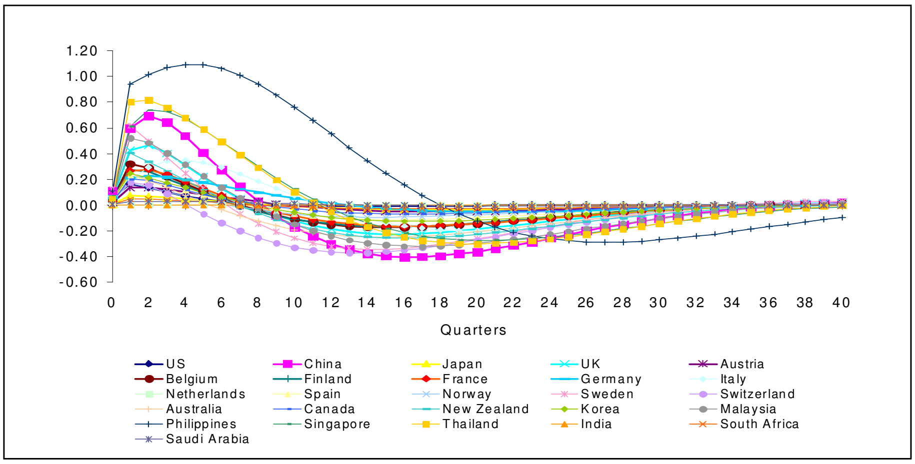
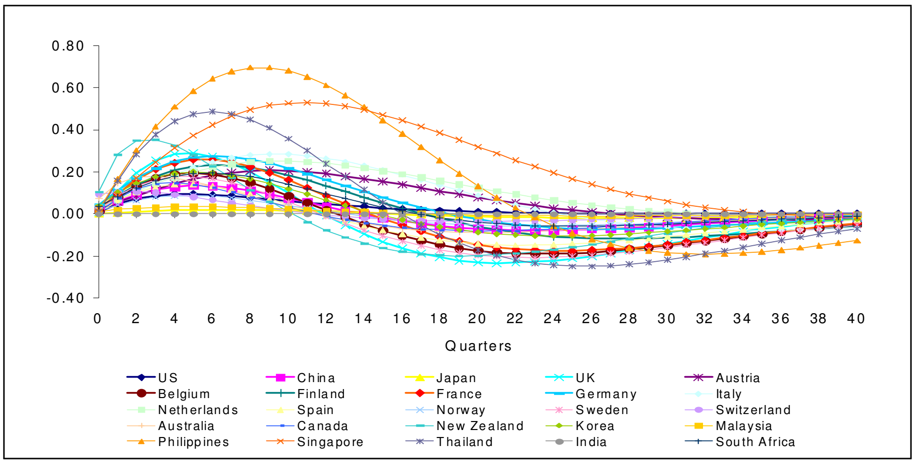


Figure 2c: Impulse responses of a demand shock on interest rates (per cent per quarter)



The demand and supply shocks show the features one would expect. The negative supply shocks increase inflation and reduce output, the positive demand shocks increase both output and inflation. In both cases interest rates rise initially to offset the higher inflation, but the rise is much bigger and more prolonged in the case of demand shocks.

7.3 Computation of bootstrap error bands

To get the bootstrap errors bands for the MCNK B bootstrap samples are generated denoted by $\tilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ from the process

$$\tilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{H}}_0^{-1}\hat{\boldsymbol{\varepsilon}}_t^{(b)},\ t = 1, 2, ..., T, \tag{26}$$

by resampling the structural residuals, $\hat{\varepsilon}_t$, and setting $\tilde{\mathring{\mathbf{x}}}_0^{(b)} = \tilde{\mathring{\mathbf{x}}}_0$ and $\tilde{\mathring{\mathbf{x}}}_{-1}^{(b)} = \tilde{\mathring{\mathbf{x}}}_{-1}$, where $\tilde{\mathring{\mathbf{x}}}_0$ and $\tilde{\mathring{\mathbf{x}}}_{-1}$ are the observed initial data vectors that include the US real exchange rate (or equivalently the US price level). Recall that the multi-country rational expectations model is solved in terms of the US price level rather than the US inflation.

The structural shocks, $\hat{\varepsilon}_t$, are initially orthogonalised by using the inverse of the Cholesky factor, $\widetilde{\mathbf{P}}$, associated with the Cholesky decomposition of the shrinkage covariance matrix, $\hat{\Sigma}_\varepsilon(0.6)$. This way we obtain the $k \times 1$ orthogonal vector $\hat{\upsilon}_t = \widetilde{\mathbf{P}}^{-1}\hat{\varepsilon}_t$ where its j^{th} element $\hat{\upsilon}_{jt}$, $j = 1, 2, ..., k$, has unit variance. The bootstrap error vector is then obtained as $\varepsilon_t^{(b)} = \widetilde{\mathbf{P}}\hat{\upsilon}_t^{(b)}$, where $\hat{\upsilon}_t^{(b)}$ is the $k \times 1$ vector of re-sampled values from $\{\hat{\upsilon}_{jt}\}_{j=1,2,...,k;t=1,2,...,T}$. Prior to any resampling the structural residuals are recentered to ensure that their bootstrap population mean is zero.

Once a set of $\tilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ are generated, US inflation is computed from the US price level so that $\tilde{\mathbf{x}}_{it}^{(b)}$ is constructed, with the corresponding foreign variables, $\tilde{\mathbf{x}}_{it}^{*(b)}$, computed using the trade weights. For each bootstrap replication the individual country models are then estimated by the inequality constrained IV procedure, ensuring that any constraint which binds for the estimates based on historical realisations are also imposed on the bootstrap estimates.

The country specific models in terms of $\tilde{\mathbf{x}}_{it}^{(b)}$ are given by

$$\hat{\mathbf{A}}_{i0}^{(b)}\tilde{\mathbf{x}}_{it}^{(b)} = \hat{\mathbf{A}}_{i1}^{(b)}\tilde{\mathbf{x}}_{i,t-1}^{(b)} + \hat{\mathbf{A}}_{i2}^{(b)}E_{t-1}(\tilde{\mathbf{x}}_{i,t+1}^{(b)}) + \hat{\mathbf{A}}_{i3}^{(b)}\tilde{\mathbf{x}}_{it}^{*(b)} + \hat{\mathbf{A}}_{i4}^{(b)}\tilde{\mathbf{x}}_{i,t-1}^{*(b)} + \boldsymbol{\varepsilon}_t^{(b)},$$

and are subsequently combined yielding the MCNK model

$$\tilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{F}}_1^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{F}}_2^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{F}}_3^{(b)}E_{t-1}(\tilde{\mathring{\mathbf{x}}}_{t+1}^{(b)}) + \hat{\mathbf{F}}_4^{(b)}E_{t-1}(\tilde{\mathring{\mathbf{x}}}_t^{(b)}) + \mathbf{u}_t^{(b)}. \tag{27}$$

Solving the quadratic matrix as described earlier, the reduced form solution of (27) follows as

$$\tilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \mathbf{u}_t^{(b)},$$

with

$$\hat{\mathbf{u}}_t^{(b)} = \tilde{\mathring{\mathbf{x}}}_t^{(b)} - \hat{\mathbf{\Phi}}_{11}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} - \hat{\mathbf{\Phi}}_{12}^{(b)}\tilde{\mathring{\mathbf{x}}}_{t-2}^{(b)}$$

and

$$\hat{\boldsymbol{\varepsilon}}_t^{(b)} = \hat{\mathbf{H}}_0^{(b)}\hat{\mathbf{u}}_t^{(b)}.$$

For the first bootstrap replication we begin the iterative back-substitution procedure, using the estimated $\hat{\Phi}$ from the actual data as an initial value to compute (26) and (27), so that for $b = 1$, $\Phi_0^{(1)} = \hat{\Phi}$. For each subsequent bootstrap replication, b, the initial value is set to the solution of (15) obtained under the preceding replication, $b-1$, so that $\Phi_0^{(b)} = \hat{\Phi}^{(b-1)}$ and $\Psi_0^{(b)} = (I_k - \hat{B}^{(b)}\hat{\Phi}^{(b-1)})^{-1}\hat{B}^{(b)}$. If for a particular bootstrap replication the iterative back-substitution procedure fails to converge after 500 iterations, the initial values for $\Phi_0^{(b)}$ and $\Psi_0^{(b)}$ are set to the identity matrix.

For each bootstrap replication $b = 1, 2, ..., B$, having estimated the individual country NK models using the simulated data $\widetilde{\tilde{\mathbf{x}}}_t^{(b)}$, the MCNK model is reconstructed as described above and the impulse responses are calculated $g^{(b)}(n)$, for $n = 0, 1, 2, ...$ These statistics are then sorted in ascending order, and the $(1-\alpha)100\%$ confidence interval is calculated by using the $\alpha/2$ and $(1-\alpha/2)$ quantiles, say $q_{\alpha/2}$ and $q_{(1-\alpha/2)}$, respectively of the bootstrap distribution of $g(n)$.

To compute the upper and lower confidence bounds we use 2000 convergent and stationary bootstrap replications. A convergent replication is defined as one where for the corresponding bootstrap sample, the iterative back-substitution procedure described above converges within 500 iterations, whether the initial values for $\Phi_0^{(b)}$ and $\Psi_0^{(b)}$ are set to the identity matrix or otherwise. Having achieved convergence, a bootstrap replication is checked to make sure that it yields a stationary solution. If any of the above two conditions is violated, a new bootstrap sample is computed. For our bootstrap results we had to carry out a total of 3475 bootstrap replications, of which 1475 where due to non-convergence of the iterative back-substitution procedure. No bootstrap replications were found to be non-stationary.

The bootstrapped 90% error bands for the impulse responses for the effects of global supply and demand shocks on the US and euro area inflation, output, and interest rates are displayed in Figures 3a and 3b. As above, the euro area impulse responses are obtained by averaging over the impulse responses of member countries using PPP GDP weights. These figures show the median (which is almost identical to the mean except for India, not shown) and the 5% and 95% quantiles of the bootstrap distribution. The results indicate that the effects of the shocks are statistically significant in the sense that the 90% bootstrap bands do not always cover zero, though in some cases the lower bounds are at zero for a number of periods. Results for other IRFs are similar.

Figure 3a: Impulse responses of a one standard error global supply shock on US and euro area inflation, output and interest rates (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)

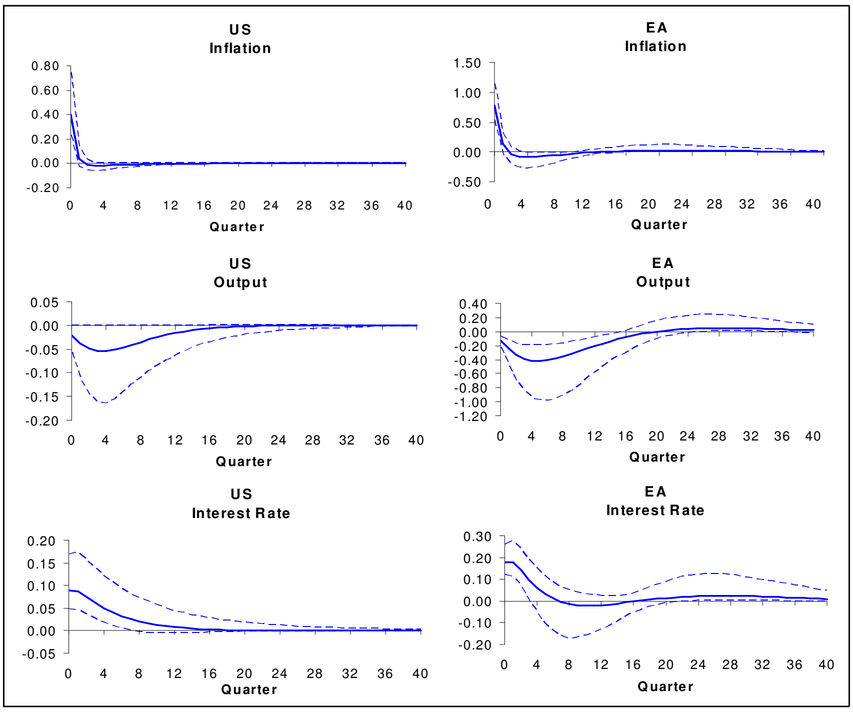
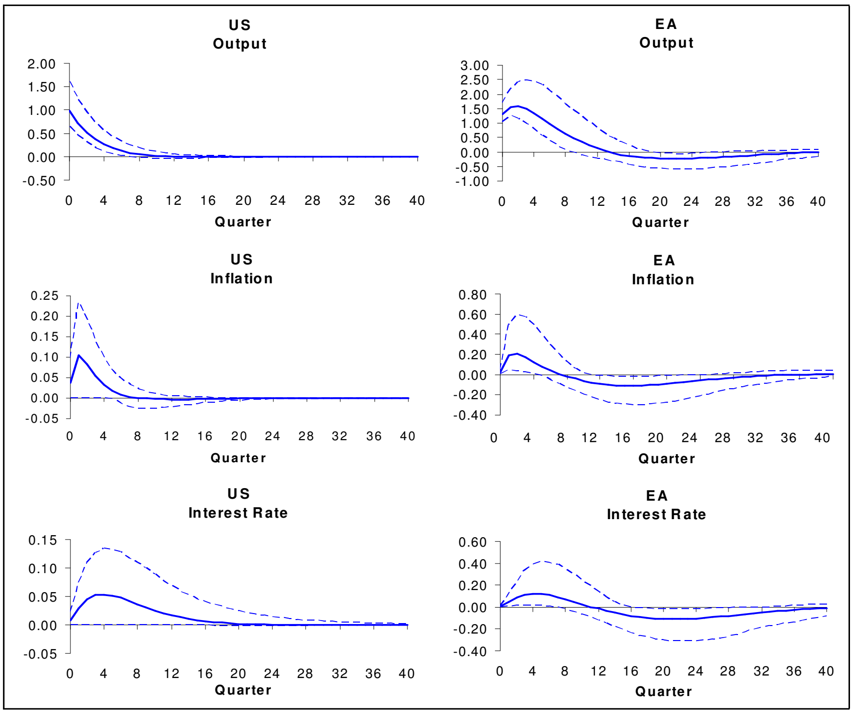


Figure 3b: Impulse responses of a one standard error global demand shock on US and euro area output, inflation and interest rates (per cent per quarter, bootstrap median estimates together with 90% bootstrap bands)



7.4 Forecast error variance decomposition

FEVDs are used to estimate the relative importance of different types of shocks in explaining the forecast error variance of different variables in the world economy. Such a decomposition can be achieved without having to specify the nature or sources of the cross-country correlations of supply or demand shocks. Additional identifying assumptions will be needed if we also wish to identify the relative importance of country-specific supply shocks, but as noted above such an exercise is beyond the scope of the present paper.

For the FEVD of global shocks we partition $\mathbf{B}_\ell = \begin{pmatrix} \mathbf{B}_{s\ell}, & \mathbf{B}_{d\ell}, & \mathbf{B}_{m\ell}, & \mathbf{B}_{e\ell} \end{pmatrix}$ in (24) conformably with the partitioning of $\boldsymbol{\varepsilon}_t^0 = (\boldsymbol{\varepsilon}_{st}', \boldsymbol{\varepsilon}_{dt}', \boldsymbol{\varepsilon}_{mt}', \boldsymbol{\varepsilon}_{et}')'$, and note that the n step ahead forecast errors can be written as

$$\tilde{\boldsymbol{\upsilon}}_{t+n} = \widetilde{\mathbf{x}}_{t+n} - E\left(\widetilde{\mathbf{x}}_{t+n} \left| \Im_{t-1} \right.\right) = \sum_{j \in shocks} \sum_{\ell=0}^{n} \mathbf{B}_{j,n-\ell} \boldsymbol{\varepsilon}_{j,t+\ell}. \tag{28}$$

Under the assumption that within country supply, demand and monetary policy shocks are orthogonal we have

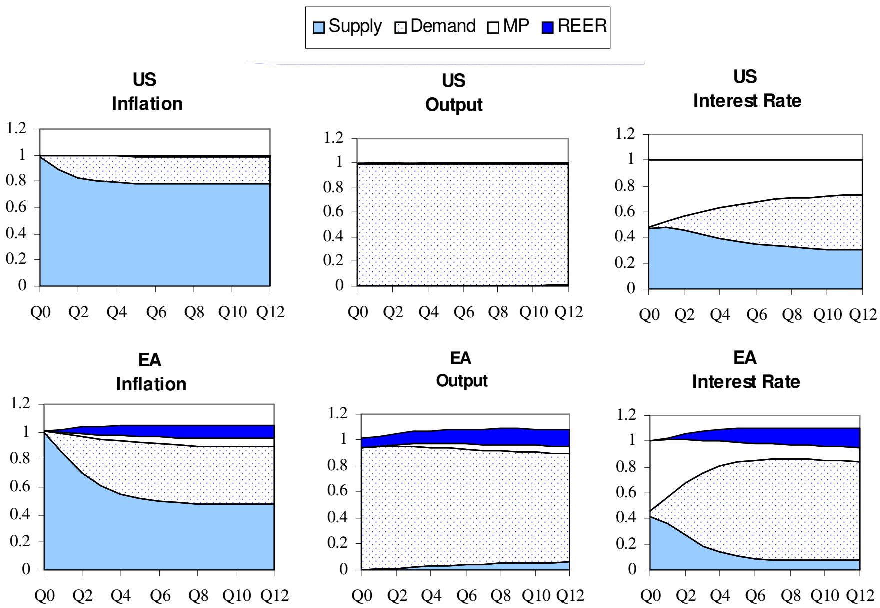
$$\begin{aligned} Var\left(\tilde{\boldsymbol{\upsilon}}_{t+n} \left| \Im_{t-1} \right.\right) \quad = \quad & \sum_{\ell=0}^{n} \mathbf{B}_{s,n-\ell} \boldsymbol{\Sigma}_{ss} \mathbf{B}_{s,n-\ell}' + \sum_{\ell=0}^{n} \mathbf{B}_{d,n-\ell} \boldsymbol{\Sigma}_{dd} \mathbf{B}_{d,n-\ell}' \\ & + \sum_{\ell=0}^{n} \mathbf{B}_{m,n-\ell} \boldsymbol{\Sigma}_{mm} \mathbf{B}_{m,n-\ell}' + \sum_{\ell=0}^{n} \mathbf{B}_{e,n-\ell} \boldsymbol{\Sigma}_{ee} \mathbf{B}_{e,n-\ell}' \\ & + \sum_{\ell=0}^{n} \mathbf{B}_{s,n-\ell} \boldsymbol{\Sigma}_{se} \mathbf{B}_{e,n-\ell}' + \sum_{\ell=0}^{n} \mathbf{B}_{e,n-\ell} \boldsymbol{\Sigma}_{es} \mathbf{B}_{s,n-\ell}' \\ & + \sum_{\ell=0}^{n} \mathbf{B}_{d,n-\ell} \boldsymbol{\Sigma}_{de} \mathbf{B}_{e,n-\ell}' + \sum_{\ell=0}^{n} \mathbf{B}_{e,n-\ell} \boldsymbol{\Sigma}_{ed} \mathbf{B}_{d,n-\ell}' \\ & + \sum_{\ell=0}^{n} \mathbf{B}_{m,n-\ell} \boldsymbol{\Sigma}_{me} \mathbf{B}_{e,n-\ell}' + \sum_{\ell=0}^{n} \mathbf{B}_{e,n-\ell} \boldsymbol{\Sigma}_{em} \mathbf{B}_{m,n-\ell}'. \end{aligned}$$

The first four terms give the contributions to the variance from each of the four shocks; the following six terms arise from the covariances between the exchange rate shocks and the three structural shocks. Using the above FEVD, one can then estimate the importance of supply shocks, demand shocks or monetary policy shocks in the world economy for the explanations of output growth, inflation, interest rates and real effective exchange rates, either for individual variables or any given linear combinations of the variables. These proportions will not add up to unity, due to the error spillover effects between the real effective exchange rates and the three structural shocks. But as we shall show below, due to the relatively small magnitudes of the covariance terms between the real exchange rates and the structural shocks, the proportion of forecast error variances explained by variances of the four shocks add to a number which is very close to unity.

Figure 4 shows the FEVDs for the US and the euro area up to 12 quarters. Because the US is modelled as a closed economy exchange rate shocks cannot account for any of the variation and the shocks add up to unity. Consider the decomposition after 24 quarters. For inflation, supply shocks account for 78% of the variation, demand 21% and monetary policy shocks 1%. For output, demand shocks account for 99% of the variation. Smets and Wouters (2007) also find that monetary policy shocks account for relatively little of the variations in output and inflation in the US. After 24 quarters supply shocks account for 29% of the variation of interest rates, demand shocks 45% and monetary policy shocks 26%.

The euro area estimates are obtained by averaging over the FEVDs of member countries using PPP GDP weights. Again consider the values of FEVDs after 24 quarters, and recall that the sum of FEVD's across shocks do not add up 100, since the underlying shocks across countries re not orthogonal. In our applications because of the positive nature of cross-country correlations the FEVDs add up to more than 100. For inflation the total is 107, made up of 39 due to supply shocks, 51 due to demand shocks, 6 due to monetary policy shocks, and 11 due to the exchange rate shocks. For output the total is 109, made up of 6 for supply, 84 for demand, 6 for monetary policy, and 13 for exchange rate shocks. For interest rates the total is 109, made up of 8 for supply, 75 for demand, 11 for monetary policy, and 15 for the exchange rate shocks. While there are differences across countries, in all cases supply and demand shocks account for most of the variations in output, inflation and interest rate in the long-run, with monetary policy shocks and exchange rate shocks accounting for relatively little of the total variations. Monetary policy shocks account for more of the variation in interest rates. On impact supply shocks account for nearly all the variation of inflation, but this drops rapidly and these shocks only account for about half of the variation of inflation in the long-run. Demand shocks account for most of the variations in output on impact, but again this figure drops quite rapidly.

Figure 4: Forecast error variance decomposition of the shocks in explaining inflation, output and interest rates for the US and the euro area



Note: Q0 refers to the values on impact.

8 Robustness

In this paper we have deviated from the empirical NK modelling literature in two important respects. First, we have estimated the steady states as long horizon expectations using an error correcting GVAR specification, as compared to using a purely statistical de-trending procedure. Second, we have allowed for international linkages across shocks and economies using a full multi-country NK model. In what follows we evaluate the importance of these innovations for our results. We also consider the sensitivity of the impulse responses to alternative specifications of the covariances of the structural shocks.

8.1 Measurement of steady states

As an alternative measure of steady states we considered the familiar Hodrick-Prescott (HP) filter for real output and following the literature assumed that the other variables, namely inflation, interest rate and the real exchange rate are stationary, and thus their steady states can be viewed as constants. We computed the HP filter of log real output using the smoothing parameter of 1600 for all countries. The output deviations based on the HP filter were then computed, which we denote by $\widetilde{y}_{it}^{HP}$, for $i = 0, 1, ..., N$. The country-specific NK models were then estimated by the IV procedure subject to the same theoretical restrictions as above, with an intercept included to allow for the assumed constant steady state values of the other three variables. The instruments used were an intercept, the lagged values of the country specific endogenous variables, $\widetilde{y}_{i,t-1}^{HP}$, $\pi_{i,t-1}, r_{i,t-1}, re_{i,t-1}$, the current values of the foreign variables $\widetilde{y}_{it}^{HP*}, \pi_{it}^{*}, r_{it}^{*}$,[7] and the first difference of the oil price variable, Δp_{it}^{o}. The results of these estimates are available in the supplement, with a summary of the main findings given in Table 5. The estimates are overall more backward looking than those obtained using GVAR deviations, with slower adjustments and near unit root autocorrelation coefficients for the real effective exchange rates. The effect of output deviations in the Phillips curve is smaller using the HP filter as compared to using the GVAR measures of the steady states - also documented in Dees et al. (2009). In the IS curve, in addition to larger estimated coefficients for the lagged variables and thus slower adjustments, domestic output deviations are less responsive to foreign output deviations when using $\widetilde{y}_{it}^{HP}$. This significantly reduces an important channel for the international transmission of shocks. The Taylor rule also adjusts more slowly, the average coefficient on the lagged interest rate is 0.8 as compared to 0.6 when using the GVAR deviations.[8]

[7] The foreign output variable based on the HP steady state values is computed as $\tilde{y}_{it}^{*HP} = \Sigma_{j=0}^{N} w_{ij} \tilde{y}_{jt}^{HP}$.

[8] We were unable to get the global MCNK model to solve when using deviations based on the HP filter. We only managed to solve the multi-country rational expectations model with HP-filtered deviations if we confined the estimation sample to the pre-crisis period (namely the subsample 1980Q1-2009Q4). Even in this case the impulse responses turned out to be very slow in returning to equilibrium, and in many cases even failing to reach their steady state values after 40 quarters. This suggests the HP-filtered deviations might still be non-stationary in the case of many variables in the global MCNK model.

Table 5: Distribution of inequality-constrained IV estimates using HP estimates of deviations from steady states: 1980Q3-2011Q2

	Mean	# Constrained	Constraint
Phillips curve - Equation (3), N=33			
β_{ib}	0.17	5	$\beta_{ib} \geq 0$
β_{if}	0.80	0	$\beta_{if} \geq 0$
β_{iy}	0.02	10	$\beta_{iy} \geq 0$
$\beta_{ib}+\beta_{if}$	0.97	24	$\beta_{ib}+\beta_{if} \leq 0.99$
IS curve - Equation (4), N=33			
α_{ib}	0.67		
α_{ir}	-0.20	3	$\alpha_{ir} \leq 0$
κ_{ir}	-0.69		
α_{ie}	-0.00		
κ_{ie}	-0.02		
α_{iy*}	0.39	2	$\alpha_{iy*} \geq 0$
κ_{iy*}	1.16		
Taylor Rule - Equation (5), N=32			
γ_{ib}	0.81		
$\gamma_{i\pi}$	0.16	0	$\gamma_{i\pi} \geq 0$
$\mu_{i\pi}$	0.89		
γ_{iy}	0.05	4	$\gamma_{iy} \geq 0$
μ_{iy}	0.51		
Exchange rates - Equation (6), N=32			
ρ_i	0.95		$\lvert\rho_i\rvert < 1$

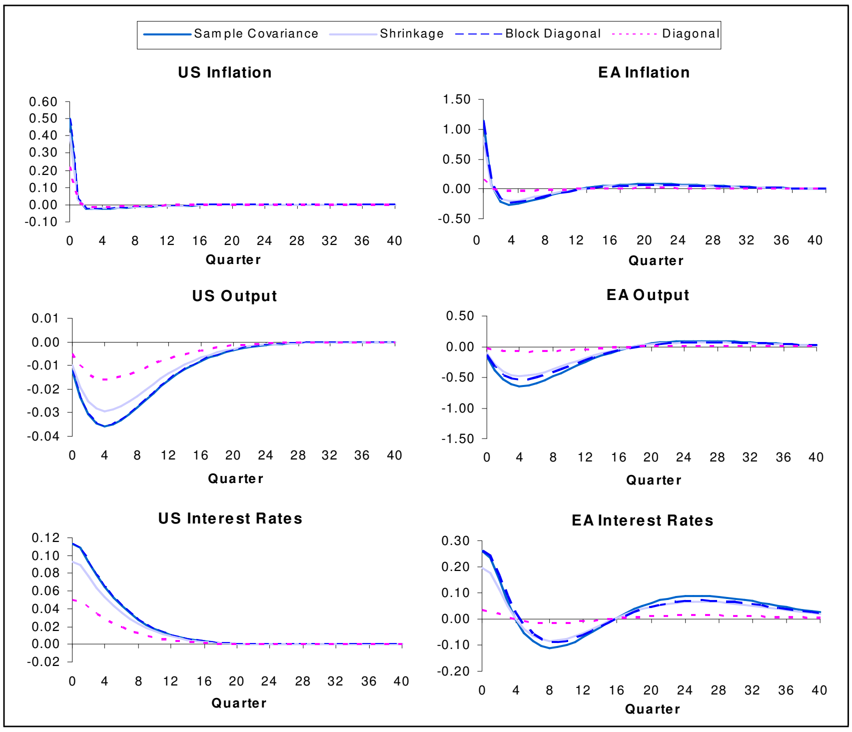
Note: The estimation period begins in t=1980Q3 and ends in 2011Q1 for PC and IS, and 2011Q2 for TR and ER. An exception is the Phillips curve in Argentina which is estimated over the sub-sample beginning in 1990Q1. N is the number of countries for which the equations are estimated. The column headed "Mean" gives the average over all estimates, constrained and unconstrained. The column headed "# Constrained" gives the number of estimates constrained at the boundary. The κ_i are the long-run coefficients in the IS curve, the μ_i the long-run coefficients in the Taylor rule. Individual country results are available in the supplement. For the IS of US, $\alpha_{ie} = \alpha_{iy*} = 0$.

8.2 Choice of error covariance matrices

The impulse responses reported so far are based on the shrinkage covariance matrix, $\hat{\Sigma}^0_\varepsilon(0.6)$, which uses a weighted average of the sample moment estimate of (20), $\hat{\Sigma}^0_\varepsilon$, with its diagonal, $diag(\hat{\Sigma}^0_\varepsilon)$. Since the choice of the weight, ϱ, is to some extent arbitrary we thought it is important to investigate the sensitivity of our results to the choice of Σ^0_ε and how it is estimated. Accordingly, here we consider four alternative estimates of the error covariance matrix: (a) the sample moment estimate, $\hat{\Sigma}^0_\varepsilon$ (b) the diagonal matrix, $diag(\hat{\Sigma}^0_\varepsilon)$, which cuts off all correlations between shocks, and (c) a block diagonal covariance matrix, $Bdiag(\hat{\Sigma}^0_\varepsilon)$ which imposes zero covariances between exchange rate and other shocks, but allows each type of shock to be correlated within a block, in addition to (d) the shrinkage estimator used above. Setting the covariances of exchange rates with the other shocks to zero, means that the shocks have no effect on exchange rates since there is no direct feedback from the other variables on the real effective exchange rates.

The impulse response functions for the effect of a global demand shock on interest rates, inflation and output, were not much affected by the choice of the four estimates of the error covariance matrices. However, there was some important differences in the case of the global supply shock, for which the impulse response functions are shown in Figure 5. It will be remembered that the highest correlations were found between supply shocks in Table 3 above. The largest effects are found using the sample covariance matrix, with the shrinkage and block diagonal quite close to those of the sample, and the diagonal covariance matrix showing the smallest effect. The results for the fully diagonal covariance matrix are interesting, because they show the effect of shutting off all international transmissions through the error spillover effects. With cross error correlations set to zero, there is no indirect instantaneous transmission between the US and the euro area. Thus in the case of the supply shocks the indirect transmission of shocks is important.

Figure 5: Impulse response functions for the effect of a global supply shock on US and euro area inflation, output and interest rates with different covariance matrices.



9 Concluding comments

This paper has examined the issues involved in specification, estimation, solution, and simulation of a multi-country rational expectations model used to estimate the effects of identified supply and demand shocks. These issues are illustrated with an MCNK model estimated for 33 countries over the period 1980Q3-2011Q2 that includes the "Great Recession".

In constructing such a model it is necessary to be cautious with regard to the assumptions made about exchange rates, particularly the treatment of the numeraire, and the patterns of cross country error spillover effects. To obtain a determinate solution and theory consistent results, it is also important that *a priori* sign and stability restrictions predicted by the theory are imposed on the parameters of the country-specific models. In the MCNK model used here as an illustration, global supply and demand shocks are the most important drivers of output, inflation and interest rates in the long run. By contrast monetary or exchange rate shocks have only a short-run role in the evolution of the world economy. Despite the uniformity of the specifications assumed across countries, there are major differences between countries in the size of the effects of the shocks. Changing the degree of international transmission, through the use of alternative error covariance matrices and foreign variables in equations can change the estimated size of the effects of the shocks. The results indicate the importance of international connections, directly as well as indirectly, through error spillover effects. Ignoring global inter-connections as country-specific models do, could give rise to misleading conclusions.

The primary objective of the current paper has been to examine the various issues involved in the analysis of large multi-country models, and illustrate them in the context of a specific model. There are a number of ways this particular model may be developed. There may be scope to allow for more global variables in the structural equations, which may reduce the cross-country correlations and allow the identification of country-specific idiosyncratic shocks. There may be advantages in including financial variables like real equity prices and long term interest rates. Less structural models, like the GVAR of DdPS indicate the importance of the international transmission of financial shocks. There is currently considerable macro-finance research to extend DSGE models to include explanations of the term premium in interest rates, the equity premium, the role of banks, government budget deficit, and the role of foreign assets, which is particularly important given the role of the US dollar as an international store of value, not just a unit of account. Another possible development is to introduce international trade variables, such as exports and imports directly rather than indirectly, as is done in this model through including the real effective exchange rate and a country-specific measure of foreign world output in the IS equation. The structure proposed in this paper provides a theoretically coherent and empirically viable framework for such extensions.

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