OPTIMAL AUTOMATIC STABILIZERS

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ABSTRACT

Should the generosity of unemployment benefits and the progressivity of income taxes depend on the presence of business cycles? This paper proposes a tractable model where there is a role for social insurance against uninsurable shocks to income and unemployment, as well as inefficient business cycles driven by aggregate shocks through matching frictions and nominal rigidities. We derive an augmented Baily-Chetty formula showing that the optimal generosity and progressivity depend on a macroeconomic stabilization term. Using a series of analytical examples, we show that this term typically pushes for an increase in generosity and progressivity as long as slack is more responsive to social programs in recessions. A calibration to the U.S. economy shows that taking concerns for macroeconomic stabilization into account raises the optimal unemployment benefits replacement rate by 13 percentage points but has a negligible impact on the optimal progressivity of the income tax. More generally, the role of social insurance programs as automatic stabilizers affects their optimal design.

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1 Introduction

The usual motivation behind large social welfare programs, like unemployment insurance or progressive income taxation, is to provide social insurance and engage in redistribution. A large literature therefore studies the optimal progressivity of income taxes typically by weighing the disincentive effect on individual labor supply and savings against concerns for redistribution and for insurance against idiosyncratic income shocks.[1] In turn, the optimal generosity of unemployment benefits is often stated in terms of a Baily-Chetty formula, which weighs the moral hazard effect of unemployment insurance on job search and creation against the social insurance benefits that it provides.[2]

For the most part, this literature abstracts from aggregate shocks, so that the optimal generosity and progressivity do not take into account business cycles. Yet, from their inception, an auxiliary justification for these social programs was that they were also supposed to automatically stabilize the business cycle.[3] Classic work that did focus on the automatic stabilizers relied on a Keynesian tradition that ignores the social insurance that these programs provide or their disincentive effects on employment. More modern work focuses on the positive effects of the automatic stabilizers, but falls short of computing optimal policies.[4]

The goal of this paper is to answer two classic questions—How generous should unemployment benefits be? How progressive should income taxes be?—but taking into account their automatic stabilizer nature. We present a model in which there is both a welfare role for social insurance as well as aggregate shocks and inefficient business cycles. Within the model, we introduce unemployment insurance and progressive income taxes as automatic stabilizers, that is programs that only depend on the aggregate state of the business cycle indirectly through their dependence on the idiosyncratic states of the household, which are employment and income.[5] We then solve for the ex ante socially optimal replacement rate of unemployment benefits and progressivity of personal income taxes in the presence of uninsured income risks, precautionary savings motives, labor market frictions, and

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[1] Mirrlees (1971) and Varian (1980) are classic references, and more recently see Benabou (2002), Conesa and Krueger (2006), Heathcote et al. (2014), Krueger and Ludwig (2013), and Golosov et al. (2016).

[2] See the classic work by Baily (1978) and Chetty (2006)

[3] Musgrave and Miller (1948) and Auerbach and Feenberg (2000) are classic references, while Blanchard et al. (2010) is a recent call for more modern work in this topic.

[4] See McKay and Reis (2016) for a recent model, DiMaggio and Kermani (2016) for recent empirical work, and IMF (2015) for the shortcomings of the older literature.

[5] Landais et al. (2015) and Kekre (2015) instead treat these social programs as active policy choices that vary directly with the business cycle.

nominal rigidities.

Our first main contribution is to provide a formal, theory-grounded definition of an automatic stabilizer. We show that a business-cycle variant of the Baily-Chetty formula for unemployment insurance and a similar formula for the optimal choice of progressivity of the tax system are both augmented by a new macroeconomic stabilization term. This term equals the expectation of the product of the welfare gain from eliminating economic slack with the elasticity of slack with respect to the replacement rate or tax progressivity. Even if the economy is efficient on average, economic fluctuations may lead to more generous unemployment insurance or more progressive income taxes, relative to standard analyses that ignore the automatic stabilizer properties of these programs. This terms captures the automatic stabilizer nature of social insurance programs.

The second contribution is to characterize this macroeconomic stabilization term analytically to understand the different economic mechanisms behind it. Fluctuations in aggregate economic slack, measured by the unemployment rate, the output gap or the job finding rate, can lead to welfare losses through four separate channels. First, they may create a wedge between the marginal disutility of hours worked and the social benefit of work. This inefficiency appears in standard models of inefficient business cycles, and is sometimes described as a result of time-varying markups (Chari et al., 2007; Galí et al., 2007). Second, when labor markets are tight, more workers are employed raising production but the cost of recruiting and hiring workers rises. The equilibrium level of unemployment need not be efficient as hiring and search decisions do not necessarily internalize these tradeoffs. This is the source of inefficiency common to search models (e.g. Hosios, 1990). Third, the state of the business cycle alters the extent of uninsurable risk that households face both in unemployment and income risk. This is the source of welfare costs of business cycles that has been studied by Storesletten et al. (2001), Krebs (2003, 2007), and De Santis (2007). Finally, with nominal rigidities, slack affects inflation and the dispersion of relative prices, as emphasized by the new Keynesian business cycle literature (Woodford, 2010; Gali, 2011). Our measure isolates these four effects cleanly in terms of separate additive terms in the condition determining the optimal extent of the social insurance programs.

In turn, the effects of benefits and progressivity of taxes on economic slack depends on their direct effect on aggregate demand, as well as on the impact of aggregate demand on equilibrium output. We show that considering macroeconomic stabilization raises the optimal replacement rate

of unemployment insurance. This is because, in recessions, economic activity is inefficiently low and aggregate slack is more responsive to the replacement rate for two reasons. First, there are more unemployed workers with high marginal propensities to consume receiving the transfers. Second, the effect of social insurance on precautionary savings motives is larger when there is a greater risk of unemployment. A similar argument applies to progressive taxation because income risk is counter-cyclical. Our analysis also incoroprates the effect of other aggregate demand policies and we show there is little role for fiscal policy to stabilize the business cycle if prices are very flexible or if monetary policy is very aggressive.

Our third and final contribution is to calculate the optimal automatic stabilizers quantitatively in the presence of business cycles. We do so by measuring how much more generous is unemployment insurance and how much more progressive are income taxes in a calibrated economy with aggregate shocks relative to one where these shocks are turned off. We find a large effect on unemployment insurance: with business cycles, the optimal unemployment replacement rate rises from 36 to 49 percent. However, the level of tax progressivity has very little stabilizing effect on the business cycle so the presence of aggregate shocks has almost no effect on the optimal degree of progressivity.

There are large literatures on the three topics that we touch on: business cycle models with incomplete markets and nominal rigidities, social insurance and public programs, and automatic stabilizers. Our model of aggregate demand has some of the key features of new Keynesian models with labor markets (Gali, 2011) but that literature focuses on optimal monetary policy, whereas we study the optimal design of the social insurance system. Our model of incomplete markets builds on McKay and Reis (2016), Ravn and Sterk (2013), and Heathcote et al. (2014) to generate a tractable model of incomplete markets and automatic stabilizers. This simplicity allows us to analytically express optimality conditions for generosity and progressivity, and to, even in the more general case, easily solve the model numerically and so be able to search for the optimal policies. Finally, our paper is part of a surge of work on the interplay of nominal rigidities and precautionary savings, but this literature has mostly been positive whereas this paper's focus is on optimal policy.[6]

On the generosity of unemployment insurance, our work is closest to Landais et al. (2015) and Kekre (2015). They also generalize the standard Baily-Chetty formula by considering the general equilibrium effects of unemployment insurance. The main difference is that while they study how

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[6]See Oh and Reis (2012); Guerrieri and Lorenzoni (2011); Auclert (2016); McKay et al. (2016); Kaplan et al. (2016); Werning (2015).

benefits should vary over the business cycle, we see how the presence of business cycles affects the ex ante fixed level of benefits.[7] In this sense, our focus is on automatic stabilizers, an ex ante passive policy, while they consider active stabilization policy. Moreover, our model includes aggregate uncertainty and we also study income tax progressivity.

On income taxes, our work is closest to Benabou (2002) and Bhandari et al. (2013). Our dynamic heterogeneous-agent model with progressive income taxes is similar to the one in Benabou (2002), but our focus is on business cycles, so we complement it with aggregate shocks and nominal rigidities. Bhandari et al. (2013) are one of the very few studies of optimal income taxes with aggregate shocks and, like us, they emphasize the interaction between business cycles and the desire for redistribution.[8] However, they do not consider unemployment benefits and restrict themselves to flat taxes over income. Moreover, they solve for the Ramsey optimal fiscal policy, which adjusts the tax instruments every period in response to shocks, while we choose the ex ante optimal rules for generosity and progressivity. This is consistent with our focus on automatic stabilizers, which are ex ante fiscal systems, rather than counter-cyclical policies.

Finally, this paper is related to the modern study of automatic stabilizers and especially our earlier work in McKay and Reis (2016). There, we considered how the actual automatic stabilizers implemented in the US alter the dynamics of the business cycle. Here we are concerned with the optimal fiscal system as opposed to the observed one.

The paper is structured as follows. Section 2 presents the model, and section 3 discusses its equilibrium properties. Section 4 derives the macroeconomic stabilization term in the optimality conditions for the two social programs. Section 5 discusses its qualitative properties, the economic mechanisms that it depends on, and its likely sign. Section 6 calibrates the model, and quantifies the effects of the automatic stabilizer effect. Section 7 concludes.

2   The Model

The main ingredients in the model are uninsurable income and employment risks, social insurance programs, and nominal rigidities so that aggregate demand matters for equilibrium allocations. The model makes a series of particular assumptions, which we explain in this section, in order to

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[7]See also Mitman and Rabinovich (2011), Jung and Kuester (2015), and Den Haan et al. (2015).

[8]Werning (2007) also studies optimal income taxes with aggregate shocks and social insurance.

generate a tractability that is laid out in the next section. Time is discrete and indexed by t.

2.1 Agents and commodities

There are two groups of private agents in the economy: households and firms.

Households are indexed by i in the unit interval, and their type is given by their productivity $\alpha_{i,t} \in \mathbb{R}_0^+$ and employment status $n_{i,t} \in \{0,1\}$. Every period, an independently drawn share δ dies, and is replaced by newborn households with no assets and productivity normalized to $\alpha_{i,t} = 1$.[9] Households derive utility from consumption, $c_{i,t}$, and publicly provided goods, G_t, and derive disutility from working for pay, $h_{i,t}$, searching for work, $q_{i,t}$, and being unemployed according to the utility function:

$$\mathbb{E}_0 \sum_t \beta^t \left[\log(c_{i,t}) - \frac{h_{i,t}^{1+\gamma}}{1+\gamma} - \frac{q_{i,t}^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi (1 - n_{i,t}) \right]. \tag{1}$$

The parameter β captures the joint discounting effect from time preference and mortality risk, while ξ is a non-pecuniary cost of being unemployed.[10]

The final consumption good is provided by a competitive final goods sector in the amount Y_t that sells for price p_t. It is produced by combining varieties of goods in a Dixit-Stiglitz aggregator with elasticity of substitution $\mu/(\mu - 1)$. Each variety $j \in [0,1]$ is monopolistically provided by a firm with output $y_{j,t}$ by hiring labor from the households and paying the wage w_t per unit of effective labor.

2.2 Asset markets and social programs

Households can insure against mortality risk by buying an annuity, but they cannot insure against risks to their individual skill or employment status. The simplest way to capture this market incompleteness is by assuming that households only trade a single risk-free bond, that has a gross real return R_t and is in zero net supply. Moreover, we assume that households cannot borrow, so

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[9]The mortality risk allows for a stationary cross-sectional distribution of productivity along with permanent shocks in section 6, but otherwise plays no significant role in the analysis and so will be assumed away in sections 4 and 5.

[10]If $\hat{\beta}$ is pure time discounting, then $\beta \equiv \hat{\beta}(1 - \delta)$.

if $a_{i,t}$ measures their asset holdings:[11]

$$a_{i,t} \geq 0. \tag{2}$$

The government provides two social insurance programs. The first is a progressive income tax such that if $z_{i,t}$ is pre-tax income, the after-tax income is $\lambda_t z_{i,t}^{1-\tau}$. $\lambda_t \in [0,1]$ determines the overall level of taxes, which together with the size of government purchases G_t, will pin down the size of the government. The object of our study is instead the automatic stabilizer role of the government, so our focus is on $\tau \in [0,1]$. This determines the progressivity of the tax system. If $\tau = 0$, there is a flat tax at rate $1 - \lambda_t$, while if $\tau = 1$ everyone ends up with the same after-tax income. In between, a higher τ implies a more convex tax function, or a more progressive income tax system.

The second social program is unemployment insurance. A household qualifies as long as it is unemployed ($n_{i,t} = 0$) and collects benefits that are paid in proportion to what the unemployed worker would earn if she were employed. Suppose the worker's productivity is such that she would earn pre-tax income $z_{i,t}$ if she were employed, then her after-tax unemployment benefit is $b\lambda_t z_{i,t}^{1-\tau}$.[12] Our focus is on the replacement rate $b \in [0,1]$, with a more generous program understood as having a higher b.[13]

Our goal is to characterize the optimal fixed levels of b and τ. Importantly, we consider the ex ante design problem, so b and τ do not depend on time or on the state of the business cycle. This corresponds to our focus on their role as automatic stabilizers, programs that can automatically stabilize the business cycle without policy intervention. We follow the tradition in the literature on automatic stabilizers that makes a sharp distinction between built-in properties of programs as opposed to feedback rules or discretionary choices that adjust these programs in response to current

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[11]A standard formulation for asset markets that gives rise to these annuity bonds is the following: A financial intermediary sells claims that pay one unit if the household survives and zero units if the household dies, and supports these claims by trading a riskless bond with return $\tilde{R}$. If a_i are the annuity holdings of household i, the law of large numbers implies the intermediary pays out in total $(1 - \delta) \int a_i di$, which is known in advance, and the cost of the bond position to support it is $(1 - \delta) \int a_i di / \tilde{R}$. Because the riskless bond is in zero net supply, then the net supply of annuities is zero $\int a_i di = 0$, and for the intermediary to make zero profits, $R_t = \tilde{R}_t / (1 - \delta)$.

[12]It would be more realistic, but less tractable, to assume that benefits are a proportion of the income the agent earned when she lost her job. But, given the persistence in earnings, both in the data and in our model, our formulation will not be quantitatively too different from this case. Also, in our notation, it may appear that unemployment benefits are not subject to the income tax, but this is just the result of a normalization: if they were taxed and the replacement rate was $\tilde{b}$, then the model would be unchanged and $b \equiv \tilde{b}^{1-\tau}$.

[13]In our model, focusing on the duration of unemployment benefits instead of the replacement rate would lead to similar trade offs, so we refer to b more generally as the generosity of the program.

and past information.[14]

2.3   Key frictions

There are three key frictions in the economy that create the policy trade-offs that we analyze.

2.3.1   Productivity risk

Labor income for an employed household is $\alpha_{it} w_t h_{it}$, where $\alpha_{i,t}$ is an idiosyncratic productivity or skill. The productivity of households evolves as

$$\alpha_{i,t+1} = \alpha_{i,t} \epsilon_{i,t+1} \text{ with } \epsilon_{i,t+1} \sim F(\epsilon; x_t), \tag{3}$$

and where $\int \epsilon dF(\epsilon, x_t) = 1$ for all t, which implies that the average idiosyncratic productivity in the population is constant and equal to one.[15]

The distribution of shocks varies over time so that the model generates cyclical changes in the distribution of earnings risks, as documented by Storesletten et al. (2004) or Guvenen et al. (2014). We capture this dependence through the variable x_t, which captures the aggregate slack in the economy. A higher x_t implies that the economy is tighter, the output gap is positive, or that the economy is closer to capacity or booming. In the next section we will map x_t to concrete measures of the state of the business cycle like the unemployment rate or the job finding rate. For concreteness, a simple case that maps to some empirical estimates is to have $F(.)$ be log-normal with $\text{Var}(\log \epsilon) = \sigma^2(x)$ and $\mathbb{E}(\log \epsilon) = -0.5\sigma(x)^2$.

2.3.2   Employment risk

The second source of risk is employment. We make a strong assumption that unemployment is distributed i.i.d. across households. Given the high (quarterly) job-finding rates in the US, this is not such a poor approximation, and it reduces the state space of the model. At the start of the period, a fraction υ of households loses employment and must search to regain employment. Search effort $q_{i,t}$ leads to employment with probability $M_t q_{i,t}$, where M_t is the job-finding rate per unit of search effort and the probability of resulting in a match is the same for each unit of search effort.

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[14]Perotti (2005) among many others.

[15]Since newborn households have productivity 1, the assumption is that they have average productivity.

Therefore, if all households make the same search effort, then aggregate hiring will be $v M_t q_t$ and as a result the unemployment rate will be:

$$u_t = v(1 - q_t M_t). \tag{4}$$

Each firm begins the period with a mass $1 - v$ of workers and must post vacancies at a cost to hire additional workers. As in Blanchard and Galí (2010), the cost per hire is increasing in aggregate labor market tightness, which is just equal to the ratio of hires to searchers, or the job-finding rate M_t. The hiring cost per hire is $\psi_1 M_t^{\psi_2}$, denominated in units of final goods where ψ_1 and ψ_2 are parameters that govern the level and elasticity of the hiring costs. Since aggregate hires are the difference between the beginning of period non-employment rate v and the realized unemployment rate u_t, aggregate hiring costs are:

$$J_t \equiv \psi_1 M_t^{\psi_2}(v - u_t). \tag{5}$$

We assume a law of large numbers within the firm so the average productivity of hires is 1.

In this model of the labor market, there is a surplus in the employment relationship since, on one side, firms would have to pay hiring costs to replace the worker and, on the other side, a worker who rejects a job becomes unemployed and foregoes the opportunity to earn wages this period. This surplus creates a bargaining set for wages, and there are many alternative models of how wages are chosen within this set, from Nash bargaining to wage stickiness, as emphasized by Hall (2005).

We assume a convenient wage rule:

$$w_t = \bar{w} A_t (1 - J_t/Y_t) x_t^\zeta. \tag{6}$$

The real wage per effective unit of labor depends on three variables, aside from a constant $\bar{w}$. First, it increases proportionately with aggregate effective productivity A_t, as it would in a frictionless model of the labor market. Second, it falls when aggregate hiring costs are higher, so that some of these costs are passed from firms to workers. The justification is that when hiring costs rise, the economy is poorer and this raises labor supply, which the fall in wages exactly offsets. Since these costs are quantitatively small, in reality and in our calibrations, this assumption has little effect

in the predictions of the model but allows us to not have to carry this uninteresting wealth effect on labor supply throughout the analysis.[16] Third, when the labor market is tighter, wages rise, with an elasticity of ζ. Standard Nash bargaining models lead to a positive dependence between economic activity and wages, while sticky wage models can be approximated by $\zeta = 0$.

The purpose of this wage rule is to simplify the analysis of the intensive margin of labor supply. Our analytical results do not depend crucially on the wage rule. Appendix A discusses this at length, showing that even for a general wage rule, that nests Nash bargaining and many others, would lead to very similar results. In fact, if labor supply were fixed on the intensive margin, as in most search models of the labor market, our results would be completely unchanged.

2.3.3 Nominal rigidities

The firm that produces each variety uses the production function $y_{j,t} = \eta_t^A h_{j,t} l_{j,t}$, where $h_{j,t}$ are hours per worker and $l_{j,t}$ the workers in the firm, subject to exogenous productivity shocks η_t^A.[17] The firm's marginal cost is

$$\frac{w_t + \psi_1 M_t^{\psi_2}/h_t}{\eta_t^A}.$$

Marginal costs are the sum of the wage paid per effective unit of labor and the hiring costs that had to be paid, divided by productivity. Under flexible prices, the firm would set a constant markup, μ, over marginal cost. The aggregate profits of these firms are distributed among employed workers in proportion to their skill, which can be thought of as representing bonus payments in a sharing economy.

However, individual firms cannot choose their actual price to equal this desired price every period because of nominal rigidities. We consider two separate simple models of nominal rigidities. In the numerical study of section 6, we assume Calvo (1983) pricing, so that every period a fraction θ of randomly drawn firms are allowed to change their price, with the remaining $1-\theta$ having to keep

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[16]Moreover, in the special cases of the model studied in section 5, J_t/Y_t is a function of x_t so this term gets absorbed by the next term after a redefinition of ζ.

[17]Given the structure of the labor market, employed workers set their hours taking the hourly wage as given. We show below they all make the same choice h_t. The firm then chooses how many workers to hire. Marginal cost is then the cost of increasing the number of workers to produce one more unit of output.

their price unchanged from the last period. This leads to the dynamics for inflation $\pi_t \equiv p_t/p_{t-1}$:

$$\pi_t = \left[(1-\theta)/\left[1 - \theta \left(\frac{p_t^*}{p_t} \right)^{1/(1-\mu)} \right] \right]^{1-\mu} \tag{7}$$

where p_t^* is the price chosen by firms that adjust their price in period t.

In the analytical study of sections 4 and 5 we assume instead a simpler and more transparent canonical model of nominal rigidities, where every period an i.i.d. fraction θ of firms can set their prices $p_{j,t} = p_t^*$, while the remaining set their price to equal what they expected their optimal price would be: $p_{j,t} = \mathbb{E}_{t-1} \, p_t^*$. Mankiw and Reis (2010) show that most of the qualitative insights from New Keynesian economics can be captured by this simple sticky-information formulation.

2.4   Other government policy

Aside from the two social programs that are the focus of our study, the government also chooses policies for nominal interest rates, government purchases, and the public debt. Starting with the first, we assume a standard Taylor rule for nominal interest rates I_t:

$$I_t = \bar{I} \pi_t^{\omega_\pi} x_t^{\omega_x} \eta_t^I, \tag{8}$$

where $\omega_\pi > 1$ and $\omega_x \geq 0$. The exogenous η_t^I represent shocks to monetary policy.[18]

Turning to the second, government purchases follow the Samuelson (1954) rule such that, absent η_t^G shocks, the marginal utility benefit of public goods offsets the marginal utility loss from diverting goods from private consumption:

$$G_t = \chi C_t \eta_t^G. \tag{9}$$

Finally, we assume that the government runs a balanced budget:

$$G_t = \int n_{i,t} \left(z_{i,t} - \lambda_t z_{i,t}^{1-\tau} \right) - (1 - n_{i,t}) \, b \lambda_t z_{i,t}^{1-\tau} di, \tag{10}$$

where $z_{i,t}$ is the income of household i should they be employed. This strong assumption deserves some explanation. It is well known, at least since Aiyagari and McGrattan (1998), that in an

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[18] As usual, the real and nominal interest rates are linked by the Fisher equation $R_t = I_t / \mathbb{E}_t \, [\pi_{t+1}]$.

incomplete markets economy like ours, changes in the supply of safe assets will affect the ability to accumulate in precautionary savings. Deficits or surpluses may stabilize the business cycle by changing the cost of self-insurance. In the same way that we abstracted above from the stabilizing properties of changes in government purchases, this lets us likewise abstract from the stabilizing property of public debt, in order to focus on our two social programs.

In previous work (McKay and Reis, 2016), we found that allowing for deficits and public debt had little effect on the effectiveness of stabilizers. This is because, in order to match the concentration of wealth in the data, almost all of the public debt is held by richer households who are already close to fully self insured. Still, in previous versions of this paper, we investigated this further by assuming instead that there are public deficits, but financed by borrowing from abroad. As long as changes in b and τ do not affect the amount of debt issued, all of our results are unchanged.

3   Equilibrium and the role of policy

Our model combines idiosyncratic risk, incomplete markets, and nominal rigidities, and yet it is structured so as to be tractable enough to investigate optimal policy. This section highlights how our assumptions, with their virtues and limitations, lead to this tractability. We also highlight the role for social insurance policy in the economy, as well as the distortions it creates.

3.1   Inequality and heterogeneity

The following result follows from the particular assumptions we made on the decision problems of different agents and plays a crucial role in simplifying the analysis:

Lemma 1. *All households choose the same asset holdings, hours worked, and search effort, so $a_{i,t} = 0$, $h_{i,t} = h_t$, and $q_{i,t} = q_t$ for all i.*

To prove this result, note that the decision problem of a household searching for a job at the start of the period is:

$$V^s(a, \alpha, \mathcal{S}) = \max_q \left\{ MqV(a, \alpha, 1, \mathcal{S}) + (1 - Mq)V(a, \alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1 + \kappa} \right\}, \qquad (11)$$

where we used $\mathcal{S}$ to denote the collection of aggregate states. The decision problem of the household

at the end of the period is:

$$V(a, \alpha, n, \mathcal{S}) = \max_{c,h,a' \geq 0} \left\{ \log c - \frac{h^{1+\gamma}}{1+\gamma} + \chi \log(G) - \xi(1-n) + \right.$$

$$\left. \beta \mathbb{E} \left[(1-\upsilon)V(a', \alpha', 1, \mathcal{S}') + \upsilon V^s(a', \alpha', \mathcal{S}') \right] \right\}, \tag{12}$$

$$\text{subject to: } a' + c = Ra + \lambda \left(n + (1-n)b \right) \left[\alpha(wh+d) \right]^{1-\tau}. \tag{13}$$

Starting with asset holdings, since no agent can borrow and bonds are in zero net supply, then it must be that $a_{i,t} = 0$ for all i in equilibrium because there is no gross supply of bonds for savers to own, a result also used by Krusell et al. (2011) and Ravn and Sterk (2013). Turning to hours worked, the intra-temporal labor supply condition for an employed household is

$$c_{i,t} h_{i,t}^{\gamma} = (1-\tau)\lambda_t z_{i,t}^{-\tau} w_t \alpha_{i,t}, \tag{14}$$

where the left-hand side is the marginal rate of substitution between consumption and leisure, and the right-hand side is the after-tax return to working an extra hour to raise income $z_{i,t}$. More productive agents want to work more. However, they are also richer and want to consume more. The combination of our preferences and the budget constraint imply that these two effects exactly cancel out so that in equilibrium all employed households work the same hours:

$$h_t^{\gamma} = \frac{(1-\tau)w_t}{w_t h_t + d_t}, \tag{15}$$

where d_t is aggregate dividends per employed worker.[19]

Finally, the optimality condition for search effort is:

$$q_{i,t}^{\kappa} = M_t \left[V(a_{i,t}, \alpha_{i,t}, 1, \mathcal{S}) - V(a_{i,t}, \alpha_{i,t}, 0, \mathcal{S}) \right]. \tag{16}$$

Intuitively, the household equates the marginal disutility of searching on the left-hand side to the expected benefit of finding a job on the right-hand side, which is the product of the job-finding probability M_t and the increase in value of becoming employed. Appendix B.1 shows that this increase in value is independent of $\alpha_{i,t}$. The key assumption that ensures this is that

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[19]To derive this, substitute $z_{i,t} = \alpha_{i,t}(w_t h_{i,t} + d_t)$ and $c_{i,t} = \lambda_t z_{i,t}^{1-\tau}$ into (14).

unemployment benefits are indexed to income $z_{i,t}$ so the after-tax income with and without a job scales with idiosyncratic productivity in the same way. This then implies that $q_{i,t}$ is the same for all households.

The lemma clearly limits the scope of our study. We cannot speak to the effect of policy on asset holdings, and differences in labor supply are reduced to having a job or not, which ignores diversity in part-time jobs and overtime. At the same time, it has the substantial payoff of implying that $\mathcal{S}$ contains only aggregate variables, so we do not need to keep track of cross-sectional distributions to characterize an equilibrium. Thus, our model can be studied analytically and numerical solutions are easy to compute. Moreover, arguably the social programs that we study are more concerned with income, rather than wealth inequality, and the vast majority of studies of the automatic stabilizers also ignores any direct effects of wealth inequality (as opposed to income inequality) on the business cycle.

In our model, there is a rich distribution of income and consumption driven by heterogeneity in employment status $n_{i,t}$ and skill $a_{i,t}$. In section 6, we are able to fit the more prominent features of income inequality in the United States by parameterizing the distribution $F(\epsilon, x)$. Moreover, in our model, there is a rich distribution of individual prices and output across firms, $(p_{j,t}, y_{j,t})$, driven by nominal rigidities. And finally, the exogenous aggregate shocks to productivity, monetary policy, and government purchases, $(\eta_t^A, \eta_t^I, \eta_t^G)$, affect all of these distributions, which therefore vary over time and over the business cycle. In spite of the simplifications and their limitations, our model still admits a rich amount of inequality and heterogeneity.

3.2 Quasi-aggregation and consumption

Define $\tilde{c}_t$ as the consumption of the average-skilled ($\alpha_{i,t} = 1$), employed agent. This is related to aggregate consumption, C_t, according to (see Appendix B.2):

$$\tilde{c}_t = \frac{C_t}{\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] (1 - u_t + u_t b)}. \tag{17}$$

Funding higher replacement rates requires larger taxes on those employed, so it reduces their consumption. Likewise, the amount of revenue raised by the progressive tax system depends on the distribution of income as summarized by $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]$. More dispersed incomes generate higher

revenues and allow for lower taxes for a given level of income.

The next property that simplifies our model is proven in Appendix B.2.

Lemma 2. *Aggregate consumption dynamics can be computed from*

$$\frac{1}{\tilde{c}_t} = \beta R_t \, \mathbb{E}_t \left\{ \frac{1}{\tilde{c}_{t+1}} Q_{t+1} \right\}, \tag{18}$$

$$\text{with: } Q_{t+1} \equiv \left[(1 - u_{t+1}) + u_{t+1} b^{-1} \right] \mathbb{E} \left[\epsilon_{i,t+1}^{-(1-\tau)} \right] \tag{19}$$

and equation (17).

Without uncertainty on productivity or unemployment, $Q_{t+1} = 1$, and this would be a standard Euler equation from intertemporal choice stating that expected consumption growth is inversely related to the product of the discount factor and the real interest rate.

The variable Q_{t+1} captures how heterogeneity affects aggregate consumption dynamics through precautionary savings motives. The more uncertain is income, the larger is Q_{t+1} and so the larger are savings motives leading to steeper consumption growth. A more generous unemployment insurance system and a more progressive income tax lower the dispersion of after-tax income growth and reduce the effect of this Q_{t+1} term.

3.3 Policy distortions and redistribution over the business cycle

Social policies not only affect aggregate consumption, but also all individual choices in the economy, introducing both distortions and redistribution.

Combining the optimality condition for hours with the wage rule gives (see Appendix B.3):

$$h_t = [\bar{w}(1 - \tau)]^{\frac{1}{1+\gamma}} x_t^{\frac{\zeta}{1+\gamma}}. \tag{20}$$

A more progressive income tax lowers hours worked by increasing the ratio of the marginal tax rate to the average tax rate.

Moving to search effort, one can show that (see Appendix B.3)

$$q_t^\kappa = M_t \left[\xi - \frac{h_t^{1+\gamma}}{1+\gamma} - \log(b) \right]. \tag{21}$$

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This states that the marginal disutility of searching for a job is equal to the probability of finding a job times the increase in utility of having a job. This increase is equal to the difference between the non-pecuniary pain from being unemployed and the disutility of working, minus the loss in utility units of losing the unemployment benefits. More generous benefits therefore lower search effort. Intuitively, they lower the value of finding a job, so less effort is expended looking for one.

The distribution of consumption in the economy is given by a relatively simple expression:

$$c_{i,t} = \left[\alpha_{i,t}^{1-\tau} (n_{i,t} + (1 - n_{i,t})b) \right] \tilde{c}_t \tag{22}$$

The expression in brackets shows that more productive and employed households consume more, as expected. Combined with $\tilde{c}_t$, this formula also shows how social policies redistribute income and equalize consumption. A higher b requires larger contributions from all households, lowering $\tilde{c}_t$, but only increases the term in brackets for unemployed households. Therefore, it raises the consumption of the unemployed relative to the employed. In turn, a higher τ lowers the cross-sectional dispersion of consumption because it reduces the income of the rich more than that of the poor. The state of the business cycle affects the extent of the redistribution by driving both unemployment and the cross-sectional distribution of productivity risk.

Finally, social programs also affect price dispersion and inflation. Recalling that $A_t \equiv Y_t / [h_t \int l_{jt} dj]$, average aggregate labor productivity, then integrating over the individual production functions and using the demand for each variety it follows immediately that $A_t = \eta_t^A / S_t$ where the new variable is price dispersion:

$$S_t \equiv \int (p_t(j)/p_t)^{\mu/(1-\mu)} \, dj \geq 1 \tag{23}$$

$$= (1 - \theta) S_{t-1} \pi_t^{-\mu/(1-\mu)} + \theta \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \quad \text{if Calvo,} \tag{24}$$

$$= \left(\frac{p_t^*}{p_t} \right)^{\mu/(1-\mu)} \left[\theta + (1 - \theta) \left(\frac{\mathbb{E}_{t-1} p_t^*}{p_t^*} \right)^{\mu/(1-\mu)} \right] \quad \text{if sticky information.} \tag{25}$$

Nominal rigidities lead otherwise identical firms to charge different prices, and this relative-price dispersion lowers productivity and output in the economy. The social insurance system will alter the dynamics of aggregate demand leading to different dynamics for nominal marginal costs, inflation, and price dispersion.

3.4 Slack and equilibrium

The missing ingredient to close the model is a concrete definition of how to measure economic slack x_t.

For the analytical results in the next two sections, we will take a convenient assumption:

$$x_t = M_t. \tag{26}$$

That is, we measure the state of the business cycle by the tightness of the labor market, as captured by the job-finding rate. This is not such a strong assumption since, in the model, (h_t, q_t) are functions of only M_t and parameters, as we can see in equations (20) and (21). Moreover, in the special cases considered in sections 4 and 5, the unemployment rate and the output gap (the difference between actual output and that which arises with flexible prices) are also functions of M_t as the single endogenous variable. Finally, when we take the model to the data in section 6, we find that in simulations, using instead one minus the unemployment rate as the measure of slack leads to essentially identical results. We assume equation (26) because it makes the analytical derivations in the next two sections more transparent, allowing us to carry fewer cross-terms that are of little interest. Most of the results would extend easily to other measures of slack, like the unemployment rate or hours worked, but with longer and more involved algebraic expressions.

An aggregate equilibrium in our economy is then a solution for 18 endogenous variables together with the exogenous processes η_t^A, η_t^G, and η_t^I. Appendix B.4 lays out the entire system of equations that defines this equilibrium.

4 Optimal policy and insurance versus incentives

All agents in our economy are identical ex ante, making it natural to take as the target of policy the utilitarian social welfare function. Using equation (22) and integrating the utility function in equation (1) gives the objective function for policy $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t$, where period-welfare is:

$$W_t = \mathbb{E}_i \log \left(\alpha_{i,t}^{1-\tau} \right) - \log \left(\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] \right) + u_t \log b - \log \left(1 - u_t + u_t b \right)$$
$$+ \log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1 + \gamma} - \upsilon \frac{q_t^{1+\kappa}}{1 + \kappa} + \chi \log(G_t) - \xi u_t. \tag{27}$$

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The first line shows how inequality affects social welfare. Productivity differences and unemployment introduce costly idiosyncratic risk, which is attenuated by the social insurance policies. The second line captures the usual effect of aggregates on welfare. While these would be the terms that would survive if there were complete insurance markets, recall that the incompleteness of markets also affects the evolution of aggregates, as we explained in the previous section.

The policy problem is then to pick b and τ to maximize equation (27) subject to the equilibrium conditions, at date 0 once and for all. As already discussed, in this and the next section, we make the following simplifications on the general problem: (i) log-normal productivity shocks, (ii) no mortality, (iii) sticky information, and (iv) no government spending shocks.[20]

4.1 Optimal unemployment insurance

Appendix C derives the following optimality condition for b:

Proposition 1. *The optimal choice of the generosity of unemployment insurance b satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ u_t \left(\frac{1}{b} - 1 \right) \left. \frac{\partial \log (b\tilde{c}_t)}{\partial \log b} \right|_{x,q} + \frac{d \log \tilde{c}_t}{d \log u_t} \left. \frac{\partial \log u_t}{\partial b} \right|_x + \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = 0. \qquad (28)$$

Equation (28) is closely related to the Baily-Chetty formula for optimal unemployment insurance. The first term captures the social insurance value of changing the replacement rate. It is equal to the percentage difference between the marginal utility of unemployed and employed agents times the elasticity of the consumption of the unemployed with respect to the benefit. If unemployment came with no differences in consumption, this term would be zero, and likewise if giving higher benefits to the unemployed had no effect on their consumption. But as long as employed agents consume more, and raising benefits closes some of the consumption gap, then this term will be positive and call for higher unemployment benefits.

The second term gives the moral hazard cost of unemployment insurance. Is is equal to the product of the elasticity of the consumption of the employed with respect to the unemployment rate, which is negative, and the elasticity of the unemployment rate with respect to the benefit that

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[20]To be clear, none of these assumptions are essential: relaxing (i) would lead to similar expressions with expectations against $F(.)$ in place of $\sigma^2(.)$, relaxing (ii) would result in more complicated expressions for the effect of skill risk on welfare without any qualitative change in the results, substituting (iii) for sticky prices would require integrating the effects of policy on S_t over time, and relaxing (iv) would lead to an additional term in all the expressions equal to the difference between the marginal utility of public expenditures and the resource cost of financing those expenditures.

arises out of reduced search effort. Higher replacement rates induce agents to search less, which raises equilibrium unemployment, and leads to higher taxes to finance benefits.

In the absence of general equilibrium effects, these would be the only two terms, and they capture the standard trade-off between insurance and incentives in the literature averaged across states and time. With business cycles and general equilibrium effects, there is an extra macroeconomic stabilization term. The larger this term is, the more generous optimal unemployment benefits should be. We explain this shortly, but first, we turn to the income tax.

4.2 Optimal progressivity of the income tax

Appendix C shows the following:

Proposition 2. *The optimal progressivity of the tax system τ satisfies:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{\beta(1-\tau)}{(1-\beta)} \right] \sigma^2(x_t) - \left(\frac{A_t}{C_t} - h_t^\gamma \right) \frac{h_t(1-u_t)}{(1-\tau)(1+\gamma)} + \frac{d\log \tilde{c}_t}{d\log u_t} \left. \frac{\partial \log u_t}{\partial \tau} \right|_x + \frac{dW_t}{dx_t} \frac{dx_t}{d\tau} \right\} = 0.$$

(29)

The first three terms again capture the familiar trade-off between insurance and incentives. The first term gives the welfare benefits of reducing the dispersion in after-tax incomes, which is increasing in the extent of pre-tax inequality as reflected by $\sigma^2(x_t)$. The second and third term give the incentive costs of raising progressivity. The second term is the labor wedge, the gap between the marginal product of labor and the marginal disutility of labor. More progressive taxes raise the wedge by discouraging labor supply, as explained earlier. The third term reflects the effect of the tax system on the unemployment rate taking slack as given. The tax system affects the relative rewards to being employed and therefore alters household search effort and the unemployment rate.

Finally, the fourth term captures the concern for macroeconomic stabilization in a very similar way to the term for unemployment benefits. A larger stabilization term in (29) justifies a larger labor wedge and therefore a more progressive tax.

4.3 The macroeconomic stabilization term

The two previous propositions clearly isolate the automatic-stabilizing role of the social insurance programs in a single term. It equals the product of the welfare benefit of changing slack and the

response of slack to policy. If business cycles are efficient, the macroeconomic stabilization term is zero. That is, if the economy is always at an efficient level of slack, so that $dW_t/dx_t = 0$, then there is no reason to take macroeconomic stabilization into account when designing the stabilizers. Intuitively, the business cycle is of no concern for policymakers in this case.

Even if business cycles are efficient on average though, the automatic stabilizers can still play a role. This is because:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{dW_t}{dx_t} \frac{dx_t}{db} \right\} = \sum_{t=0}^{\infty} \beta^t \left\{ \mathbb{E}_0 \left[\frac{dW_t}{dx_t} \right] \mathbb{E}_0 \left[\frac{dx_t}{db} \right] + \text{Cov} \left[\frac{dW_t}{dx_t}, \frac{dx_t}{db} \right] \right\},$$

so that even if $\mathbb{E}_0 [dW_t/dx_t] = 0$, a positive covariance term would still imply a positive aggregate stabilization term and an increase in benefits (or more progressive taxes). Our model therefore provides a sharp definition of the the hallmark of a social policy that serves as an automatic stabilizer: it stimulates the economy more in recessions, when slack is inefficiently high. The stronger this effect, the larger the program should be. In the next section, we discuss the sign of this covariance and what affects it.

5   Inspecting the macroeconomic stabilization term

Understanding the automatic stabilizer nature of social program requires understanding separately the effect of slack on welfare, dW_t/dx_t, and the effect of the social policies on slack, dx_t/db and $dx_t/d\tau$. Instead of trying to measure the covariance between these two unobservables in the data, a daunting task, we proceed by characterizing their structural determinants instead in terms of familiar economic channels that have been measure elsewhere.

5.1   Slack and welfare

There are five separate channels through which the business cycle may be inefficient in our model, characterized in the following result:

Proposition 3. *The effect of macroeconomic slack on welfare can be decomposed into:*

$$\frac{dW_t}{dx_t} = \underbrace{(1-u_t)\left[\frac{A_t}{C_t} - h_t^\gamma\right]\frac{dh_t}{dx_t}}_{labor\text{-}wedge} - \underbrace{\frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t}}_{price\text{-}dispersion} + \underbrace{\frac{1}{C_t}\frac{dC_t}{du_t}\frac{du_t}{dx_t} - \frac{1}{C_t}\left.\frac{\partial J_t}{\partial x_t}\right|_u}_{Hosios} \tag{30}$$

$$\underbrace{-\left(\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma}\right)\left.\frac{\partial u_t}{\partial x_t}\right|_q + \frac{1-b}{1-u_t+u_t b}\frac{du_t}{dx_t}}_{unemployment\text{-}risk} - \underbrace{\frac{\beta(1-\tau)^2}{2(1-\beta)}\frac{d\sigma^2(x_t)}{dx_t}}_{income\text{-}risk}$$

The first term captures the effect of the labor wedge or markups. In the economy, A_t/C_t is the marginal product of an extra hour worked in utility units, while h_t^γ is the marginal disutility of working. If the first exceeds the second, the economy is underproducing, and increasing hours worked would raise welfare.

The second term captures the effect of slack on price dispersion. Because of nominal rigidities, aggregate shocks will lead to price dispersion. In that case, changes in aggregate slack will affect inflation, via the Phillips curve, and so price dispersion. This is the conventional channel in new Keynesian models through which the output gap affects inflation and its welfare costs.

The third and fourth terms capture the standard Hosios (1990) trade-off of hiring more workers. On the one hand, the extra hire lowers unemployment and raises consumption. On the other hand, it increases hiring costs. If hiring is efficient, so the Hosios condition holds, then $(dC_t/du_t)(du_t/dx_t) = \partial J/\partial x_t$ at all dates, but otherwise changes in slack will affect hirings, unemployment and welfare.

The terms in the second line of equation (30) fix aggregate consumption and focus on inequality and its effect on welfare. If the extent of income risk is cyclical, which the literature since Storesletten et al. (2004) has extensively demonstrated, then raising economic activity reduces income risk and so raises welfare. In our model, there is both unemployment and income risk, so this works through two channels.

The fourth and fifth term capture the effect of slack on unemployment risk. For a given aggregate consumption, more unemployment has two effects on welfare. First there are more unemployed who consume a lower amount. The term $\xi - \log b - h_t^{1+\gamma}/(1+\gamma)$ is the utility loss from becoming unemployed. Second, those who are employed consume a larger share (dividing the pie among fewer employed people). These are the two effects of unemployment risk. The sixth and final term shows that cutting slack also lowers the variance of skill shocks, which lowers income inequality.

5.2 Three special cases

To better understand these different channels of stabilization, and link them to the literature before us, we consider three special cases that correspond to familiar models of fluctuations.

5.2.1 Frictional unemployment

Consider the special case where prices are flexible ($\theta = 1$), there is no productivity risk ($\sigma^2 = 0$), and labor supply does not vary on the intensive margin because hours worked are constant ($\gamma = \infty$). The only source of inequality is then unemployment, which in our model becomes a result of the search and matching paradigm. Therefore, equation (30) becomes:

$$\frac{dW_t}{dx_t} = \frac{1}{C_t}\frac{dC_t}{du_t}\frac{du_t}{dx_t} - \frac{1}{C_t}\left.\frac{\partial J_t}{\partial x_t}\right|_u - \left(\xi - \log b - \frac{h_t^{1+\gamma}}{1+\gamma}\right)\left.\frac{\partial u_t}{\partial x_t}\right|_q + \frac{1-b}{1-u_t+u_tb}\frac{du_t}{dx_t}. \tag{31}$$

as only the Hosios effect and the unemployment risk are now present.

In this special case, our model captures the main effects in Landais et al. (2015). They discuss the macroeconomic effects of unemployment benefits from the perspective of their effect on labor market tightness by changing the worker's bargaining position and wages on the one hand and, on the other hand, their impact on dissuading search effort.

5.2.2 Real Business Cycle effects

Next, we consider the case of flexible prices ($\theta = 1$), constant search effort ($\kappa = \infty$), and exogenous job finding (M_t exogenous).[21] With nominal rigidities and search removed, what is left is the labor wedge and the effect of cyclical income risk on welfare, so equation (30) simplifies to

$$\frac{dW_t}{dx_t} = (1-u_t)\left[\frac{A_t}{C_t} - h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{\beta}{1-\beta}(1-\tau)^2\frac{d}{dx_t}\frac{\sigma^2(x_t)}{2}. \tag{32}$$

In this case, our paper fits into the standard analysis of business cycles in Chari et al. (2007) through the first term, and into the study the costs of business cycles due to income inequality emphasized by Krebs (2003) through the second term.

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[21]When M_t is constant we need to define slack differently from $x_t = M_t$. In this case, the role of x_t is to change the wage and change labor supply on the intensive margin. The wage will need to adjust to clear the labor market as in the three-equation New Keynesian model and then the wage rule, equation (6), becomes the definition of x_t.

5.2.3 Aggregate-demand effects

Traditionally, the literature on automatic stabilizers has focussed on aggregate demand effects following a Keynesian tradition. When there is no productivity risk ($\sigma^2 = 0$), job search effort is constant ($\kappa = \infty$) and the labor market's matching frictions are constant (M_t is constant), equation(30) simplifies to:

$$\frac{dW_t}{dx_t} = (1 - u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dx_t} - \frac{Y_t}{C_t S_t} \frac{dS_t}{dx_t},$$

so only the markup effects are present, both through the labor wedge and through price dispersion.

Appendix D.2 shows that a second-order approximation of W_t around the flexible-price, socially-efficient level of aggregate output Y_t^* and consumption C_t^* transforms this expression into

$$\frac{dW_t}{dx_t} = \left(\frac{Y_t^*}{C_t^*} \right) \left[\left(\frac{1}{C_t^*} + \frac{\gamma}{Y_t^*} \right) \left(\frac{Y_t^* - Y_t}{Y_t^*} \right) \frac{dY_t}{dx_t} + \left(\frac{1 - \theta}{\theta} \frac{\mu}{\mu - 1} \right) \left(\frac{\mathbb{E}_{t-1} p_t - p_t}{\mathbb{E}_{t-1} p_t} \right) \frac{dp_t}{dx_t} \right],$$

In this case, our model fits into the new Keynesian framework with unemployment developed in Blanchard and Galí (2010) or Gali (2011). Raising slack affects the output gap and the price level, through the Phillips curve, and this affects welfare through the two conventional terms in the expression. The first is the effect on the output gap, and the second the effect on surprise inflation. These are the two sources of welfare costs in this economy.

5.3 Social programs and slack

We now turn attention to the second component of the macroeconomic stabilization term, either dx_t/db in the case of unemployment benefits or $dx_t/d\tau$ in the case of tax progressivity. We make a few extra simplifying assumptions to obtain analytical expressions that are easy to interpret. First, we assume that aggregate shocks only occur at date 0 and there is no aggregate uncertainty after that, so that the analysis can be contained to a single period. Second, we assume that household search effort is exogenous and constant ($\kappa = \infty$) as in sections 5.2.2 and 5.2.3, since the role of job search is well described by Landais et al. (2015).
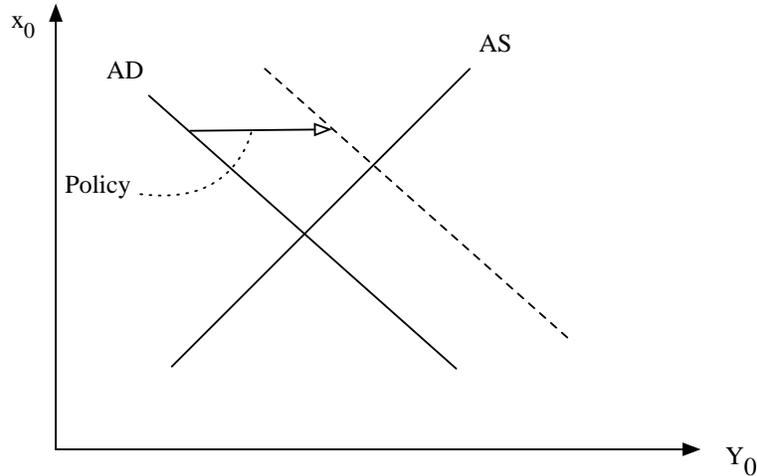
Figure 1: Determination of x_0.

5.3.1 An AD-AS interpretation

With these simplifications, the analysis of the model becomes static, and the household decisions are reduced to consumption and hours worked. Appendix D.3 shows that combining equilibrium in the labor market with the resource constraint and production functions gives an upward sloping relationship between slack x and output Y, which we will call the "aggregate supply curve", for lack of a better term. Likewise combining the aggregate Euler equation, the monetary rule and the Phillips curve gives an "aggregate demand curve." Both are plotted in figure 1, so that equilibrium is at the intersection of the two curves.

The impact of the social policies on equilibrium slack depends on two features of the diagram. First, how much does a change in policy horizontally shift the AD curve? A bigger shift in AD will lead to a larger effect on slack. Second, what are the slopes of the two curves? If both the AD and the AS are steeper, then a given horizontal shift in the AD brought about by the policies leads to a larger change in slack at the new equilibrium. We discuss shifts and slopes separately.

5.3.2 Unemployment benefits and slack

Appendix D.4 proves the following:

Proposition 4. *Under the assumptions of section 5.3:*

$$\frac{d \log x_0}{d \log b} = \Lambda^{-1} \left[\frac{u_0 b}{1 - u_0 + u_0 b} + \frac{u_1 b^{-1}}{1 - u_1(1 - b^{-1})} - \frac{u_1 b}{1 - u_1(1 - b)} \right] \tag{33}$$

where Λ is defined below in lemma 3.

There are two direct effects of raising unemployment benefits on economic slack. The first shows the usual logic of the unemployment insurance system as an automatic stabilizer based on redistribution: aggregate demand responds more strongly to benefits if there are more unemployed workers. This effect arises because the unemployed have a high marginal propensity to consume so the extra benefits lead to an increase in demand.

Second, there is an additional effect coming from expected marginal utility in the future, which depends on the unemployment rate in the next period ($t = 1$). Expected marginal utility is determined by two components. The first is the social insurance that UI provides, lowering uncertainty and precautionary savings and so pushing up aggregate demand today. The second is the higher taxes in order to finance the higher benefits, which reduce consumption. For unemployment rates less than 50% the former effect dominates and the sum of these terms is increasing in the unemployment rate u_1 (see appendix D.5).

These two effects, captured by the expression in square brackets, shift the AD curve rightwards. As the unemployment rate increases in recessions, both of these effects point towards a counter-cyclical elasticity. The overall effect on tightness is then tempered by the slopes of AD and AS, through the term Λ. We explain this effect below, but since it is common to the effect of more progressive taxes, we turn to that first.

5.3.3   Tax progressivity and slack

Appendix D.6 proves:

Proposition 5. *Under the assumptions of section 5.3:*

$$\frac{d \log x_0}{d \log \tau} = \Lambda^{-1} \left[2\sigma_\epsilon^2(x_0)(1 - \tau)\tau - \left. \frac{\partial \log R_0}{\partial \log \tau} \right|_x + \left. \frac{\partial \log S_0}{\partial \log \tau} \right|_x + \frac{d \log Y_1}{d \log x_1} \frac{d \log x_1}{d \log \tau} \right] \tag{34}$$

where Λ is defined below in lemma 3.

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The first term in between square brackets reflects the effect of social insurance on aggregate demand. When households face uninsurable skill risk, a progressive tax system will raise aggregate demand by reducing the precautionary savings motive. This term will be counter-cyclical if risk increases in a recession as has been documented by Guvenen et al. (2014).

The remaining terms inside the brackets reflect the effect of a change in τ on marginal cost holding x_t fixed. These terms arise because hiring costs are spread over the hours of the worker so that even at given levels of w_t and M_t, a more progressive tax would lower hours worked per worker and raise hiring costs per hour worked. If labor supply were fixed on the intensive margin, these terms would not be present. With a choice of hours of work, we see that higher marginal costs put upward pressure on prices which is reflected in real interest rates via the monetary policy rule and in price dispersion. For similar reasons, an increase in τ puts upward pressure on marginal cost in period 1. As prices are flexible in period 1 this force leads to a reduction in labor market tightness in period 1 so that marginal cost is again equal to the inverse of the desired markup. Because these effects all operate through rescaling the hiring costs, which are themselves small, they are not likely to be important quantitatively.

5.3.4 Slopes of AS and AD

Finally, we turn to Λ, which reflects the slopes of the AS and AD curves. As the previous two propositions showed, a larger Λ attenuates the effect of the social policies on slack, because it makes both AS and AD flatter. The next lemma, proven in appendix D.7, describes what determines it:

Lemma 3. *Under the assumptions of section 5.3:*

$$
\begin{aligned}
\Lambda = & \frac{d \log R_0}{d \log x_0} + (1-\tau)^2 \sigma_\epsilon^2(x_0) \frac{d \log \sigma_\epsilon^2(x_0)}{d \log x_0} + \frac{1-b}{1-u_0+u_0 b} u_0 \frac{d \log u_0}{d \log x_0} \\
& + \frac{d \log h_0}{d \log x_0} + \frac{d \log(1-u_0)}{d \log x_0} + \frac{d \log(1-J_0/Y_0)}{d \log x_0} - \frac{d \log S_0}{d \log x_0}
\end{aligned}
\tag{35}
$$

The first line has the three terms that affect the slope of the AD curve. The first effect involves the real interest rate. A booming economy leads to higher nominal interest rates, both directly via the Taylor rule and indirectly via higher inflation. With nominal rigidities, this raises the real interest rate, which dampens the effectiveness of any policy on equilibrium slack. In other words, a more aggressive monetary policy rule (or more flexible prices) makes AD flatter and so attenuates

the effectiveness of social policies. An extreme example of this is when the economy is at the zero lower bound, which magnifies the effectiveness of the automatic stabilizers.[22]

The second term refers to income risk. If risk is counter-cyclical, this term is negative, so it makes social programs more powerful in affecting slack. The reason is that there is a destabilizing precautionary savings motive that amplifies demand shocks. In response to a reduction in aggregate demand, labor market tightness falls, leading to an increase in risk and an increase in the precautionary savings motive and so a further reduction in aggregate demand. These reinforcing effects make the AD curve steeper so that social policies become more effective.[23]

The third term reflects the impact of economic expansions on the number of employed households, who consume more than unemployed households. Therefore, aggregate demand rises as employment rises in a tighter labor market. This consumption multiplier makes the AD curve steeper and increases the effectiveness of social programs.

The second line in the lemma has the four terms that affect the slope of the AS. Increasing slack raises hours worked or employment, this makes the AS flatter as output increases by relatively more, so it raises Λ and attenuates the effect of social programs. This occurs net of hiring costs, since the fact that they increase with slack works in the opposite direction. Finally, if in a booming economy the efficiency loss from price dispersion increases, so S increases, then the AS is likewise steeper and Λ is lower.

5.4 Summary and likely sign

To summarize, there are two main channels through which unemployment benefits or income tax progressivity raise aggregate demand and thereby eliminate slack. These channels are redistribution and social insurance, and both are increasing in the unemployment rate. As unemployment and income risks are counter-cyclical, these forces push for counter-cyclical elasticities of slack to the social programs, since they dampen the counter-cyclical fluctuations in the precautionary savings motive. If business cycles are inefficient in the sense that tightness is inefficiently low in a recession, then we expect a positive covariance between dW_t/dx_t and the elasticities of tightness with respect to policy. This positive covariance implies a positive aggregate stabilization term and more generous

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[22] McKay and Reis (2016) and Kekre (2015) show that automatic stabilizers and unemployment benefits, respectively, have stronger stimulating effects when the economy is at the zero lower bound.

[23]Similar reinforcing dynamics arise out of unemployment risk in Ravn and Sterk (2013), Den Haan et al. (2015), and Heathcote and Perri (2015).

unemployment benefits and a more progressive tax system even if the business cycle is efficient on average.

6 Quantitative analysis

We have shown that business cycles lead to a macroeconomic stabilization term that affects the optimal generosity of unemployment insurance and the progressivity of income taxes, and that this term likely makes these programs more generous and progressive, respectively, through multiple channels. We now ask whether these effects are quantitatively significant. To do so, we solve and calibrate the model in sections 2 and 3, without the simplifications made in sections 4 to 5 used for analytical characterizations.

6.1 Calibration and solution of the model

In our baseline case, we define slack as $x_t = (1-u_t)/(1-\bar{u})$ where $\bar{u}$ is the steady state unemployment rate, instead of the job-finding rate as before. This makes the calibration more transparent since the unemployment rate is a more commonly used indicator of labor market conditions.[24] For similar reasons, we use the Calvo model of price stickiness. We solve the model using global methods as described in Appendix E.2, so that we can accurately compute social welfare, assuming that the economy starts at date 0 at the deterministic steady state. We then numerically search for the values of b and τ that maximize the social welfare function, and compare these with the maximal values in a counterfactual economy without aggregate shocks, but otherwise identical.

Table 1 shows the calibration of the model, dividing the parameters into different groups. The first group has parameters that we set ex ante to standard choices in the literature. Only the last one deserves some explanation. ψ_2 is the elasticity of hiring costs with respect to labor market tightness, and we set it at 1 as in Blanchard and Galí (2010), in order to be consistent with an elasticity of the matching function with respect to unemployment of 0.5 as suggested by Petrongolo and Pissarides (2001).

Panel B contains parameters individually calibrated to hit some time-series moments. For the preference for public goods, we target the observed average ratio of government purchases to GDP

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[24]Quantitatively though, it makes no difference as in our baseline specification there is an almost perfect relationship between the job finding rate and this measure of slack so with an appropriate recalibration of the model we could treat the job-finding rate as the measure of slack.

Symbol	Value	Description	Target
Panel A. Parameters chosen ex ante			
δ	1/200	Mortality rate	50 year expected lifetime
$\frac{\mu}{\mu-1}$	6	Elasticity of substitution	Basu and Fernald (1997)
θ	1/3.5	Prob. of price reset	Klenow and Malin (2010)
$1/\gamma$	1/2	Frisch elasticity	Chetty (2012)
ψ_2	1	Elasticity of hiring cost	Blanchard and Galí (2010)
Panel B. Parameters individually calibrated			
χ	0.262	Preference for public goods	$G/Y = 0.207$
ω_π	1.66	Mon. pol. response to π	Estimated Taylor rule
ω_u	0.133	Mon. pol. response to u	Estimated Taylor rule
υ	0.153	Job separation rate	Average value
$F(\epsilon,.)$	mix-normals	Skill-risk process	Guvenen and McKay (2016)
Panel C. Parameters jointly calibrated to steady-state moments			
β	0.974	Discount factor	3% annual real interest rate
$\bar{w}$	0.809	Average wage	Unemployment rate $= 6.1\%$
ψ_1	0.0309	Scale of hiring cost	Recruiting costs of 3% of pay
$1/\kappa$	0.0810	Search effort elasticity	$d\log u/d\log b|_x = 0.5$
ξ	0.230	Pain from unemployment	Leisure benefit of unemployment
Panel D. Parameters jointly calibrated to volatilities			
ρ	0.9	Autocorrelation of shocks	0.9
$\text{StDev}(\eta^A)$	0.724%	TFP innovation	$\text{StDev}(u_t) = 1.59\%$
$\text{StDev}(\eta^I)$	0.244%	Monetary policy innovation	$\text{StDev}(Y|\eta^A) = \text{StDev}(Y|\eta^I)$
$\text{StDev}(\eta^G)$	4.63%	Government purchases innovation	$\text{StDev}(G_t/Y_t) = 1.75\%$
ζ	1.68	Elasticity of wage w.r.t. x	$\text{StDev}(h_t)/\text{StDev}(1-u_t) = 0.568$
Panel E: Automatic stabilizers			
b	0.783	UI replacement rate	Rothstein and Valletta (2014)
τ	0.151	Progressivity of tax system	Heathcote et al. (2014)

Table 1: Calibrated parameter values and targets

in the US in 1984-2007. For the monetary policy rule, we use OLS estimates of equation (8). The parameter υ determines the probability of not having a job in our model, which we set equal to the sum of quarterly job separation rate, which we construct following Shimer (2012), and the average unemployment rate. Finally, Guvenen and McKay (2016) estimate a version of the income innovation process that we have specified in equation (3) using a mixture of normals as a flexible parameterization of the distribution $F(\epsilon'; \cdot)$. Two of the mixtures shift with a measure of slack to match the observed pro-cyclical skewness of earnings growth rates documented by Guvenen et al. (2014), and we take the unemployment rate to be the measure of slack in our implementation. Appendix E.1 provides additional details.

Panel C instead has parameters chosen jointly to target a set of moments. We target the average unemployment rate between 1960 to 2014 and recruiting costs of 3 percent of quarterly pay, consistent with Barron et al. (1997). The parameter κ controls the marginal disutility of effort searching for a job, and we set it to target a micro-elasticity of unemployment with respect to benefits of 0.5 as reported by Landais et al. (2015). Last in the panel is ξ, the non-pecuniary costs of unemployment. In the model, the utility loss from unemployment is $\log(1/b) - h^{1+\gamma}/(1+\gamma) + \xi$, reflecting the loss in consumption, the gain in leisure, and other non-pecuniary costs of unemployment. We set $\xi = h^{1+\gamma}/(1+\gamma)$ in the steady state of our baseline calibration so that the benefit of increased leisure in unemployment is dissipated by the non-pecuniary costs.

Panel D calibrates the three aggregate shocks in our model that perturb productivity, monetary policy, and the rule for public expenditures. In each case we assume that the exogenous process is an AR(1) in logs with common autocorrelation. We set the variances to match three targets: (i) the standard deviation of the unemployment rate, (ii) the standard deviation of G_t/Y_t, and (iii) equal contributions of productivity and monetary shocks to the variance of aggregate output. Finally, we set $\zeta = 1.68$ to match the standard deviation of hours per worker relative to the standard deviation of the employment-population ratio.

Finally, panel E has the baseline values for the automatic stabilizers. For τ we adopt the estimate of 0.151 from Heathcote et al. (2014). For b we choose $b^{1/(1-\tau)} = 0.75$, consistent with a 20-25% decline in household income during unemployment reported by Rothstein and Valletta (2014). Note that this implies interpreting a household as having two workers only one of whom becomes unemployed so $b^{1/(1-\tau)} = 0.75$ corresponds to a 50 percent replacement rate of one worker's
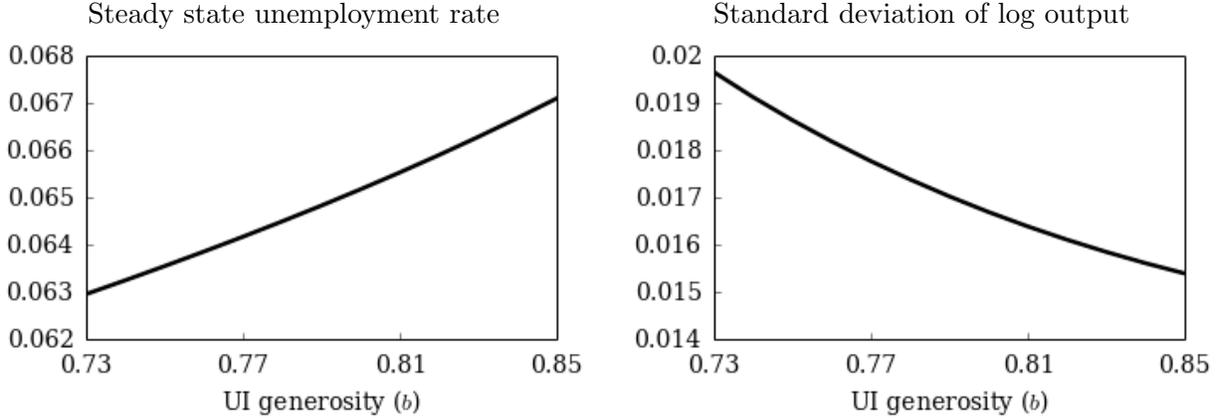
Figure 2: Effects of changing b for $\tau = 0.26$

income. Our focus is on the effect that aggregate shocks have on these choices, not on their levels.

As a check on the model's performance, we computed the standard deviation of hours, output, and inflation in the model, which are 0.74%, 1.67%, and 0.65%. The equivalent moments in the US data 1960-2014 are 0.84%, 1.32%, and 0.55%.

6.2 Optimal unemployment insurance

Our first main quantitative result is that aggregate shocks increase the optimal b to 0.805 from 0.752 in the absence of aggregate shocks. Converting the values for b into pre-tax UI replacement rates based on a two-earner household then we have an optimal replacement rate of 49 percent with aggregate shocks as compared to 36 percent without.[25]

Figure 2 provides a first hint for why this effect is so quantitatively large. It shows the effects of raising b on the steady state unemployment rate and on the volatility of log output. Raising the generosity of unemployment benefits hurts the incentives for working, so unemployment rises somewhat. However, it has a strong macroeconomic stabilizing effect.

Figure 3 provides a different way to look at this result. Equation (27), repeated her for conve-

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[25]The conversion we apply here is to solve for x in $(x/2 + 1/2)^{1-\tau} = b$.

nience:

$$\begin{aligned}
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t W_t = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\mathbb{E}_i \log \left(\alpha_{i,t}^{1-\tau} \right) - \log \left(\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] \right) \right] \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[u_t \log b - \log \left(1 - u_t + u_t b \right) \right] \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_t) - (1 - u_t) \frac{h_t^{1+\gamma}}{1+\gamma} - \upsilon \frac{q_t^{1+\kappa}}{1+\kappa} + \chi \log(G_t) - \xi u_t \right],
\end{aligned}$$

shows that welfare is the sum of three components: the welfare loss from income inequality that results from skill risk, $\alpha_{i,t}$, the welfare loss from income inequality as a result of unemployment, and the utility that derives from aggregates. Figure 3 plots each of these components in consumption equivalent units and their sum, as we vary b, both with and without business cycles.[26]

As expected, higher b provides insurance by lowering idiosyncratic income risk (top-left panel) and unemployment risk (top-right panel). More interesting, the presence of aggregate shocks has a large effect on the first effect, but a negligible one on the second. The reason is that, by stabilizing the business cycle, a higher b leads to less pre-tax income inequality. Idiosyncratic income risk is persistent and it increases in recessions. The contribution of skill risk to welfare is strongly non-linear in the state of the business cycle and mitigating the most severe recessions leads to a considerable gain in welfare. Instead, the welfare loss from unemployment risk is close to linear in the state of the business cycle so the presence of aggregate risk and the stabilizing effects of benefits have little effect on this component of welfare.

The next two panels confirm that uninsured skill risk drives the results. The welfare coming from aggregates, in the lower-left panel, falls with b because of the moral hazard effect on the unemployment rate, but the presence of aggregate shocks has a modest effect on the slope of the relation between welfare and b. The sum of social welfare, plotted in the lower-right panel of figure 3 mimics the patterns in the top-left panel. The optimal unemployment benefit is substantially larger with aggregate shocks because it stabilizes the business cycle leading to welfare gains by

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[26]The consumption equivalent units are relative to the social welfare function in the deterministic steady state associated with the policy parameters that are optimal without aggregate shocks. Suppose the additively decomposed social welfare function for a given policy is $V^A + V^B + V^C$. Let $\tilde{V}^A + \tilde{V}^B + \tilde{V}^C$ be the social welfare function of in the steady state with the policy parameters that are optimal without aggregate shocks. If we rescaled the consumption of all households by a factor $1+\Delta$, the latter would become $\tilde{V}^A + \tilde{V}^B + \tilde{V}^C + \log(1+\Delta)/(1-\beta)$. The top-left panel plots $\Delta^A = \exp\{(1-\beta)(V^A - \tilde{V}^A)\} - 1$. The bottom-right panel plots $\Delta = \exp\{(1-\beta)(V^A + V^B + V^C - \tilde{V}^A - \tilde{V}^B - \tilde{V}^C)\} - 1$. Notice that $1 + \Delta = (1 + \Delta^A)(1 + \Delta^B)(1 + \Delta^C)$.
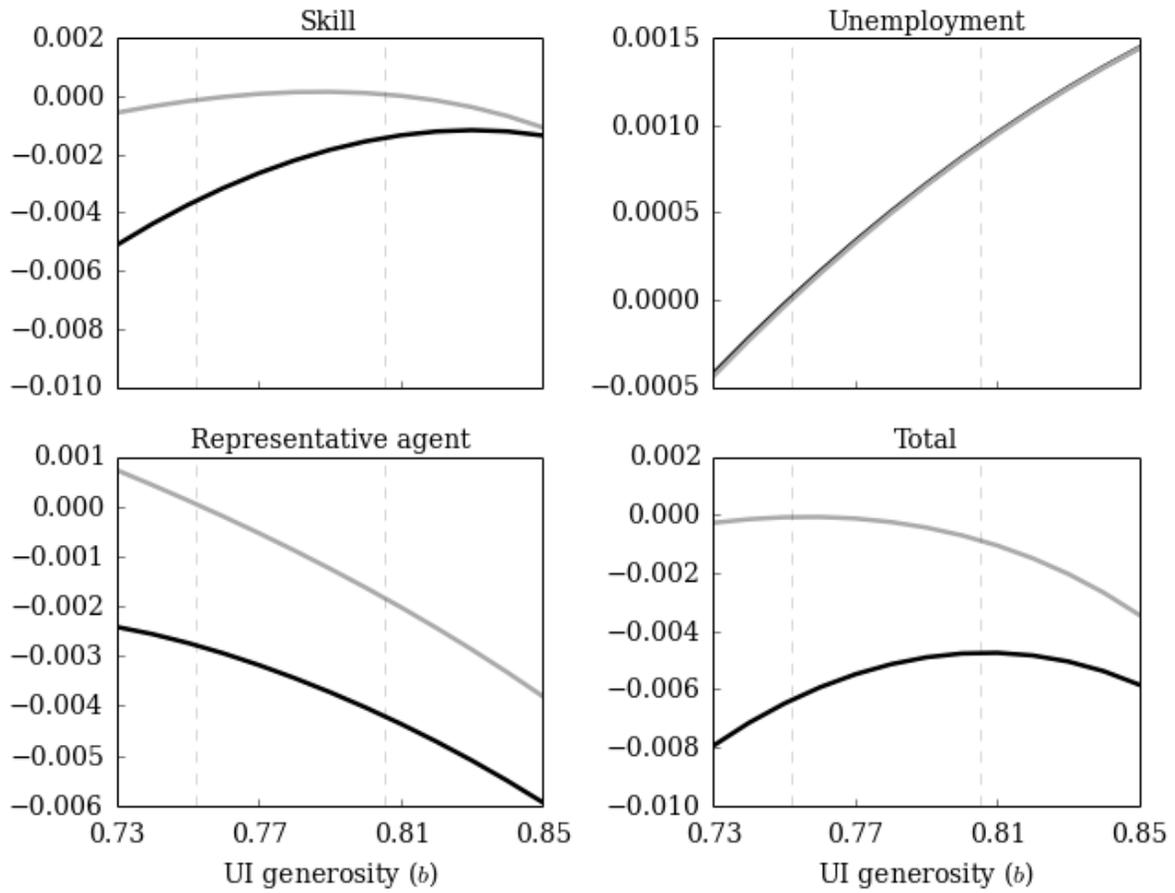
Figure 3: Decomposition of welfare in consumption equivalent units. $\tau = 0.26$. With aggregate shocks (black) and without (gray). Vertical lines show the optimal b with and without aggregate shocks.
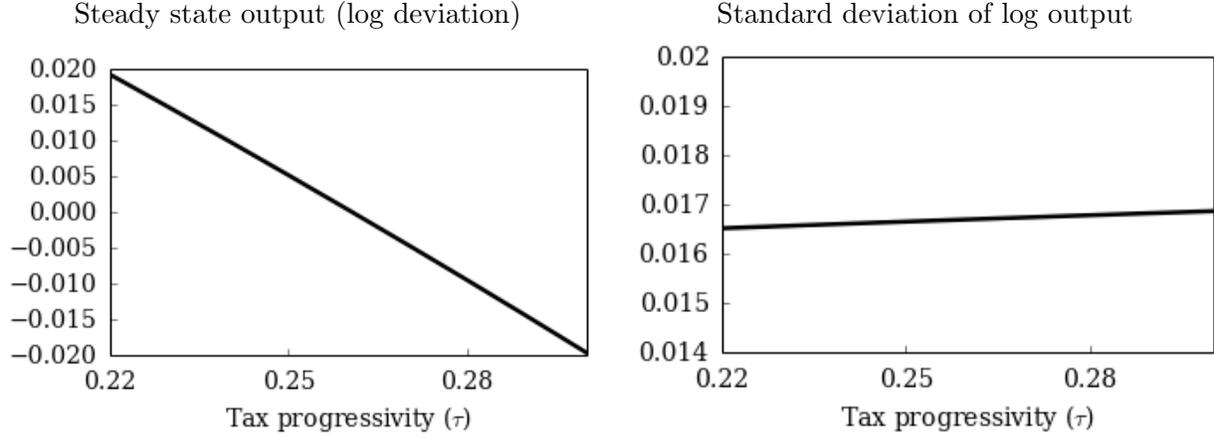
Steady state output (log deviation)   Standard deviation of log output

Figure 4: Effects of changing τ for $b = 0.80$. The left panel shows log output as a deviation from the value at $\tau = 0.265$. The right panel uses the same vertical axis as figure 2 for comparison.

reducing the fluctuations in uninsured risk.

6.3 Optimal income tax progressivity

Our second main quantitative result is that the optimal tax progressivity is almost unchanged at 0.264 relative to 0.265 without aggregate shocks.

The left panel of figure 4 shows the steady state level of output as a log deviation from the level under the optimal policy without aggregate shocks. There is a strong negative effect of progressivity on output that works through the disincentive effect on labor supply. At the optimal policy without aggregate shocks, the welfare loss from reducing aggregate output is balanced by the welfare gain from insuring skill risk. When we introduce aggregate shocks, the level of tax progressivity has essentially no effect on the volatility of the business cycle as shown in the right-panel of figure 4 and therefore there is no stabilization benefit of raising progressivity.

6.4 Alternative specifications

Table 2 repeats our optimal policy calculation for alternative calibrations of the model in order to confirm what is driving the conclusions and assess their robustness. Row (i) repeats our baseline results for comparison.

Row (ii) shows that macroeconomic stabilization concerns have almost no effect on the optimal b and τ when prices are flexible. This confirms the important role of aggregate demand and inefficient

	Aggregate shocks................	b		τ		replacement rate	
		without	with	without	with	without	with
(i)	Baseline	0.752	0.805	0.265	0.264	0.36	0.49
(ii)	Flexible prices	0.752	0.751	0.265	0.265	0.36	0.36
(iii)	Aggressive monetary policy	0.752	0.768	0.265	0.265	0.36	0.40
(iv)	Acyclical purchases	0.752	0.797	0.265	0.257	0.36	0.47
(v)	Volatile business cycle	0.752	0.809	0.265	0.246	0.36	0.51
(vi)	Smaller demand shocks	0.752	0.814	0.265	0.256	0.36	0.52
(vii)	Larger demand shocks	0.752	0.792	0.265	0.243	0.36	0.47
(viii)	Positive wage elasticity	0.492	0.708	0.241	0.276	–	0.24

Table 2: Optimal policies under alternative specifications. Replacement rates are calculated based on two-worker households as described in footnote 25.

business cycles.

Rows (iii) and (iv) investigate the interaction with other policies. Row (iii) increases the coefficient on inflation in the monetary policy rule to 2.50 from 1.66. With this more aggressive monetary policy rule there is less of a need for fiscal policy to manage aggregate demand. Therefore, stabilization plays a smaller role in the design of the optimal social insurance system than in the baseline calibration. Another way to understand these results is in terms of the slope of the AD curve in figure 1. Flexible prices or aggressive monetary policy make the real interest rate responds strongly to changes in slack. From Lemma 3 then, the AD curve will be flatter, and so the elasticities of slack with respect to the social programs is small leading to a small automatic-stabilizer role.

Row (iv) changes instead the policy rule for government purchases. Our baseline specification, following the Samuelson rule, makes government purchases pro-cyclical. Row (iv) considers instead acyclical government purchases, as we observe in the data, by using instead the rule: $G_t = \bar{G}\eta_{lt}^G$. Because the pro-cyclical rule amplified the business cycle it left a larger role for the automatic stabilizers, but the effect is quantitatively minor.

Rows (v), (vi), and (vii) show that the calibration of the aggregate shocks is not crucial to our results. Row (v) raises the standard deviations of the productivity and monetary policy shocks by 25%. As one might expect, when the aggregate shocks are more volatile, stabilization plays a larger role in the choice of b and τ. In our baseline calibration, the productivity and monetary policy shocks contribute equally to the variance of log output. In row (vi) we consider the case where productivity shocks account for 75% of the variance and monetary policy shocks account for

25% and in row (vii) we consider the reverse case where productivity shocks account for 25%. As the ratio of the variance of monetary relative to productivity shocks rises, the automatic stabilizer role of social insurance programs diminishes but only slightly. To understand these findings, again consider Lemma 3. The precautionary savings motive rises as the unemployment rate increases, which leads to a steeper AD curve. This implies that any shifts in the AS or AD curves will have larger effects on slack. This time-varying precautionary motive is an important part of our quantitative analysis and unemployment insurance has a strong stabilizing effect on the economy by interfering with this self-reinforcing mechanism. As unemployment insurance flattens the AD curve, it leads to smaller fluctuations in slack for any type of shock.

Finally, row (viii) considers an alternative specification of the wage mechanism in which $\bar{w}$ responds to increases in the unemployment benefit. Our baseline specification assumes that the steady state wage is essentially unchanged across policy regimes.[27] This specification is consistent with empirical work that fails to find an effect of unemployment benefits on wages.[28] However, some authors have found large effects of unemployment benefits on the equilibrium unemployment rate possibly reflecting general equilibrium effects operating through wages (Hagedorn et al., 2013). We explore how our analysis is affected when the steady state wage is increasing in the unemployment benefit.

In this specification we assume that $\bar{w}$ has an elasticity of 0.1 with respect to b. As one would expect, with a positive wage elasticity the steady state unemployment rate increases with the benefits more strongly than before. Higher benefits raise wages, which lowers the incentives for firms to hire workers leading to a lower job-finding rate. Moreover, the incentives for searching for a job fall both due to the decline in the job-finding rate and because of the usual moral hazard effect. With a positive wage elasticity, the greater sensitivity of the steady state unemployment rate to the unemployment benefit leads to a much lower optimal benefit in the absence of aggregate shocks: 0.49 as opposed to 0.71. However, when there are aggregate shocks, a very low benefit leads to strong de-stabilizing dynamics because of the precautionary savings motive. Such low levels of the benefit lead to very costly fluctuations, so the automatic-stabilizer nature of unemployment benefits is much stronger. As a result, in this case we find that aggregate shocks lead to a large

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[27]It is not exactly constant due to changes in the term $1 - J_t/Y_t$ although in practice this term is small.

[28]Card et al. (2007), Van Ours and Vodopivec (2008), Lalive (2007), and Johnston and Mas (2015) find no evidence that UI generosity affects earnings upon re-employment.

change in the optimal benefit relative to what is optimal in steady state. A positive wage elasticity with respect to benefits leads to lower replacement rates overall, but a bigger effect of the business cycle on the optimal policy.

7   Conclusion

Policy debates take as given that there are stabilizing benefits of unemployment insurance and income tax progressivity, but there are few systematic studies of what factors drive these benefits and how large they are. The study of these social programs rarely takes into account this macroeconomic stabilization role, instead treating it a fortuitous side benefits.

This paper tried to remedy this situation. Our theoretical study provided a theoretical characterization of what is an automatic stabilizer. In general terms, an automatic stabilizer is a fixed policy for which there is a positive covariance between the effect of slack on welfare, and the effect of the policy on slack. If a policy tool has this property of stimulating the economy more in recessions when slack is inefficiently high, then its role in stabilizing the economy calls for expanding the use of the policy beyond what would be appropriate in a stationary environment. We showed what factors drive this covariance, through which economic channels they operate, and what makes them larger or smaller. Overall, we found that the role of social insurance programs as automatic stabilizers affects their optimal design and, in the case of unemployment insurance, it can lead to substantial differences in the generosity of the system.

Our focus on the automatic stabilizing nature of existing social programs led us to take a Ramsey approach to the ex ante design of fiscal policy. A different question is whether macroeconomic stabilization concerns interfere with the incentive problems that give rise to these programs, as in the Mirrleesian literature. Another question is how these fiscal policy programs can adjust to the state of the business cycle, taking into account measurement difficulties, time inconsistency, and the many other issues that the literature on monetary policy has faced. There is already some progress on these two topics, and hopefully our analysis will provide some insights to its further development.

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Appendix

This appendix contains five sections, which: describe the role of the wage rule introduced in section 2; provide auxiliary steps to some results stated in section 3; prove the propositions in sections 4; derive the AD-AS apparatus and prove the propositions in section 5; and describe the methods used to solve the model in section 6.

A   The role of the wage rule in our analysis

We assumed that the wage was determined by equation (6). A more general specification uses a generic wage function:

$$w_t = w(\eta_t, x_t, b, \tau),$$

that maps the aggregate shocks, slack (or labor market tightness), and policy parameters into a wage. Nash bargaining is an example of a wage mechanism that would take this form.

In this more general case, starting from (44) we can write h_t as:

$$h_t = \left\{ (1 - \tau) \left[\frac{\eta_t^A}{S(x_t)} \left(1 - \frac{J_t}{Y_t} \right) \right]^{-1} w(\eta_t^A, x_t, b, \tau) \right\}^{1/(1+\gamma)}$$

where $S(x_t)$ is the level of price dispersion associated with that level of slack. Using the generic wage rule and equations (4), (5), (21), (25), (42), and (44), we can write J_t/Y_t as a function of (η_t, x_t, b, τ) to yield:

$$h_t = \{ (1 - \tau) H(\eta_t, x_t, b, \tau) \}^{1/(1+\gamma)},$$

where $H_t \equiv \left[\frac{\eta_t}{S(x_t)} \left(1 - \frac{J_t}{Y_t} \right) \right]^{-1} w(\eta_t, x_t, b, \tau)$. The purpose of assuming the wage rule in equation (6) in the paper is that H_t simplifies to x_t^ζ.

However, not making this simplification, and so carrying the new $H(.)$ term along only adds to our results a few additional terms. To start, in addition to those terms that appear in equation

(28) (Proposition 1), we would now need to add:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dH_t} \left. \frac{\partial H_t}{db} \right|_x .$$

Similarly, we would need to modify equation (29) (Proposition 2) to include a term:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 - u_t) \left[\frac{A_t}{C_t} - h_t^\gamma \right] \frac{dh_t}{dH_t} \left. \frac{\partial H_t}{d\tau} \right|_x .$$

Both of these terms relate to the adjustment of hours on the intensive margin. Intuitively, the wage plays two roles in the economy: it determines the marginal cost of the firms and therefore the incentives for hiring and it determines the incentives for labor supply on the intensive margin. The first effect is captured in our analysis through dx_t/db. Using our wage rule, the second effect is captured through $\frac{dh_t}{dx_t} \frac{dx_t}{db}$. With a more general wage rule brings in these new effects of policy on intensive margin labor supply. While they result in a different value for dx_t/db and $dx_t/d\tau$, they are not qualitatively different considerations than the ones we focus on already.

B   Additional steps for deriving the results in section 3

B.1   The value of employment

In equilibrium, $a_{i,t} = 0$, and search effort is determined by comparing the value of working and not working according to equation (16). This section of the appendix derives the two key steps that make this difference independent of the household's type, so that all households choose the same search effort.

Lemma 4. *Suppose the household's value function has the form*

$$V(\alpha, n, \mathcal{S}) = V^\alpha(\alpha, \mathcal{S}) + V^n(a, n, \mathcal{S})$$

for some functions V^α and V^n. The choice of search effort is then the same for all searching households regardless of α.

Proof: The household's search problem is

$$V^s(\alpha, \mathcal{S}) = \max_q \left\{ MqV(\alpha, 1, \mathcal{S}) + (1 - Mq)V(\alpha, 0, \mathcal{S}) - \frac{q^{1+\kappa}}{1+\kappa} \right\}.$$

Substitute for the value functions to arrive at

$$V^s(\alpha, \mathcal{S}) = \max_q \left\{ Mq\left[V^\alpha(\alpha, \mathcal{S}) + V^n(a, 1, \mathcal{S})\right] + (1 - Mq)\left[V^\alpha(\alpha, \mathcal{S}) + V^n(a, 0, \mathcal{S})\right] - \frac{q^{1+\kappa}}{1+\kappa} \right\}$$

$$V^s(\alpha, \mathcal{S}) = V^\alpha(\alpha, \mathcal{S}) + \max_q \left\{ MqV^n(1, \mathcal{S}) + (1 - Mq)V^n(0, \mathcal{S}) - \frac{q^{1+\kappa}}{1+\kappa} \right\}, \tag{36}$$

where we have brought $V^\alpha(\alpha, \mathcal{S})$ outside the max operator as it appears in an additively separable manner. As there is no α inside the max operator, the optimal q is independent of α. $\qquad\square$

Lemma 5. *The household's value function can be written as*

$$V(\alpha, n, \mathcal{S}) = \frac{1 - \tau}{1 - \beta} \log(\alpha) + n \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right] + \bar{V}(\mathcal{S}) \tag{37}$$

for some function $\bar{V}$.

Proof. Suppose that the value function is of the form given in (37). We will establish that the Bellman equation maps functions in this class into itself, which implies that the fixed point of the Bellman equation is in this class by the contraction mapping theorem. V^s will then be

$$V^s(\alpha, \mathcal{S}) = \frac{1 - \tau}{1 - \beta} \log(\alpha) + Mq^* \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right] - \frac{q^{*1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S})$$

and the choice of q^* is independent of α by Lemma 4. Regardless of employment status, the continuation value is

$$(1 - \upsilon)V(\alpha', 1, \mathcal{S}') + \upsilon V^s(\alpha', \mathcal{S}')$$

$$= (1 - \upsilon)\left[\frac{1 - \tau}{1 - \beta} \log(\alpha') + \left[\log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi \right] + \bar{V}(\mathcal{S}') \right]$$

$$+ \upsilon \left[\frac{1 - \tau}{1 - \beta} \log(\alpha') + M'q^{*'} \left[\log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} + \xi \right] - \frac{q^{*'1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}') \right]$$

$$= \frac{1 - \tau}{1 - \beta} \log(\alpha') + g(\mathcal{S}'),$$

where

$$g(\mathcal{S}) \equiv (1 - \upsilon + \upsilon M' q^{*\prime}) \left[\log \frac{1}{b} - \frac{h'^{1+\gamma}}{1+\gamma} \right] - (1 - \upsilon) \frac{q^{*\prime 1+\kappa}}{1+\kappa} + \bar{V}(\mathcal{S}').$$

Turning to the Bellman equation, we have

$$V(\alpha, n, \mathcal{S}) = \log \left[\lambda(n + (1-n)b) \left(\alpha(wh+d) \right)^{1-\tau} \right] - n \frac{h^{1+\gamma}}{1+\gamma} - (1-n)\xi + \beta \mathbb{E} \left[\frac{1-\tau}{1-\beta} \log(\alpha') + g(\mathcal{S}') \right]$$

$$= \frac{1-\tau}{1-\beta} \log(\alpha) + n \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1+\gamma} + \xi \right]$$

$$+ \log \left[\lambda (wh+d)^{1-\tau} \right] + \log b - \xi + \beta \frac{1-\tau}{1-\beta} \mathbb{E} \left[\log(\epsilon') \right] + \beta \mathbb{E} \left[g(\mathcal{S}') \right].$$

Finally, $\bar{V}(\mathcal{S})$ is given by the second row of the expression above. $\qquad\square$

B.2 Proof of lemma 2

First, the Euler equation for a household is

$$\frac{1}{c_{i,t}} \geq \beta R_t \mathbb{E} \left[\frac{1}{c_{i,t+1}} \right].$$

as usual. At the same time, as we showed in the text, the consumption of an individual is

$$c_{i,t} = \lambda_t \alpha_{i,t}^{1-\tau} \left(w_t h_t + d_t \right)^{1-\tau} (n_{i,t} + (1 - n_{i,t})b). \tag{38}$$

Replacing individual consumption into the Euler equation, and rewriting gives:

$$\frac{1}{\lambda_t \left(w_t h_t + d_t \right)^{1-\tau}} \geq \beta R_t \mathbb{E}_t \left\{ \frac{(1 - u_{t+1}) + u_{t+1} b^{-1}}{\lambda_{t+1} \left(w_{t+1} h_{t+1} + d_{t+1} \right)^{1-\tau}} \right\} \mathbb{E}_t \left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] [n_{i,t} + (1 - n_{i,t})b]. \tag{39}$$

Notice that $\mathbb{E} \left[\frac{\alpha_{i,t}^{1-\tau}}{\alpha_{i,t+1}^{1-\tau}} \right] = \mathbb{E} \left[\epsilon_{i,t+1}^{\tau-1} \right]$ is common across households. This Euler equation only differs across households due to the final term involving $n_{i,t}$. Assuming the unemployment benefit replacement rate is less than one, this term will be larger for employed than unemployed so in equilibrium all unemployed will be constrained and the Euler equation will hold with equality for all employed.[29]

---

[29]Here we follow Krusell et al. (2011), Ravn and Sterk (2013), and Werning (2015) in assuming that the Euler equation of the employed/high-income household holds with equality. This household is up against its constraint $a' = 0$ so there could be other equilibria in which the Euler equation does not hold with equality. The equilibrium

Summing equation (38) across households gives:

$$C_t = \lambda_t \left(w_t h_t + d_t\right)^{1-\tau} \left(1 - u_t + u_t b\right) \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right]. \tag{40}$$

Solve (40) for $\lambda_t \left(w_t h_t + d_t\right)^{1-\tau}$ and substitute it into (38) to arrive at:

$$c_{i,t} = \alpha_{i,t}^{1-\tau}(n_{i,t} + (1 - n_{i,t})b) \frac{C_t}{(1 - u_t + u_t b)\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right]}. \tag{41}$$

Equations (17) and (22) follow from (41) and the definition of $\tilde{c}_t$.

To derive equations (18) and (19), solve (40) for $\lambda_t \left(w_t h_t + d_t\right)^{1-\tau}$ and substitute it into (39) holding with equality and $n_{i,t} = 1$ to arrive at:

$$\frac{(1 - u_t + u_t b)\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right]}{C_t} = \beta R_t \mathbb{E}_t \left\{ \left[(1 - u_{t+1}) + u_{t+1}b^{-1}\right] \frac{(1 - u_{t+1} + u_{t+1}b)\mathbb{E}_i \left[\alpha_{i,t+1}^{1-\tau}\right]}{C_{t+1}} \right\} \mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right]$$

and substitute in $\tilde{c}_t$ using (17):

$$\frac{1}{\tilde{c}_t} = \beta R_t \mathbb{E}_t \left\{ \left[(1 - u_{t+1}) + u_{t+1}b^{-1}\right] \frac{1}{\tilde{c}_{t+1}} \right\} \mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right].$$

Rearranging gives eqations (18) and (19) as $\mathbb{E}\left[\epsilon_{i,t+1}^{\tau-1}\right]$ is known at date t.

B.3 Optimal hours worked and search effort

We start by deriving equation (20). Using labor market clearing and the definition of A_t we can write the aggregate production function as

$$Y_t = A_t h_t (1 - u_t). \tag{42}$$

---

we focus on is the limit of the unique equilibrium as the borrowing limit approaches zero from below. See Krusell et al. (2011) for further discussion of this point.

Output net of hiring costs is paid to employed workers in the form of wage and dividend payments. As the average $\alpha_{i,t}$ is equal to one we have

$$Y_t - J_t = (w_t h_t + d_t)(1 - u_t). \tag{43}$$

Multiply both sides of (42) by $(Y_t - J_t)/Y_t$ and substitute for $Y_t - J_t$ using (43) to give:

$$w_t h_t + d_t = A_t h_t \frac{Y_t - J_t}{Y_t}.$$

Substitute this for $w_t h_t + d_t$ in equation (15) to arrive at

$$h_t^\gamma = \frac{(1 - \tau)w_t}{A_t h_t \frac{Y_t - J_t}{Y_t}}. \tag{44}$$

Finally, use the wage rule in equation (6) to arrive at (20).

We turn next to derive optimal search effort in equation (21). By Lemma 5, the results of Lemma 4 can be applied. Proceed from equation (36) using the functional form for the value function in Lemma 5. This leads to

$$\max_q \left\{ Mq \left[\log \frac{1}{b} - \frac{h^{1+\gamma}}{1 + \gamma} + \xi \right] - \frac{q^{1+\kappa}}{1 + \kappa} \right\}.$$

The first order condition yields equation (21).

B.4 Equilibrium definition

We first state some addition equilibrium conditions and then state a definition of an equilibrium. Here we focus on the Calvo-pricing version of the model for concreteness.

The aggregate resource constraint is:

$$Y_t - J_t = C_t + G_t. \tag{45}$$

The Fisher equation is:

$$R_t = I_t / \mathbb{E}_t [\pi_{t+1}]. \tag{46}$$

The price-setting first order condition leads to:

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s h_s + \psi_1 M_s^{\psi_2}\right)/(A_s h_s)}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}. \tag{47}$$

The link between $\tilde{c}_t$ and C_t depends on $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right]$. This evolves according to:

$$\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right] = (1-\delta)\mathbb{E}_i \left[\alpha_{i,t-1}^{1-\tau}\right] \mathbb{E}_i \left[\epsilon_{i,t}^{1-\tau}\right] + \delta.. \tag{48}$$

An equilibrium of the economy can be calculated from a system equations in 17 variables and three exogenous processes. The variables are

$$C_t, \tilde{c}_t, u_t, \mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right], Q_t, R_t, I_t, \pi_t, Y_t, G_t, h_t, w_t, S_t, \frac{p_t^*}{p_t}, J_t, q_t, M_t.$$

And the equations are: (4), (5), (6), (7), (8), (9), (17) (18), (19), (20), (21), (24), (42), (45), (46), (47), and (48). The exogenous processes are η_t^A, η_t^G, and η_t^I.

C   Proofs for section 4

C.1   Skill dispersion with log normality

Lemma 6. *Under no mortality, $\delta = 0$, and a log normal income process:*

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, \mathbb{E}_i \left[\log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau}\right]\right)\right] = \log\left(\alpha_{i,0}^{1-\tau}\right) - \log\left(\mathbb{E}_i \left[\alpha_{i,0}^{1-\tau}\right]\right) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\beta}{1-\beta}(1-\tau)^2 \frac{\sigma^2(x_t)}{2}$$

Proof: When there is no mortality, $\delta = 0$, we can compute the cumulative welfare effect of a change in $F(\epsilon_{i,t+1}, x_t)$ including the effects on current and future skill dispersion. In particular

$$\begin{aligned}
\mathbb{E}_i \log\left(\alpha_{i,t}^{1-\tau}\right) &= \mathbb{E}_i \log\left(\alpha_{i,t-1}^{1-\tau} \epsilon_{i,t}^{1-\tau}\right) \\
&= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau} \epsilon_{i,1}^{1-\tau} \cdots \epsilon_{i,t}^{1-\tau}\right) \\
&= \mathbb{E}_i \log\left(\alpha_{i,0}^{1-\tau}\right) + \mathbb{E}_i \log\left(\epsilon_{i,1}^{1-\tau}\right) + \cdots + \mathbb{E}_i \log\left(\epsilon_{i,t}^{1-\tau}\right).
\end{aligned}$$

Similarly

$$\log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) = \log\left(\mathbb{E}_i\left[\alpha_{i,t-1}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)$$

$$= \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\mathbb{E}_i\left[\alpha_{i,1}^{1-\tau}\right]\cdots\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)$$

$$= \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right) + \log\left(\mathbb{E}_i\left[\alpha_{i,1}^{1-\tau}\right]\right) + \cdots + \log\left(\mathbb{E}_i\left[\epsilon_{i,t}^{1-\tau}\right]\right)$$

Notice that in this no-mortality case, the date-t loss from skill dispersion can be written as:

$$\mathbb{E}_i\log\left(\alpha_{i,t}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\right) = \mathbb{E}_i\log\left(\alpha_{i,0}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\alpha_{i,0}^{1-\tau}\right]\right) + \sum_{s=1}^{t}\left[\mathbb{E}_i\log\left(\epsilon_{i,s}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,s}^{1-\tau}\right]\right)\right].$$

The level of slack at date t, x_t, determines the contribution $\mathbb{E}_i\log\left(\epsilon_{i,t+1}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]\right)$ that will appear in W_s for all future dates $s > t$. Using the log-normal income process, we have

$$\mathbb{E}_i\log\left(\epsilon_{i,t+1}^{1-\tau}\right) - \log\left(\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}\right]\right) = -(1-\tau)\frac{\sigma^2(x_t)}{2} + (1-\tau)\frac{\sigma^2(x_t)}{2} - (1-\tau)^2\frac{\sigma^2(x_t)}{2} = -(1-\tau)^2\frac{\sigma^2(x_t)}{2}.$$

So the effect of x_t on $\sum_{t=0}^{\infty}\beta^t W_t$ is given by

$$-\frac{\beta}{1-\beta}(1-\tau)^2\frac{\sigma^2(x_t)}{2}, \tag{49}$$

proving the result.

□

C.2   Proof of Proposition 1

For this proof, in addition to the social welfare function, (27), the relevant equations of the model are (4), (42), (5), (45), (21), and (20).

The first-order condition of the social welfare function with respect to b is

$$\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{u_t}{b} - \frac{u_t}{1-u_t+u_t b} + \frac{dW_t}{du_t}\left.\frac{\partial u_t}{\partial b}\right|_x - \upsilon q_t^{\kappa}\left.\frac{\partial q_t}{\partial b}\right|_x + \frac{dW_t}{dx_t}\frac{dx_t}{db}\right\} = 0. \tag{50}$$

The first two terms in (50) can be expressed as

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left(\frac{1}{b} - 1 + 1 - \frac{1}{1 - u_t + u_t b} \right)$$

$$= u_t \left(\frac{1}{b} - 1 \right) \left(1 - \frac{u_t b}{1 - u_t + u_t b} \right)$$

and note that

$$\left. \frac{\partial \log (b \tilde{c}_t)}{\partial \log b} \right|_{x,q} = \frac{\partial}{\partial \log b} \log \left(\frac{b C_t}{\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right] (1 - u_t + u_t b)} \right) \tag{51}$$

$$= 1 - \frac{u_t b}{1 - u_t + u_t b}$$

where the partial derivative on the right hand side of (51) is with respect to b alone.[30] So, we have:

$$\frac{u_t}{b} - \frac{u_t}{1 - u_t + u_t b} = u_t \left(\frac{1}{b} - 1 \right) \left. \frac{\partial \log (b \tilde{c}_t)}{\partial \log b} \right|_{x,q} \tag{52}$$

For the third and fourth terms in (50), start by noting that:

$$\frac{dW_t}{du_t} = \log b + \frac{1 - b}{1 - u_t + u_t b} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t} + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi$$

$$= \left(\log b + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \tag{53}$$

where

$$\frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} = \frac{1 - b}{1 - u_t + u_t b} - \frac{A_t h_t}{C_t} + \frac{\psi_1 M_t^{\psi_2}}{C_t}.$$

Then, it follows that:

$$\left. \frac{dW_t}{du_t} \frac{\partial u_t}{\partial b} \right|_x - v q_t^\kappa \left. \frac{\partial q_t}{\partial b} \right|_x = \left[\left(\log b + \frac{h_t^{1+\gamma}}{1 + \gamma} - \xi \right) + \frac{1}{\tilde{c}_t} \frac{d\tilde{c}_t}{du_t} \right] \left. \frac{\partial u_t}{\partial b} \right|_x - v q_t^\kappa \left. \frac{\partial q_t}{\partial b} \right|_x.$$

---

[30] As $\mathbb{E}_i \left[\alpha_{i,t}^{1-\tau} \right]$ is an endogenous state that depends on the history of x, we are taking the partial derivative holding fixed this history.

Using equation (21), this becomes

$$\frac{dW_t}{du_t}\frac{\partial u_t}{\partial b}\bigg|_x - vq_t^\kappa\frac{\partial q_t}{\partial b}\bigg|_x = \left[\left(\log b + \frac{h_t^{1+\gamma}}{1+\gamma} - \xi\right) + \frac{1}{\tilde{c}_t}\frac{d\tilde{c}_t}{du_t}\right]\frac{\partial u_t}{\partial b}\bigg|_x + \left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right)\frac{du_t}{dq_t}\frac{\partial q_t}{\partial b}\bigg|_x$$

$$= \frac{1}{\tilde{c}_t}\frac{d\tilde{c}_t}{du_t}\frac{\partial u_t}{\partial b}\bigg|_x$$

$$= \frac{d\log\tilde{c}_t}{d\log u_t}\frac{\partial\log u_t}{\partial b}\bigg|_x. \tag{54}$$

Substituting (52) and (54) into (50) yields the result.

C.3 Proof of Proposition 2

We proceed as in the proof of Proposition 1. The first-order condition of the social welfare function with respect to τ is:

$$\mathbb{E}_0\sum_{t=0}^\infty \beta^t\left\{\frac{\beta}{1-\beta}(1-\tau)\sigma^2(x_t) + \frac{dW_t}{dh_t}\frac{\partial h_t}{\partial\tau}\bigg|_x + \frac{dW_t}{dx_t}\frac{dx_t}{d\tau}\right\} = 0. \tag{55}$$

From (27), (42), and (45) we have

$$\frac{dW_t}{dh_t} = \frac{dW_t}{du_t}\frac{du_t}{dq_t}\frac{dq_t}{dh_t} + \frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma - vq_t^\kappa\frac{dq_t}{dh_t}.$$

From (21) we have

$$\frac{\partial h_t}{\partial\tau}\bigg|_x = -\frac{h_t}{(1-\tau)(1+\gamma)}$$

combining these and using (21), (53), and $du_t/dq_t = -vM_t$ we arrive at

$$\frac{dW_t}{dh_t}\frac{\partial h_t}{\partial\tau}\bigg|_x = \frac{dW_t}{du_t}\frac{du_t}{dq_t}\frac{dq_t}{dh_t}\frac{\partial h_t}{\partial\tau}\bigg|_x + \frac{A_t(1-u_t)}{C_t}\frac{\partial h_t}{\partial\tau}\bigg|_x - (1-u_t)h_t^\gamma\frac{\partial h_t}{\partial\tau}\bigg|_x - vq_t^\kappa\frac{dq_t}{dh_t}\frac{\partial h_t}{\partial\tau}\bigg|_x$$

$$= \frac{1}{\tilde{c}_t}\frac{d\tilde{c}_t}{du_t}\frac{du_t}{dq_t}\frac{dq_t}{dh_t}\frac{\partial h_t}{\partial\tau}\bigg|_x - \left[\frac{A_th_t(1-u_t)}{C_t} - (1-u_t)h_t^{1+\gamma}\right]\frac{1}{(1-\tau)(1+\gamma)}$$

$$= \frac{d\log\tilde{c}_t}{d\log u_t}\frac{\partial\log u_t}{\partial\tau}\bigg|_x - \left[\frac{A_t}{C_t} - h_t^\gamma\right]\frac{h_t(1-u_t)}{(1-\tau)(1+\gamma)}.$$

Substituting this into (55) yields the desired result.

D Proofs for section 5

D.1 Proof of Proposition 3

Proceeding as in the proof of Proposition 1 we have

$$\frac{dW_t}{dx_t} = \frac{dW_t}{du_t}\frac{du_t}{dx_t} - vq_t^\kappa \frac{dq_t}{dx_t} + \left[\frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{1}{C_t}\left.\frac{\partial J_t}{\partial x_t}\right|_u \tag{56}$$
$$- \frac{\beta}{1-\beta}(1-\tau)^2\frac{d}{dx_t}\frac{\sigma^2(x_t)}{2} - \frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t}$$

where dW_t/du_t is given by (53). We rearrange (56) to arrive at the desired result. First, note that:

$$\frac{du_t}{dx_t} = -vq_t\frac{dM_t}{dx_t} - vM_t\frac{dq_t}{dx_t}. \tag{57}$$

Using this, (21), and (53), equation (56) then becomes

$$\frac{dW_t}{dx_t} = \left(-\left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right) + \frac{1}{\tilde{c}_t}\frac{d\tilde{c}_t}{du_t}\right)\left(-vq_t\frac{dM_t}{dx_t} - vM_t\frac{dq_t}{dx_t}\right) - vM_t\left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right)\frac{dq_t}{dx_t}$$
$$+ \left[\frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{1}{C_t}\left.\frac{\partial J_t}{\partial x_t}\right|_u - \frac{\beta}{1-\beta}(1-\tau)^2\frac{d}{dx_t}\frac{\sigma^2(x_t)}{2} - \frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t}$$
$$= vq_t\left(-\log b - \frac{h_t^{1+\gamma}}{1+\gamma} + \xi\right)\frac{dM_t}{dx_t} - \frac{Y_t}{C_t S_t}\frac{dS_t}{dx_t} + \frac{1}{\tilde{c}_t}\frac{d\tilde{c}_t}{du_t}\frac{du_t}{dx_t}$$
$$+ \left[\frac{A_t(1-u_t)}{C_t} - (1-u_t)h_t^\gamma\right]\frac{dh_t}{dx_t} - \frac{1}{C_t}\left.\frac{\partial J_t}{\partial x_t}\right|_u - \frac{\beta}{1-\beta}(1-\tau)^2\frac{d}{dx_t}\frac{\sigma^2(x_t)}{2}.$$

This can be rearranged to the desired result by making use of

$$\frac{1}{\tilde{c}_t}\frac{d\tilde{c}_t}{du_t} = \frac{1}{C_t}\frac{dC_t}{du_t} + \frac{1-b}{1-u_t+u_t b},$$

and

$$-vq_t\frac{dM_t}{dx_t} = \left.\frac{\partial u_t}{\partial x_t}\right|_q.$$

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D.2 Deriving the special case in section 5.2.3

We can normalize $\mathbb{E}_{t-1}p_t = 1$. A fraction θ of firms set the price p_t^* and the remaining set the price $\mathbb{E}_{t-1}p_t^* = 1$. From the definition of the price index we can solve for

$$\left(\frac{p_t^*}{p_t}\right)^{\mu/(1-\mu)} = \left(\frac{1-(1-\theta)p_t^{-1/(1-\mu)}}{\theta}\right)^{\mu}.$$

Substituting into (25) we arrive at

$$S_t = (1-\theta)p_t^{\mu/(\mu-1)} + \theta^{1-\mu}\left(1-(1-\theta)p_t^{1/(\mu-1)}\right)^{\mu}.$$

This makes clear that S_t is a function of p_t. Differentiating this function we arrive at

$$S'(1) = (1-\theta)\frac{\mu}{\mu-1}\left[p_t^{1/(\mu-1)} - \theta^{1-\mu}\left(1-(1-\theta)p_t^{1/(\mu-1)}\right)^{\mu-1}p_t^{(2-\mu)/(\mu-1)}\right] = 0$$

$$S''(1) = \frac{1-\theta}{\theta}\frac{\mu}{\mu-1}.$$

Next, rewrite the welfare function as

$$W_t = \log(C_t) - (1-u_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \chi\log(G_t) + \text{t.i.p.},$$

$$C_t = Y_t - J_t - G_t,$$

$$Y_t = \frac{\eta_t^A}{S(p_t)}h_t(1-u_t)$$

$$G_t = \chi C_t.$$

Observe that u_t and J_t are exogenous in this case as M_t and q_t are exogenous. Use the production function to rewrite h_t in terms of Y_t

$$h_t = \frac{Y_t S(p_t)}{\eta_t^A(1-u_t)}.$$

Now rewrite W_t in terms of Y_t and p_t

$$W_t = (1+\chi)\log\left(\frac{Y_t - J_t}{1+\chi}\right) - \frac{1-u_t}{1+\gamma}\left(\frac{Y_t S(p_t)}{\eta_t^A(1-u_t)}\right)^{1+\gamma}.$$

With flexible prices we have $p = 1$ and $S = 1$. The flexible-price, socially-efficient level of

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output, Y^*, satisfies

$$\frac{\eta^A}{Y_t^* - J_t - G_t} = \left(\frac{Y_t^*}{\eta_t^A(1 - u_t)}\right)^\gamma.$$

Treating W_t as a function of Y_t and p_t, we take a second-order Taylor approximation around the point $(Y_t^*, 1)$. The first-derivatives are

$$W_Y(Y_t^*, 1) = \frac{1}{Y_t - J_t - G_t} - \left(\frac{Y_t S(p_t)}{\eta_t^A(1 - u_t)}\right)^\gamma \frac{S(p_t)}{\eta_t^A}$$

$$W_p(Y_t^*, 1) = -(1 - u_t)\left(\frac{Y_t S(p_t)}{\eta_t^A(1 - u_t)}\right)^\gamma \frac{Y_t}{\eta_t^A(1 - u_t)} S'(p_t)$$

and both are zero, the former because Y^* is optimal and the latter because $S'(1) = 0$. The second derivatives are

$$W_{YY}(Y_t^*, 1) = -\frac{1}{(C^*)^2} - \gamma(Y_t^*)^{\gamma-1}\left(\frac{S(p_t)}{\eta_t^A}\right)^{1+\gamma}\left(\frac{1}{1 - u_t}\right)^\gamma,$$

$$W_{Yp}(Y_t^*, 1) = -\left(\frac{Y_t^*}{1 - u_t}\right)^\gamma\left(\frac{1}{\eta_t^A}\right)^{1+\gamma}(1 + \gamma)S(p_t)^\gamma S'(p_t),$$

$$W_{pp}(Y_t^*, 1) = -(1 - u_t)\left(\frac{Y_t^*}{\eta_t^A(1 - u_t)}\right)^{1+\gamma}\left[\gamma S(p_t)^{\gamma-1} S'(p_t) + S(p_t)^\gamma S''(p_t)\right].$$

Using $S(1) = 1, S'(1) = 0$, the expression for $S''(1)$ above, and the optimality condition for Y_t^* we arrive at

$$W_{YY}(Y_t^*, 1) = -\frac{1}{(C^*)^2}\left[1 + \gamma\frac{C_t^*}{Y_t^*}\right],$$

$$W_{Yp}(Y_t^*, 1) = 0,$$

$$W_{pp}(Y_t^*, 1) = -\frac{Y_t^*}{C_t^*}\frac{1 - \theta}{\theta}\frac{\mu}{\mu - 1}.$$

By Taylor's theorem we can write

$$W(Y_t, p_t) \approx \frac{1}{2}W_{YY}(Y_t^*, 1)(Y_t - Y_t^*)^2 + \frac{1}{2}W_{pp}(Y_t^*, 1)(p_t - 1)^2$$

Observe that Y and p are functions of x. So when we differentiate with respect to x we arrive at

$$\frac{dW_t}{dx_t} \approx W_{YY}(Y_t^*, 1)(Y_t - Y_t^*)\frac{dY_t}{dx_t} + W_{pp}(Y_t^*, 1)(p_t - 1)\frac{dp_t}{dx_t}.$$

Substituting for W_{YY} and W_{pp} and rearranging yields the result.

D.3 The AD-AS apparatus in section 5.3

The first step in the analysis is to express the equilibrium in date 0 in terms of as few variables as possible, namely x_0. First, it follows from equations (4), (5), (6), and (42) that we can express J, u, Y, and w as functions of x and non-policy parameters. It then follows that $j \equiv J/Y$ is a function of x. Second, a firm that sets its price in period 0 will set the optimal markup over current marginal cost because all prices will be re-optimized in subsequent periods. Therefore the optimal relative price for a firm that updates in period 0 is $\mu \left(w_0 + \psi_1 M_0^{\psi_2}/h(x_0, \tau) \right)/A_0$ where $h(x, \tau)$ is given by (20). The price level and inflation rate in period 0 are therefore functions of x_0 and τ. Third, it follows from (8) that the same is true of the nominal interest rate I_0 and S_0. The Fisher equation (46) and the assumption that the central bank sets inflation to zero in period 1 implies that R_0 is also a function of x_0 and τ.

The next step is to derive an "aggregate demand curve" from the aggregate Euler equation. Using equations (17), (18), and (19) along with the log-normal income process we arrive at:

$$\frac{(1 - u_t + u_t b)}{C_t} = \beta R_t e^{\sigma_\epsilon^2 (x_t)(1-\tau)^2} \mathbb{E}_t \left[\frac{(1 - u_{t+1}(1 - b))(1 - u_{t+1}(1 - b^{-1}))}{C_{t+1}} \right].$$

For $t \geq 0$ the expectation in the Euler equation disappears because outcomes in $t + 1$ are deterministic. We can re-write the Euler equation as

$$C_0 = \frac{1 - u_0 + u_0 b}{\beta R_0(x_0)} \frac{C_1}{(1 - u_1(1 - b))(1 - u_1(1 - b^{-1}))} e^{-\sigma_\epsilon^2 (x_0)(1-\tau)^2}.$$

Next, recall from market clearing that: $Y_0(1 - j_0) = C_0(1 + \chi \eta_0^G)$ Replacing out C_0 gives:

$$Y_0 = \frac{1 + \chi \eta_0^G}{1 + \chi \eta_1^G} \frac{1 - j_1}{1 - j_0} \frac{1 - u_0 + u_0 b}{\beta R(x_0)} \frac{Y_1}{(1 - u_1(1 - b))(1 - u_1(1 - b^{-1}))} e^{-\sigma_\epsilon^2 (M_0)(1-\tau)^2}. \tag{58}$$

This provides an "AD" curve: a negative relation between x_0 and Y_0.

The AD curve depends on Y_1. Under flexible prices, the price-setting first-order condition

implies

$$w(x_t) + \frac{\psi_1 x_t^{\psi_2}}{h(x_t, \tau)} = \frac{\eta_t^A}{\mu}.$$

This equation implies x_t is a function of η_t^A, τ, and non-policy parameters when prices are flexible. We call this function $x^{\text{Flex}}(\eta_t^A, \tau)$. Y_1 can be calculated from (42) with $x_1 = x^{\text{Flex}}(\eta_1^A, \tau)$ because all prices are flexible in period 1. This leads to

$$Y_1 = \eta_1^A (1 - u(x_1)) \left[\bar{w}(1 - \tau) \right]^{\frac{1}{1+\gamma}} (x_1)^{\frac{\zeta}{1+\gamma}} \tag{59}$$

Note that x_1 is given by $x^{\text{Flex}}(\eta_1^A, \tau)$ so (59) makes Y_1 independent of x_0 and b.

The aggregate production function, (42), is the "AS" curve: a positive relation between x_0 and Y_0. We now set AS = AD. Specifically, equations (58) and (42) lead to two expressions for Y_0. Taking the log of equations (58) and (42) and set them equal to one another yields an implicit solution for x_0:

$$\log\left(\frac{1 - \chi\eta_0^G}{1 - \chi\eta_1^G}\right) + \log(1 - j_1) - \log(1 - j_0) \tag{60}$$
$$+ \log\left(1 - u_0 + u_0 b\right) - \log\beta - \log\left(R(x_0, \tau)\right) + \log\left(Y_1(x_1^{\text{Flex}}, \tau)\right) - \log\left(1 - u_1(1 - b)\right)$$
$$- \log\left(1 - u_1(1 - b^{-1})\right) - \sigma_\epsilon^2(x_0)(1 - \tau)^2 - \log\left(\eta_0^A\right) + \log\left(S_0\right) - \log\left(h_0\right) - \log\left((1 - u_0)\right) = 0.$$

D.4 Proof of proposition 4

x_0 is implicitly defined by (60). Using the implicit function theorem we obtain

$$\frac{dx_0}{db} = -\frac{\frac{u_0}{1 - u_0 + u_0 b} - \frac{u_1}{1 - u_1(1 - b)} + \frac{u_1 b^{-2}}{1 - u_1(1 - b^{-1})}}{-\frac{1 - b}{1 - u_0 + u_0 b}\frac{du_0}{dx_0} - \frac{1}{R_0}\frac{dR_0}{dx_0} - (1 - \tau)^2\frac{d}{dx_0}\sigma_\epsilon^2(x_0) + \frac{1}{S_0}\frac{dS_0}{dx_0} - \frac{1}{h_0}\frac{dh_0}{dx_0} + \frac{1}{1 - u_0}\frac{du_0}{dx_0} + \frac{1}{1 - j_0}\frac{dj_0}{dx_0}}$$

As an elasticity we have

$$\frac{b}{x_0}\frac{dx_0}{db} = \frac{\frac{u_0 b}{1 - u_0 + u_0 b} - \frac{u_1 b}{1 - u_1(1 - b)} + \frac{u_1 b^{-1}}{1 - u_1(1 - b^{-1})}}{\frac{d\log R_0}{d\log x_0} + (1 - \tau)^2\sigma_\epsilon^2(x_0)\frac{d\log \sigma_\epsilon^2(x_0)}{d\log x_0} + \frac{1 - b}{1 - u_0 + u_0 b}u_0\frac{d\log u_0}{d\log x_0} - \frac{d\log S_0}{d\log x_0} + \frac{d\log h_0}{d\log x_0} + \frac{d\log(1 - u_0)}{d\log x_0} + \frac{d\log(1 - j_0)}{d\log x_0}}$$

$$\tag{61}$$

D.5 Proof that the numerator of (33) is increasing in u_0 and u_1 for $u < 1/2$

Start with $\frac{u_0 b}{1-u_0(1-b)}$, the derivative with respect to u_0 is

$$\frac{b(1 - u_0(1 - b)) + u_0 b(1 - b)}{[1 - u_0(1 - b)]^2} = \frac{b}{[1 - u_0(1 - b)]^2} > 0.$$

Next turn to $\frac{b u_1(1-u_1)(b^{-2}-1)}{[1+u_1(b^{-1}-1)][1+u_1(b-1)]}$. We start by dividing by $b(b^{-2} - 1)$,which is positive for $b \in (0,1)$. We will then show the derivative with respect to u_1 is positive

$$\frac{1 - 2u_1}{\{[1 + u_1(b^{-1} - 1)] [1 + u_1(b - 1)]\}^2} \left[1 + u_1(b^{-1} - 1)\right] \left[1 + u_1(b - 1)\right]$$

$$- \frac{u_1(1 - u_1)}{\{[1 + u_1(b^{-1} - 1)] [1 + u_1(b - 1)]\}^2} \left[\left(b^{-1} - 1\right) (1 + u_1(b - 1)) + (b - 1) \left(1 + u_1(b^{-1} - 1)\right)\right] > 0$$

multiply both sides by $\left\{\left[1 + u_1(b^{-1} - 1)\right] [1 + u_1(b - 1)]\right\}^2$

$$[1 - 2u_1] \left[1 + u_1(b^{-1} - 1)\right] [1 + u_1(b - 1)] - u_1(1 - u_1) \left[\left(b^{-1} - 1\right) (1 + u_1(b - 1)) + (b - 1) \left(1 + u_1(b^{-1} - 1)\right)\right] > 0$$

Rearranging

$$(1 - 2u_1) \left[1 + (u_1 - u_1^2)(b^{-1} + b - 2)\right] - u_1(1 - u_1) \left(b^{-1} + b - 2\right) (1 - 2u_1) > 0$$

divide both sides $(1 - 2u_1)$ (recall $u_1 < 1/2$) and rearrange

$$1 + u_1(1 - u_1)(b^{-1} + b - 2) - u_1(1 - u_1) \left(b^{-1} + b - 2\right) = 1 > 0.$$

D.6 Proof of proposition 5

Using the implicit function theorem to differentiate (60) we obtain

$$\frac{dx_0}{d\tau} = -\frac{2\sigma_\epsilon^2(x_0)(1 - \tau) + \frac{1}{Y_1}\frac{1}{1+\gamma}\frac{Y_1}{1-\tau} - \frac{1}{h_0}\frac{1}{1+\gamma}\frac{h_0}{1-\tau} + \frac{1}{S_0}\left.\frac{\partial S_0}{\partial \tau}\right|_x - \frac{1}{R_0}\left.\frac{\partial R_0}{\partial \tau}\right|_x + \frac{1}{Y_1}\frac{dY_1}{dx_1}\frac{dx\text{Flex}_{(A_1,\tau)}}{d\tau}}{-\frac{1-b}{1-u_0+u_0 b}\frac{du_0}{dx_0} - \frac{1}{R_0}\frac{dR_0}{dx_0} - (1-\tau)^2\frac{d}{dx_0}\sigma_\epsilon^2(x_0) + \frac{1}{S_0}\frac{dS_0}{dx_0} - \frac{1}{h_0}\frac{dh_0}{dx_0} + \frac{1}{1-u_0}\frac{du_0}{dx_0} + \frac{1}{1-j_0}\frac{dj_0}{dx_0}}$$

$$= -\frac{2\sigma_\epsilon^2(x_0)(1 - \tau) - \frac{1}{R_0}\left.\frac{\partial R_0}{\partial \tau}\right|_x + \frac{1}{S_0}\left.\frac{\partial S_0}{\partial \tau}\right|_x + \frac{1}{Y_1}\frac{dY_1}{dx_1}\frac{dx\text{Flex}_{(A_1,\tau)}}{d\tau}}{-\frac{1-b}{1-u_0+u_0 b}\frac{du_0}{dx_0} - \frac{1}{R_0}\frac{dR_0}{dx_0} - (1-\tau)^2\frac{d}{dx_0}\sigma_\epsilon^2(x_0) + \frac{1}{S_0}\frac{dS_0}{dx_0} - \frac{1}{h_0}\frac{dh_0}{dx_0} + \frac{1}{1-u_0}\frac{du_0}{dx_0} + \frac{1}{1-j_0}\frac{dj_0}{dx_0}}$$

As an elasticity we have

$$\frac{\tau}{x_0}\frac{dx_0}{d\tau} = \frac{2\sigma_\epsilon^2(x_0)\left(1-\tau\right)\tau - \left.\frac{\partial\log R_0}{\partial\log\tau}\right|_x + \left.\frac{\partial\log S_0}{\partial\log\tau}\right|_x + \frac{d\log Y_1}{d\log x_1}\frac{d\log x_1}{d\log\tau}}{\frac{d\log R_0}{d\log x_0} + (1-\tau)^2\sigma_\epsilon^2(x_0)\frac{d\log\sigma_\epsilon^2(x_0)}{d\log x_0} + \frac{1-b}{1-u_0+u_0 b}u_0\frac{d\log u_0}{d\log x_0} - \frac{d\log S_0}{d\log x_0} + \frac{d\log h_0}{d\log x_0} + \frac{d\log(1-u_0)}{d\log x_0} + \frac{d\log(1-j_0)}{d\log x_0}} \tag{62}$$

D.7 Proof of lemma 3

See (61) and (62).

E Description of methods for section 6

E.1 Estimated income process

The material in this appendix describes an implementation of the procedure of Guvenen and McKay (2016) and that paper gives more discussion and details.

The income process is as follows: $\alpha_{i,t}$ evolves as in (3). Earnings are given by $\alpha_{i,t}w_t$ when employed and zero when unemployed. Notice that here we normalize $h_t = 1$ and subsume all movements in h_t into w_t. While this gives a different interpretation to w_t it does not affect the distribution of earnings growth rates apart from a constant term. The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) = \begin{cases} N(\mu_{1,t}, \sigma_1) & \text{with prob. } P_1, \\ N(\mu_{2,t}, \sigma_2) & \text{with prob. } P_2, \\ N(\mu_{3,t}, \sigma_3) & \text{with prob. } P_3 \\ N(\mu_{4,t}, \sigma_4) & \text{with prob. } P_4 \end{cases}$$

The tails of F move over time as driven by the latent variable x_t such that

$$\mu_{1,t} = \bar{\mu}_t,$$

$$\mu_{2,t} = \bar{\mu}_t + \mu_2 - x_t,$$

$$\mu_{3,t} = \bar{\mu}_t + \mu_3 - x_t,$$

$$\mu_{4,t} = \bar{\mu}_t.$$

where $\bar{\mu}_t$ is a normalization such that $\mathbb{E}_i[exp\{\epsilon_{i,t+1}\}] = 1$ in all periods.

The model period is one quarter. The parameters are selected to match the median earnings growth, the dispersion in the right tail (P90 - P50), and the dispersion in the left-tail (P50-P10) for one, three, and five year earnings growth rates computed each year using data from 1978 to 2011. In addition we target the kurtosis of one-year and five year earnings growth rates and the increase in cross-sectional variance over the life-cycle. The moments are computed from the Social Security Administration earnings data as reported by Guvenen et al. (2014) and Guvenen et al. (2015). We use these moments to form an objective function as described in Guvenen and McKay (2016).

The estimation procedure simulates quarterly data using the observed job-finding and -separation rates and then aggregates to annual income and computes these moments. To simulate the income process, we require time series for x_t and w_t. We assume that these series are linearly related to observable labor market indicators (for details see Guvenen and McKay, 2016). Call the weights in these linear relationships β. We then search over the parameters P, μ, σ, and β subject to the restrictions $P_2 = P_3$ and $\sigma_2 = \sigma_3$.

Guvenen et al. (2014) emphasize the pro-cyclicality in the skewness of earnings growth rates. The estimated income process does an excellent job capturing this as shown in the top panel of figure 5. The estimated β implies a time-series for x_t which shifts the tails of the earnings distribution and gives rise the pro-cyclical skewness shown in figure 5. We regress this time-series on the unemployment rate and find a coefficient of 16.7.[31] The fourth component of the mixture distribution occurs with very low probability, and in our baseline specification we set it to zero. This choice is not innocuous, however, because the standard deviation σ_4 is estimated to be very large and this contributes to the high kurtosis of the earnings growth distribution. In particular,

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[31] We regress this estimated time series x_t on the unemployment rate, which we smooth with an HP filter with smoothing parameter 100,000. If we call this regression function f, we then proceed with $F(\epsilon'; f(u))$.

omitting this component leads to a substantially smaller τ as a result of having less risk in the economy. We prefer to omit this from our baseline calibration because the interpretation of these high-kurtosis terms is unclear and we are not entirely satisfied with modeling them as permanent shocks to skill.

The resulting income process that we use in our computations is as follows: The innovation distribution is given by

$$\epsilon_{i,t+1} \sim F(\epsilon; x_t) =\sim \begin{cases} N(\mu_{1,t}, 0.0403) & \text{with prob. } 0.9855, \\ N(\mu_{2,t}, 0.0966) & \text{with prob. } 0.00727, \\ N(\mu_{3,t}, 0.0966) & \text{with prob. } 0.00727 \end{cases}$$

with

$$\mu_{1,t} = \bar{\mu}_t,$$
$$\mu_{2,t} = \bar{\mu}_t + 0.266 - 16.73(u_t - u^*),$$
$$\mu_{3,t} = \bar{\mu}_t - 0.184 - 16.73(u_t - u^*),$$

where u^* is the steady state is unemployment rate in our baseline calibration. The bottom panels of figure 5 show the density of ϵ and how it changes with an increase in the unemployment rate.

E.2 Global solution method

As a first step, we need to rewrite the Calvo-pricing first-order condition recursively:

$$\frac{p_t^*}{p_t} = \frac{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s + \psi_1 M_s^{\psi_2}/h_s\right)/A_s}{\mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s}.$$

Define p_t^A as

$$p_t^A = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1} (1-\theta)^{s-t} \left(\frac{p_t}{p_s}\right)^{\mu/(1-\mu)} Y_s \mu \left(w_s + \psi_1 M_s^{\psi_2}/h_s\right)/A_s$$
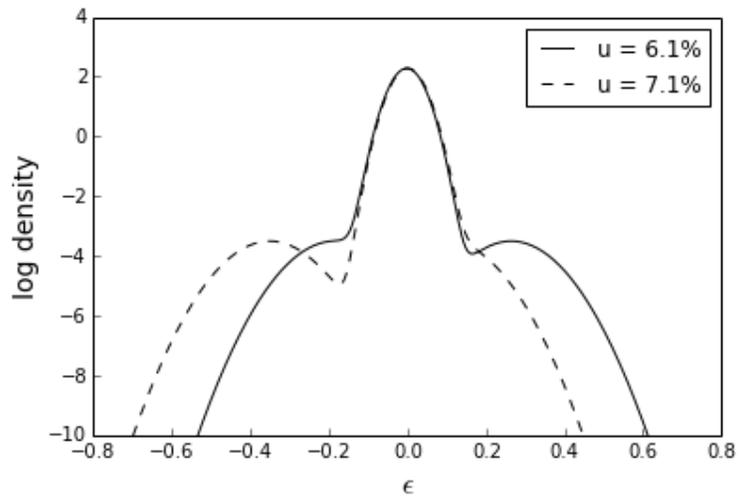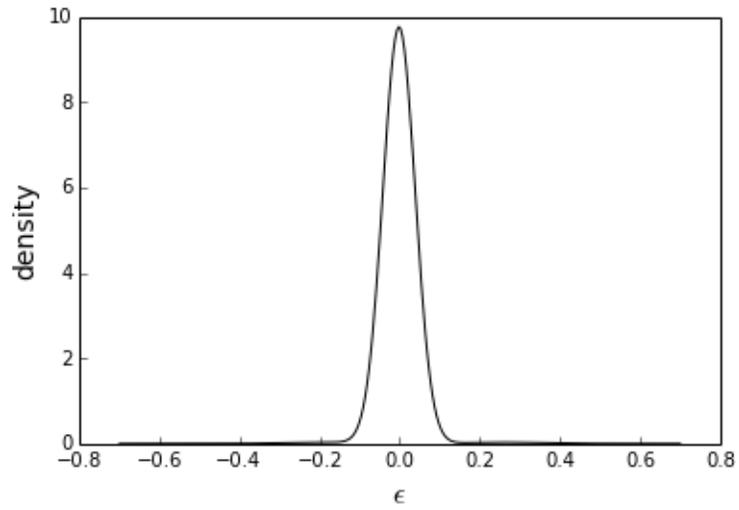
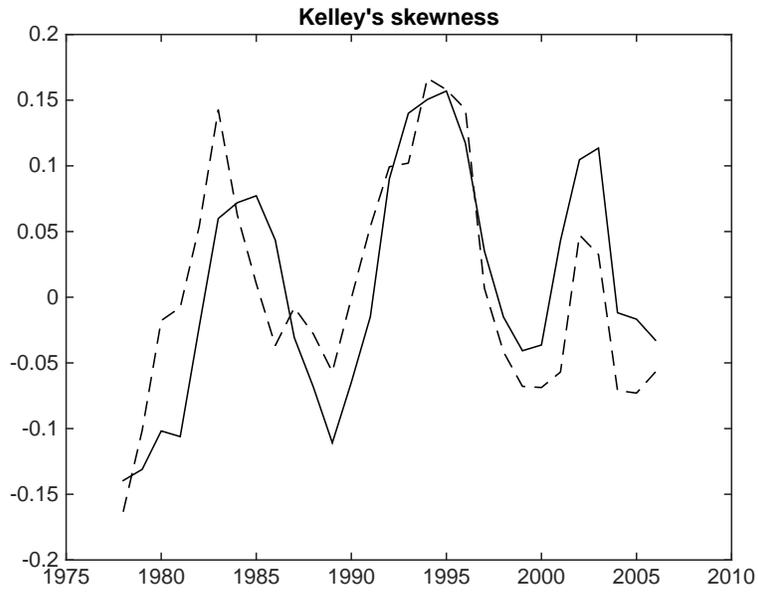Kelley's skewness

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Figure 5: Properties of $F(\epsilon)$.

and p_t^B as

$$p_t^B = \mathbb{E}_t \sum_{s=t}^{\infty} R_{t,s}^{-1}(1-\theta)^{s-t}\left(\frac{p_t}{p_s}\right)^{1/(1-\mu)} Y_s$$

such that

$$\frac{p_t^*}{p_t} = \frac{p_t^A}{p_t^B}.$$

p_t^A and p_t^B can be rewritten as

$$p_t^A = \mu Y_t\left(w_t + \psi_1 M_s^{\psi_2}/h_s\right)/A_t + (1-\theta)\mathbb{E}_t\left[\left(\frac{I_t}{\pi_{t+1}}\right)^{-1}\pi_{t+1}^{-\mu/(1-\mu)}p_{t+1}^A\right] \tag{63}$$

$$p_t^B = Y_t + (1-\theta)\mathbb{E}_t\left[\left(\frac{I_t}{\pi_{t+1}}\right)^{-1}\pi_{t+1}^{-1/(1-\mu)}p_{t+1}^B\right]. \tag{64}$$

The procedure we use builds on the method proposed by Maliar and Maliar (2015) and their application to solving a New Keynesian model. We first describe how we solve the model for a given grid of aggregate state variables and then describe how we construct the grid.

There are six state variables that evolve according to

$$\mathbb{E}_i\left[\alpha_{i,t+1}^{1-\tau}\right] = (1-\delta)\mathbb{E}_i\left[\alpha_{i,t}^{1-\tau}\right]\mathbb{E}_i\left[\epsilon_{i,t+1}^{1-\tau}u_t\right] + \delta$$

$$\mathbb{E}_i\left[\log\alpha_{i,t+1}\right] = (1-\delta)\left[\mathbb{E}_i\left[\log\alpha_{i,t}\right] + \mathbb{E}_i\left[\log\epsilon i,t+1|u_t\right]\right]$$

$$S_{t+1}^A = S_t$$

$$\log\eta_{t+1}^A = \rho^A\log\eta_t^A + \varepsilon_{t+1}^A$$

$$\log\eta_{t+1}^G = \rho^G\log\eta_t^G + \varepsilon_{t+1}^G$$

$$\log\eta_{t+1}^I = \rho^I\log\eta_t^I + \varepsilon_{t+1}^I,$$

where S^A is the level of price dispersion in the previous period and the ε terms are i.i.d. normal innovations.

There are five variables that we approximate with complete second-order polynomials in the state: $(1/C_t)$, p_t^A, p_t^B, J_t, and V_t, where V_t is the value of the social welfare function. We use (17) and (18) to write the Euler equation in terms of C_t and this equation pins down $1/C_t$. p_t^A and p_t^B

satisfy (63) and (64). V_t satisfies

$$V_t = W_t + \beta \, \mathbb{E}_t \left[V_{t+1} \right].$$

J_t satisfies $J_t = \psi_1 M_t^{\psi_2}(v - u_t)$. Abusing language slightly, we will refer to these variables that we approximate with polynomials as "forward-looking variables."

The remaining variables in the equilibrium definition can be calculated from the remaining equations and all of which only involve variables dated t. We call these the "static" variables.

To summarize, let $\mathcal{S}_t$ be the state variables, $\mathcal{X}_t$ be the forward-looking variables, and $\mathcal{Y}_t$ be the static variables. The three blocks of equations are

$$\mathcal{S}' = \mathcal{G}^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$$
$$\mathcal{X} = \mathbb{E}\, \mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$$
$$\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$$

where $\mathcal{G}^{\mathcal{S}}$ are the state-transition equations, $\mathcal{G}^{\mathcal{X}}$ are the forward-looking equations and $\mathcal{G}^{\mathcal{Y}}$ are the state equations. Let $\mathcal{X} \approx \mathcal{F}(\mathcal{S}, \Omega)$ be the approximated solution for the forward-looking equations for which we use a complete second-order polynomial with coefficients given by Ω. We then operationalize the equations as follows: given a value for $\mathcal{S}$, we calculate $\mathcal{X} = \mathcal{F}(\mathcal{S}, \Omega)$ and $\mathcal{Y} = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}, \mathcal{X})$. We then take an expectation over ε' using Gaussian quadrature. For each value of ε' in the quadrature grid, we compute $\mathcal{S}' = G^{\mathcal{S}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \varepsilon')$, $\mathcal{X}' = \mathcal{F}(\mathcal{S}', \Omega)$ and $\mathcal{Y}' = \mathcal{G}^{\mathcal{Y}}(\mathcal{S}', \mathcal{X}')$. We can now evaluate $\mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$ for this value of ε' and looping over all the values in the quadrature grid we can compute $\hat{\mathcal{X}} = \mathbb{E}\, \mathcal{G}^{\mathcal{X}}(\mathcal{S}, \mathcal{X}, \mathcal{Y}, \mathcal{S}', \mathcal{X}', \mathcal{Y}')$. $\hat{\mathcal{X}}$ will differ from the value of $\mathcal{X}$ that was obtained initially from $\mathcal{F}(\mathcal{S}, \Omega)$. We repeat these steps for all the values of $\mathcal{S}$ in our grid for the aggregate state space. We then adjust the coefficients Ω part of the way towards those implied by the solutions $\hat{\mathcal{X}}$. We then iterate this procedure to convergence of Ω.

Evaluating some of the equations of the model involves taking integrals against the distribution of idiosyncratic skill risk $\epsilon_{i,t+1} \sim F(\epsilon_{i,t+1}, u_t - u^*)$. We do this using Gaussian quadrature within each of the components of the mixture distribution. We compute expectations over aggregate shocks using Gaussian quadrature as well.

We use a two-step procedure to construct the grid on the aggregate state space. We have six aggregate states so we choose the grid to lie in the region of the aggregate state space that is visited

by simulations of the solution. We create a box of policy parameters $[b_L, b_H] \times [\tau_L, \tau_H]$. For each of the four corners of this box, we use the procedure of Maliar and Maliar (2015) to construct a grid and solve the model. This procedure iterates between solving the model and simulating the solution and constructing a grid in the part of the state space visited by the simulation. This gives us four grids, which we then merge and eliminate nearby points using the techniques of Maliar and Maliar (2015). This leaves us with one grid that we use to solve the model when we evaluate policies. Each of the grids that we construct have 100 points.