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A cointegration model of money and wealth

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Abstract

Extending the data set used in Beyer (2009) from 2007 to 2017, we estimate $I(1)$ and $I(2)$ money demand models for euro area M3. We find that the elasticities in the money demand and the real wealth relations identified previously in Beyer (2009) have remained remarkably stable throughout the extended sample period, once only a few additional deterministic variables in the long run relationships for the period after the start of the global financial crisis and the ECB's non-standard monetary policy measures are included. Testing for price homogeneity in the $I(2)$ model we find that the nominal-to-real transformation is not rejected for the money relation whereas the wealth relation cannot be expressed in real terms.

Keywords: Money demand, wealth, cointegration, vector error correction model, $I(2)$ analysis.

JEL codes: E41, C32, C22

Non-Technical Summary

In this paper, we estimate a cointegrating model for euro area M3 money demand over the period 1980 to 2017 that, in addition to the price level, real GDP and interest rates, also includes housing wealth. We identify two long-run relations that we interpret as a money demand and a wealth relation. This specification has originally been proposed by Beyer (2009) who, using data up to 2007, showed that shifts in velocity since the early 1980s can be explained when including housing wealth in a money demand system. We update his data set until the third quarter of 2017 (2017:3), thus including the period over which the Global Financial Crisis (GFC) and the euro area sovereign debt crisis unfolded. Overall, we find that, despite the large fluctuations in many economic and financial variables occurring during this time, the money demand relation that was identified by Beyer (2009) has remained reasonably stable once only a few additional deterministic variables are included that account for the disruption in monetary policy transmission that occured in late 2008 following the outbreak of the GFC and for the start of the ECB's unconventional monetary policy measures in summer 2014. We apply a battery of misspecification tests and recursive estimation to show that our specification remains reasonably robust also during the GFC. Estimates for the long-run as well as for the short-run adjustment coefficients remain virtually unchanged to those estimated by Beyer (2009).

In the second part of the paper we extend the analysis to the nominal variables and estimate an $I(2)$ system that allows us to investigate a richer set of dynamic adjustments. For $I(2)$ variables, different forms of cointegration can occur. First, linear combinations of $I(2)$ variables may cointegrate to form an $I(1)$ variable; for instance, nominal money and the price level, being $I(2)$ variables, can cointegrate to real money as an $I(1)$ variable. Second, this cointegrating relationship can cointegrate with other $I(1)$ variables, e.g. real income or interest rates, to form a stationary, i.e. $I(0)$, relation. These different levels of cointegration give rise to various forms of adjustment to the long-run equilibrium of the system. Testing for price homogeneity in the $I(2)$ model we find that the nominal-to-real transformation is not rejected for the money relation whereas the wealth relation cannot be expressed in real terms. The main adjustment takes place through an adjustment of the growth rates to the long-run equilibrium relations whereas the α coefficients that drive the adjustment to the polynomially cointegrated relations are numerically small.

1 Introduction

Money demand is one of the most researched relations in economics. It is particularly relevant for monetary policy makers because a stable money demand function is a prerequisite to use monetary aggregates as a target variable in order to achieve price stability. For those central banks that attach importance to the evolution of monetary aggregates, like e.g. the ECB, recurrent instabilities in estimated money demand functions have presented challenges when assessing the monetary policy stance from the perspective of the monetary analysis, the ECB's second pillar. Whereas specifications could be found up to the financial crisis that were reasonably stable,[1] the situation worsened with the financial crisis and the ensuing large shifts in macroeconomic variables such as real GDP and interest rates.

The analysis of money demand for the euro area is even more challenging than for other regions. First, the euro as a currency exists only since a relatively short period of time so that in general empirical studies have to rely on aggregated national data for the time before 1999. Second, the convergence between the euro area member states that took place before and after the introduction of the single currency might have caused changes in the dynamics of the system. Third, during the last decade the euro area was hit by large financial shocks that might have driven variables relatively far away from their equilibrium values and thus take a relatively long time to unwind.

In this paper, we revisit the money demand specification of Beyer (2009) who showed that shifts in velocity since the early 1980s can be explained when including housing wealth in a money demand system. We update the sample until the third quarter of 2017 (2017:3), thus including the period over which the Global Financial Crisis (GFC) and the euro area sovereign debt crisis unfolded. Overall, we find that, despite the large fluctuations in many economic and financial variables occurring during this time, the money demand relation that was identified by Beyer (2009) has remained reasonably stable. In addition, we investigate money demand in an $I(2)$ system that allows us to investigate a richer set of dynamic adjustments and to test price homogeneity assumptions instead of imposing them from the outset.

Motivated by the ECB's second pillar that reflects the importance of money growth for inflation developments, various money demand models have been estimated for the euro area.[2] Many of the money demand specifications that have been proposed in the early 2000s became unstable over time. Beyer (2009) obtained a stable specification in a sample extending to 2007:4 when housing wealth was considered as an additional explanatory variable. The inclusion of wealth in a money demand system can be justified by considering money demand as part of the portfolio choice (see Gerdesmeier 1996).

In fact, most of the more recent studies model money demand in a portfolio choice setting. De Santis et al. (2012), for instance, include the long-term government bond yield, the stock market price-earnings ratio and the USD/EUR exchange rate to identify three long-run relationships which they interpreted as euro area money demand, a relation for domestic assets and a relation for foreign assets. Dreger and Wolters (2015) include real housing prices as a proxy for real wealth and found a stable money demand function for a sample period spannning 1983:3 to 2010:4. Jung (2016) investigates the individual components in M3 and finds stable relationships for each subcomponent with the exception of currency in circulation.

Though money demand has also been investigated in an $I(2)$ setting, the literature

[1] See e.g. Calza et al. (2001), Brand and Cassola (2004).

[2] See Beyer (2009) for an overview.

is relatively scarce. Norrbin and Reffett (1995) analysed money demand in the US in an $I(2)$ model and concluded that, like in their $I(1)$ models, stable representations of money demand can only be found for broad monetary aggregates such as M2. Juselius (2006) estimated an $I(2)$ money demand system on Danish data and found that, though nominal money and prices move one to one in the money equation, price homogeneity was violated, which seemed to be associated with the behaviour of money, prices and income in the price and income equations. Johansen (1992*a*) estimated the demand for M1 in the United Kingdom and found broadly similar coefficients in an $I(1)$ and an $I(2)$ setting. Kongsted (2005) re-examined in an $I(2)$ setting the money demand models of Stock and Watson (1993) for M1 in the US, of Ericsson et al. (1998) for broad money in the UK and of Muscatelli and Spinelli (2000) for Italy and concluded that the nominal-to-real transformation could not be rejected for the US and UK money demand relations whereas this was not the case with the Italian data. For the euro area, Holtemöller (2004) founds that nominal M3 and prices are $I(2)$ and cointegrate to an $I(1)$ variable. As price homogeneity cannot be rejected, he performed the analysis in the $I(1)$ space. Jung and Carcel (2019) found that an $I(2)$ model for M3 including stock prices can be used for forecasting inflation at horizons between one and two years.

This paper is structured as follows: Section 2 presents the data and the methodology, Section 3 provides an update of the $I(1)$ model for euro area money demand based on the results of Beyer (2009), Section 4 discusses our empirical results for the money demand system in an $I(2)$ setting and Section 5 concludes.

2 Data and methodology

2.1 Data

The analysis in Beyer (2009) used data up to 2007:3. Model estimates based on provisory data until 2008:4 hinted, at the time, at a large structural break right at the end of the sample period. Almost ten years later, we investigate in this paper whether the model specification identified in Beyer (2009) has remained stable after the GFC. We explore how many breaks seem to be present and how they can be modelled. Overall, we find that the model still holds once these breaks are appropriately accounted for. For the empirical analysis in this paper we used the software PcGive (Doornik, 2014), CATS 3 in OxMetrics 8 (Doornik and Juselius, 2018) and SVAR (Warne, 2009).

We use the same variables as Beyer (2009), namely the euro area nominal money stock M3 (m_t); the GDP deflator (p_t), which we use to deflate the nominal variables; nominal housing wealth (wwh_t); real GDP (y_t); the three-month money market interest rate (RS_t); and the own rate for M3 (RO_t). Our data starts in 1980:2, which is the first observation available and ends in 2017:3.[3] As it is common practice, some of the macroeconomic time series have been revised backwards, even for observations covering the original sample period before 2007. To ensure comparability of the model over the extended sample with the original model, we extend Beyer's (2009) dataset by updating the levels of the original series with growth rates of the variables over the sample from 2008:1 to 2017:3. Apart from interest rates, all data are seasonally adjusted. Figures 1 and 2 show the time series.

To illustrate the enormous impact the GFC had on nominal as well as real GDP and money growth, Table 1 shows the means of Δm, $\Delta(m-p)$, Δy^n and Δy over the

[3] All data can be downloaded at the ECB's Statistical Data Warehouse webpage, see http://sdw.ecb.europa.eu/.

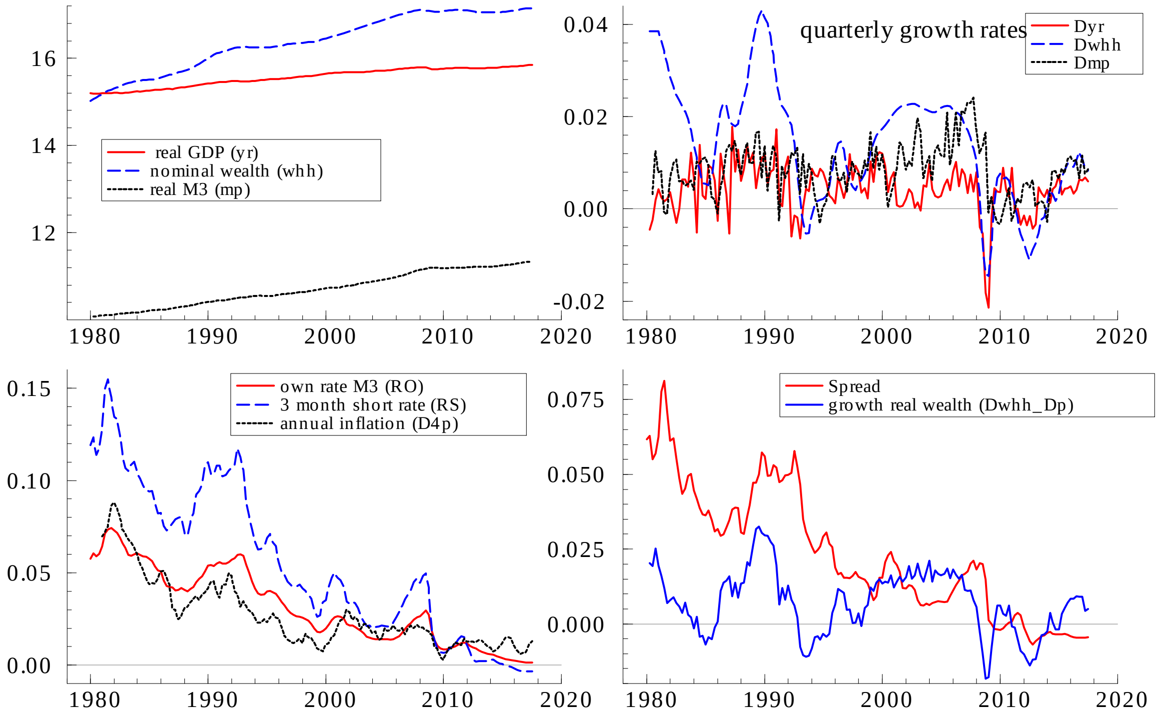


Figure 1: Time series of data

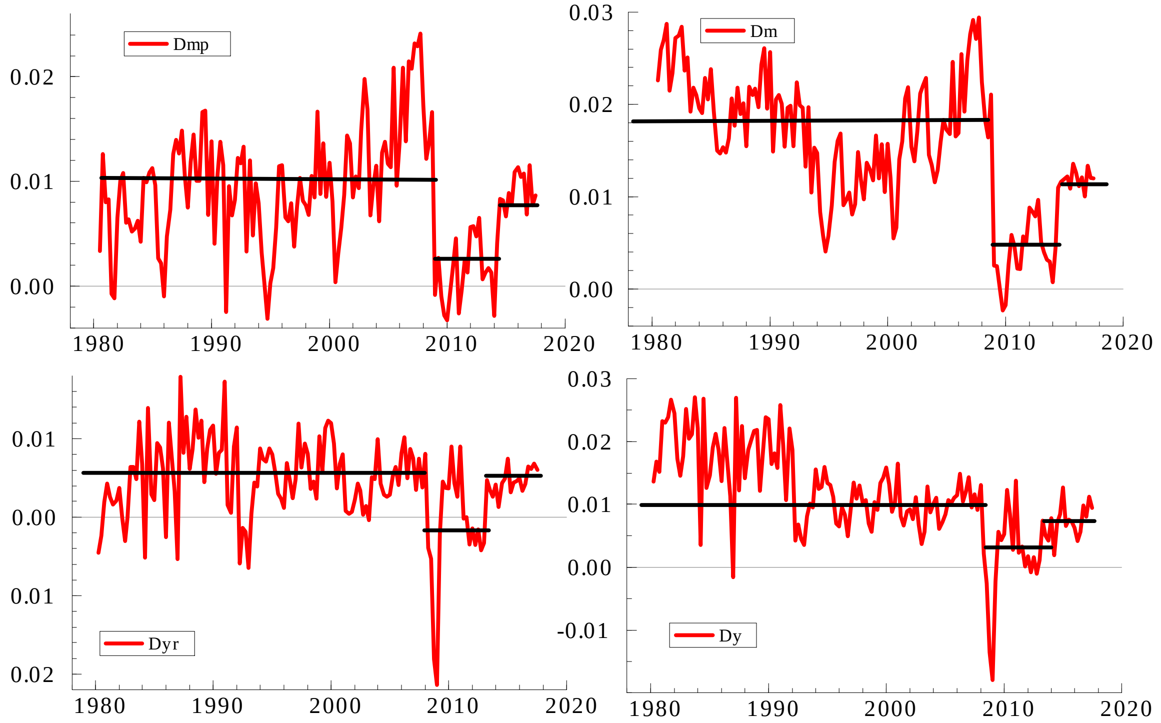


Figure 2: Real (left column) and nominal (right column) quarterly growth rates of M3 and GDP

Table 1: Annualised growth rates of nominal and real M3 and GDP

	1980:3-2017:3	1980:3-2008:3	2008:4-2014:2	2014:3-2017:3
Δm	6.18	7.22	1.84	4.79
$\Delta(m-p)$	3.40	3.89	0.81	3.65
Δy^n	4.54	5.43	0.94	3.21
Δy	1.77	2.11	-0.10	2.07

Quarter-on-quarter changes in per cent, multiplied by four.

Table 2: Augmented Dickey-Fuller tests

Variable	Data series	Order of integration
m	Nominal money stock M3	$I(2)$
p	GDP deflator	$I(2)$
$\Delta_4 p$	Annualised inflation rate	$I(1)$
$m-p$	Real money stock M3	$I(1)$
whh	Nominal housing wealth	$I(2)$
$whh-p$	Real housing wealth	$I(1)$
y	Real GDP	$I(1)$
RS	Three-month money market interest rate	$I(1)$
RO	Own rate for M3	$I(1)$

entire sample period and over subsamples. The three subsamples correspond to the pre-crisis period until 2008:3; the GFC starting with the bankruptcy of Lehman Brothers in 2008:4; and the ECB's quantitative easing period starting in 2014:3. During the second subsample, annualised nominal money and GDP growth were about 4 percentage points lower than in the pre-crisis period. Growth of real money fell by 3 percentage points, and real GDP growth by more than 2 percentage points. In the last subsample, during the period of the ECB's non-standard measures, real GDP growth recovered to its pre-crisis rate and also real money growth reached almost its pre-crisis value. Nevertheless, nominal growth rates still were around 2-2.5 percentage points lower than in the pre-crisis period, owing to protracted low inflation.

To investigate the time-series properties of our data, we first conduct univariate Augmented Dickey-Fuller (ADF) tests with a lag length that is determined by the Akaike Information Criterion (AIC). We include a trend in the test equation for the variables in levels (except for annual inflation and the interest rates) and a constant only for the first differences.

Table 2 shows that M3, the GDP deflator and nominal housing wealth are found to be $I(2)$, whereas real M3, real wealth, real GDP and the two interest rates seem to be $I(1)$. Notice that the first and fourth difference of the price level appear only borderline non-stationary, depending on the specification of the deterministic variables in the test equation.[4] These univariate ADF tests should be regarded as a first indication of the integration properties of the data. Later on, we will revisit the question of non-stationarity in the context of the estimated $I(1)$ and $I(2)$ systems. As these approaches rely on different models to test stationarity, one should not expect that they necessarily yield the same results. For the purpose of our analysis, we conclude that the first inspection of the data suggests the presence of at least two types of stochastic trends, namely a nominal one and at least one or more real trends, that potentially drive the

[4]Results for the GDP deflator were puzzling in that trend stationarity could not be rejected for the level whereas stationarity was rejected for the first differences.

dynamics of the time series.

2.2 Modelling structural breaks: deterministic variables

To account for the effects of the GFC and the subsequent impairment of the monetary transmission mechanism in the euro area, we include two broken trends in the $I(1)$ and the $I(2)$ model and a number of dummy variables that are consistent with the specific time periods in which the broken trends materialise. The first broken trend starts in 2008:4 when the bankruptcy of Lehman Brothers began to spill into the real economy and central banks around the world started to lower interest rates. The second broken trend starts in 2014:2, when the ECB began with credit easing through its Targeted Long-Term Refinancing Operations (TLTROs), reduced its policy rate to below zero and subsequently enacted its expanded asset purchase programme (see European Central Bank 2015, 2017).[5]

In the $I(1)$ model we further include two corresponding shift dummies $D_{08:4}$ and $D_{14:3}$ that take the value of unity from 2008:4 and from 2014:3 to the end of our sample and zero otherwise. These shift dummies basically reflect the first differences of the broken trend variables and take account of the large shifts in the sample means of real money growth and GDP growth in the subperiods identified in Table 1. We also use an impulse dummy that is unity in 2008:4 and zero elsewhere.[6] Our modelling strategy is to avoid the use of impulse dummy variables as much as possible. In particular we refrain from using outlier dummy variables to get a better fit or better test results.

The $I(2)$ model also includes the two broken trends. As the estimation procedure in CATS automatically includes these broken trends as first and second differences, the corresponding shift dummies and impulse dummies enter the model as well. In addition, we add a transitory impulse dummies for 2008:4 that captures the immediate effects of the GFC.[7]

To avoid quadratic or cubic trends in the level VAR, we restrict the trend t and the two broken trends to lie in the cointegrating space for the $I(1)$ and the $I(2)$ models. We leave the constant, the shift dummies $D_{08:4}$ and $D_{14:3}$ and the impulse dummy $D_{2008:4}$ unrestricted with respect to the cointegrating space in order to account for the trending behaviour of $(m-p)_t$, y_t and whh_t.[8] The transitory impulse dummy is unrestricted as well since it cumulates to zero and therefore does not exert a permanent impact on the variables.

2.3 $I(1)$ versus $I(2)$: the economic rationale

Starting point for our empirical analysis is a p-dimensional vector autoregressive model (VAR) of order k,

$$X_t = \Pi_1 X_{t-1} + \Pi_2 X_{t-2} + \ldots + \Pi_k X_{t-k} + \Phi D_t + \varepsilon_t, t = 1 \ldots T, \tag{1}$$

[5] In the $I(2)$ model, the broken trends enter the model with one lag, i.e. starting in 2009:1 and 2014:3, respectively. Results for the $I(1)$ are almost identical if the broken trends are lagged as well, since both model specifications include another impulse dummy that takes account of the large swings in many macroeconomic variables in 2008:4.

[6] See Juselius (2006) for a discussion of different types of dummies and their effects on the dynamics in the $I(1)$ and the $I(2)$ model.

[7] The transitory impulse dummy for 2008:4 is unity in 2008:4 and minus unity in 2009:1.

[8] We tried various specifications and combinations, with deterministic variables restricted or unrestricted with respect to the cointegrating space. The results concerning the likelihood, the p-value for the validity of the overidentifying restrictions on β, the point estimates and the recursive parameter stability properties are similar.

where the data vector is given by $X_t = [m_t, p_t, y_t, whh_t, RS_t, RO_t]$ as defined in Table 2 above. ε_t is a sequence of independent Gaussian error variables with zero mean and covariance matrix Ω, and D_t contains the deterministic variables as discussed above, namely, a constant, a trend, two broken trends, and a set of dummies to account for the shifts related to the effects of the GFC. The model in equation (1) will form the basis of our $I(1)$ and $I(2)$ analyses. For the $I(1)$ model, however, we will transform the variables appropriately in order to remove the $I(2)$ trends before the estimation. In the following, we will discuss some scenarios to illustrate this.

As nominal money, nominal housing wealth and the price level were found to be $I(2)$, we first discuss some scenarios for the potential driving forces of the order of integration of the variables and how these could be represented in the empirical analysis (see Juselius 2006, 2017). We assume the existence of three independent stochastic shocks, which we could think of as a nominal demand shock, u_{1t}, a real supply shock, u_{2t}, and a shock to real housing wealth, u_{3t}. Starting point is a decomposition of the data vector into its driving shocks and the trend component, where we allow for the possibility of a deterministic trend in m_t, p_t, y_t and whh_t, but not in either of two interest rates:

$$\begin{bmatrix} m_t \\ p_t \\ y_t \\ whh_t \\ RS_t \\ RO_t \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \\ 0 \\ c_{41} \\ 0 \\ 0 \end{bmatrix} \left(\sum\sum u_{1t}\right) + \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \\ d_{41} & d_{42} & d_{43} \\ d_{51} & d_{52} & d_{53} \\ d_{61} & d_{62} & d_{63} \end{bmatrix} \begin{bmatrix} \sum u_{1t} \\ \sum u_{2t} \\ \sum u_{3t} \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \\ 0 \\ 0 \end{bmatrix} t + stat.comp. \quad (2)$$

The nominal demand shock is cumulated twice and enters m_t, p_t and whh_t, inducing $I(2)$ behaviour in the variables. Long-run price homogeneity of money demand would require $c_{11} = c_{21}$, meaning that the nominal demand shocks affects money and prices in the same way in the long run, so that the $I(2)$ trends in nominal money and prices cancel out.[9] Correspondingly, we can define long-run price homogeneity for nominal housing wealth, which was also found to be $I(2)$. If $c_{41} = c_{21}$ nominal housing wealth cointegrates with the GDP deflator to $I(1)$. With these assumptions, our system in equation (2) can be reformulated as follows:

$$\begin{bmatrix} m_t - p_t \\ \Delta p_t \\ y_t \\ whh_t - p_t \\ RS_t \\ RO_t \end{bmatrix} = \begin{bmatrix} d_{11} - d_{21} & d_{12} - d_{22} & d_{13} - d_{23} \\ c_{21} & 0 & 0 \\ d_{31} & d_{32} & d_{33} \\ d_{41} - d_{21} & d_{42} - d_{22} & d_{43} - d_{23} \\ d_{51} & d_{52} & d_{53} \\ d_{61} & d_{62} & d_{63} \end{bmatrix} \begin{bmatrix} \sum u_{1t} \\ \sum u_{2t} \\ \sum u_{3t} \end{bmatrix} + \begin{bmatrix} g_1 - g_2 \\ 0 \\ g_3 \\ g_4 - g_2 \\ 0 \\ 0 \end{bmatrix} t + stat.comp. \quad (3)$$

where the inflation rate is solely affected by the once-cumulated nominal shock whereas all the other variables can be affected by all three shocks in the medium run. While equation (3) assumes long-run price homogeneity, it allows for money and prices to deviate over the business cycle, i.e. real money is $I(1)$ as the three shocks cumulated once can enter the real money equation. This seems reasonable as e.g. real GDP might be affected differently by, for instance, the supply shocks, $\sum u_{2t}$, than real wealth or real M3, i.e. $d_{32} = d_{42} - d_{22} = d_{12} - d_{22}$ is unlikely to hold. The same is true for the medium-run impact of the real wealth shock on the variables.

If we suspect that the GDP deflator is not ideally suited to deflate real wealth, we alternatively can reformulate the system (2) by imposing price homogeneity for the

[9] See Juselius (2006).

money equation but including nominal wealth in first differences, as in Beyer (2009), which would lead to different coefficient restrictions, see equation (4).

$$\begin{bmatrix} m_t - p_t \\ \Delta p_t \\ y_t \\ \Delta whh_t \\ RS_t \\ RO_t \end{bmatrix} = \begin{bmatrix} d_{11}-d_{21} & d_{12}-d_{22} & d_{13}-d_{23} \\ c_{21} & 0 & 0 \\ d_{31} & d_{32} & d_{33} \\ c_{41} & 0 & 0 \\ d_{51} & d_{52} & d_{53} \\ d_{61} & d_{62} & d_{63} \end{bmatrix} \begin{bmatrix} \sum u_{1t} \\ \sum u_{2t} \\ \sum u_{3t} \end{bmatrix} + \begin{bmatrix} g_1 - g_2 \\ 0 \\ g_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} t + stat.comp. \quad (4)$$

In the $I(2)$ analysis, we will start with the levels of the variables and investigate further the nominal-to-real transformation whereas the $I(1)$ analysis takes equation(4) as the starting point.[10]

3 The statistical $I(1)$ model

Assuming long-run price homogeneity, we proceed treating the process $\{X_t\} = [(m-p)_t, \Delta_4 p_t, y_t, \Delta whh_t, RS_t, RO_t]$ as $I(1)$. This deviates slightly from the scenario in equation (4) as we use the annual difference of inflation to make it comparable to nominal interest rates that are also measured on an annual basis. The VAR model in equation (1) can be represented as a vector error correction model (VECM)

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \varepsilon_t \quad (5)$$

where $\Pi = \sum_{i=1}^{k} \Pi_i - I$ and $\Gamma_i = -\sum_{j=i+1}^{k} \Pi_j$. If the variables in the model are cointegrated, $\Pi = \alpha\beta'$, where α and β are $(p \times r)$ matrices having full rank r, with β being the cointegrating vectors and α the loading coefficients. We assume that the characteristic polynomial

$$A(z) = I - \sum_{i=1}^{k} \Pi_i z^i \quad (6)$$

satisfies the condition that if $|A(z)| = 0$, then either$|z| > 1$ or $z = 1$.

3.1 Determining the lag length

As the next step, we determine the lag order k of the VAR model. The usual information criteria indicate a lag length of two. We start with a lag length of four and test whether all variables dated $t-4$ can be excluded from the system. This hypothesis is rejected at the 5% level of significance, but not at the 1% level, with a likelihood ratio (LR) test statistic of $\chi^2(36) = 55.11$ (p-value of 0.03). By contrast, reducing the VAR(4) to a VAR(2) is rejected with a p-value of zero ($\chi^2(72) = 126.24$). Moreover, we find that results from misspecification tests did not improve when more lags were included, which lead us to opt for the VAR(3) model in order to conserve degrees of freedom.

Examining the residuals (see Table 3) shows that absence of serial correlation and heteroskedasticity (autoregressive conditional or squared regressors) are mostly not rejected. However, normality is strongly rejected in the GDP and interest rate equations due to some large outliers that occur mainly around 2009 (GDP) and 2011 (interest

[10]Results in Section 4 show that price homogeneity for wealth is rejected.

Table 3: Misspecification tests for the VAR(3) model

	$(m-p)$	y	Δwhh	$\Delta_4 p$	RS	RO
AR(1-3), F(3,116)	0.67	**5.97**	1.69	1.70	0.38	1.71
p-value	[0.56]	[0.00]	[0.17]	[0.16]	[0.76]	[0.16]
$ARCH$(1-3), F(3,138)	0.29	0.99	1.24	**4.50**	0.11	0.26
p-value	[0.86]	[0.39]	[0.29]	[0.01]	[0.95]	[0.84]
Hetero X_i^2, F(44,98)	0.92	**1.84**	**1.74**	1.11	1.44	**2.28**
p-value	[0.60]	[0.01]	[0.01]	[0.32]	[0.06]	[0.00]
Normality, $\chi^2(2)$	1.85	**21.16**	1.65	**9.51**	**17.44**	**23.19**
p-value	[0.39]	[0.00]	[0.43]	[0.01]	[0.00]	[0.00]

AR(1-3): residual autocorrelation up to 3 lags; $ARCH$(1-3): conditional heteroskedasticity; HeteroX_i^2 unconditional heteroscedasticity (squared product of regressors). Significant test statistics at the 5% level are indicated in bold.

rates). Yet, we refrain from adding more impulse dummies (in addition to the one included for 2008:4). Instead we proceed with the estimation as inference is generally reasonably robust to the presence of kurtosis, thereby justifying the assumption of asymptotic normality; see e.g. the discussion in Hendry (1995) or Spanos (1986).

We hence choose a lag length of three, i.e. in the VECM formulation our model will include two lagged differences of the variables. We will use the same lag length also for the $I(2)$ model, as it includes the same variables, though in slightly different transformations. Performing the lag length tests with the variables for the $I(2)$ model yields broadly identical results.

3.2 Determining the cointegration rank

Next, we estimate the cointegration rank. For the null hypothesis "rank $\Pi \leq r$" Table 4 shows the trace-test statistic and the Bartlett-corrected trace-test statistic, marked with an asterisk, together with the 95% quantiles and the corresponding p-values. Bartlett-corrected trace statistics are applied to take into account small sample biases which often yield over-sized tests, see Johansen (2000, 2002). As can be seen from Table 4, the standard test statistic clearly rejects the hypotheses of at most zero ($r = 0$) or at most one cointegrating vector ($r = 1$). The existence of at most two cointegrating vectors is rejected too, pointing towards a third relationship. Applying the Bartlett correction, the existence of at most two cointegrating vectors is rejected at the 5% level of significance but not at the 1% level.

It is well known that if non-standard deterministic variables are restricted to lie in the cointegrating space, standard tables for the trace test to determine the cointegrating rank are not valid. Instead, p-values have to be simulated from a bootstrap distribution. The simulated critical values are reported in Table 5. Overall, it turns out that also the simulated critical values point to the same choice of cointegration rank.

In practice, the decision about the cointegrating rank generally is a pragmatic one. Often not all criteria point to the same rank. Additional important information for the choice of the cointegrating rank is therefore the largest non-unit eigenvalue of the companion matrix of the characteristic polynomial in equation (6) after imposing $(p-r)$ unit roots, corresponding to cointegrating rank r. It is desirable for the largest non-unit eigenvalue to be as small as possible, as the inclusion of a cointegration vector that is in fact non-stationary would lead to a large characteristic root close to the unit circle see

Table 4: Trace test statistics for cointegrating rank

$p-r$	r	Eig.value	Trace	Trace*	Frac95	p-value	p-value*	Root[a]
6	0	0.40	214.31	186.32	138.95	0.00	0.00	0.99
5	1	0.27	140.46	118.54	106.84	0.00	0.01	0.85
4	**2**	**0.25**	**94.72**	**81.14**	**78.97**	**0.00**	**0.04**	**0.85**
3	3	0.16	52.56	45.26	55.27	0.08	0.26	0.93
2	4	0.13	27.61	23.61	34.29	0.21	0.40	0.92
1	5	0.05	8.07	4.53	17.53	0.56	0.89	0.98

[a] Largest non-unit eigenvalue of the companion matrix of the characteristic polynomial when $(p-r)$ unit roots have been imposed. Trace* and p-value* are the Bartlett-corrected trace statistics and the corresponding p-values.

Table 5: Quantiles of the simulated rank test distribution

$p-r$	r	50%	75%	80%	85%	90%	95%	97.5%	99%
6	0	114.86	13.99	114.41	123.75	126.29	129.29	133.39	138.95
5	1	86.17	12.05	85.63	94.07	96.16	98.82	102.09	106.84
4	2	61.05	10.35	60.36	67.70	69.70	71.84	74.59	78.97
3	3	40.12	8.53	39.52	45.31	46.92	48.85	51.24	55.27
2	4	22.63	6.42	22.02	26.50	27.69	29.23	31.23	34.29
1	5	9.38	4.34	8.67	11.72	12.57	13.73	15.05	17.53

e.g. Juselius (2006, p. 142). The larger the modulus of the largest unrestricted root, the more persistence is left in the model, indicating that a return to the estimated long-run relations occurs only slowly. Table 4 reports those roots in the last column. It shows that for $r = 2$ (i.e. four unit roots imposed) the largest unconstrained root is much smaller (0.85) than when imposing three unit roots (0.93, corresponding to three cointegrating vectors). Hence, we follow Beyer (2009) and continue our analysis imposing $r = 2$.[11]

Before proceeding to the identification of a long-run structure for the model, we complement our preliminary ADF test analysis and present formal unit root tests for stationarity of the individual variables in X_t. Table 6 shows χ^2 test statistics for each variable under the null hypothesis that a variable is a single cointegrating vector in itself. For $r = 2$ this is rejected for all variables, also for inflation (with a p-value just above one percent) which was only borderline non-stationary according to the ADF tests shown above. This confirms our $I(1)$ assumption for all variables in X_t. Next, we will identify and estimate two cointegrating relationships.

3.3 Identifying a long-run structure

The estimates of the overidentified cointegrating vectors are shown in Table 7. As in Beyer (2009) we impose overidentifying restrictions that are motivated by interpreting the long-run relations as a money demand and a real wealth equilibrium relationship.

To facilitate comparison, we also reproduce the estimates of Beyer (2009) for the sample period that ends in 2007:4. Parameters in bold are restricted and standard errors are shown in parentheses. We restricted the broken trends to lie in the cointegration

[11] When all deterministic variables are unrestricted, the trace test yields basically the same results but the assumption of $r = 2$ is slightly more supported. Notice that the roots of the eigenvalues of the companion matrix are identical irrespectively of whether the broken trends are restricted to lie in the cointegration space or not.

Table 6: Test for stationarity of individual variables as single cointegrating vectors

rank r	d.o.f.	5% c.v.	$m-p$	y	Δwhh	$\Delta_4 p$	RO	RS
1	5	11.07	44.20 [0.00]	48.92 [0.00]	45.55 [0.00]	41.58 [0.00]	42.23 [0.00]	42.65 [0.00]
2	**4**	**9.49**	**19.81 [0.00]**	**25.99 [0.00]**	**24.17 [0.00]**	**13.65 [0.01]**	**17.54 [0.00]**	**18.66 [0.00]**
3	3	7.82	23.90 [0.00]	24.68 [0.00]	20.55 [0.00]	10.39 [0.02]	14.76 [0.00]	16.26 [0.00]
4	2	5.99	9.79 [0.01]	6.86 [0.03]	7.64 [0.02]	2.88 [0.24]	6.67 [0.04]	5.76 [0.06]
5	1	3.84	3.47 [0.06]	2.30 [0.13]	3.15 [0.08]	0.03 [0.86]	0.03 [0.85]	0.25 [0.62]

Likelihood ratio test, $\chi^2(6-r)$, p-values in brackets.

space and, in line with the return of real GDP growth rates to their pre-crisis level, we imposed a zero restriction on the coefficient of $t_{2014:2}$ in the second cointegrating relation.

The seven overidentifying restrictions are not rejected with an asymptotically $\chi^2(7)$ distributed test statistic of 10.42 and an asymptotic p-value of 0.16. Based on simulated critical values for the restrictions on β, the corresponding $\chi^2(6)$ test statistic is also safely not rejected with a $p-$value of 0.63.[12]

For the specification of β in Table 7, Table 8 shows the corresponding α vectors with p-values in brackets. Again, the point estimates and the significance of the loading coefficients are almost identical to those of the pre-crisis model. A formal test for weak exogeneity of individual variables (i.e. none of the two cointegrating vectors enters the corresponding VECM equation) is reported at bottom of Table 8. As in the pre-crisis model, nominal wealth growth clearly turns out to be weakly exogenous.

As already motivated above, we interpret the first β vector in Table 7 as a money demand relation,

$$(m-p)^*_t = 1.72y_t - 4.49(\Delta whh - \Delta_4 p)_t - 0.004t_{2008:4} + 0.011t_{2014:2} \tag{7}$$

and the second β vector as a real wealth relation.

$$(\Delta whh - \Delta_4 p)^*_t = 0.85y_t - 0.005t + 0.003t_{2008:4} - 1.35(RS - RO)_t \tag{8}$$

The point estimates for the updated model are remarkably similar compared to the pre-crisis model for which the sample ends in 2007. The economic interpretation obviously remains the same as in Beyer (2009). Equation (7) suggests that the income elasticity of money demand is positive and greater than unity, while the coefficient on real housing wealth is negative, indicating a substitution relation between money and real housing wealth, consistent with a portfolio balance approach. The zero restriction on the interest rate spread in the money relation is not rejected. Beyer (2009) discusses potential reasons for why this might be the case. The interest rate spread, however, enters the real wealth relation, and because linear combinations of cointegrating vectors are also potential valid cointegrating relationships, this does not rule out an influence from interest rates on money demand velocity.

Beyer (2009) includes a wealth variable to account for the 2003 break in velocity that occurred in standard quantity theory based money demand models and is depicted by the green dashed line in Figure 3. Quantity theory based velocity is defined as

$$v_{QT} = -(m-p-y). \tag{9}$$

[12] Notice that for the bootstrap simulations we had to relax the restriction that the broken trend coefficients lie within β. Therefore we estimate β by only imposing six overidentifying restrictions.

Table 7: Over-identified cointegrating relationships

End of sample	2007:4	2017:3	2007:4	2017:3
$(m-p)$	1.00 (–)	1.00 (–)	0.00 (–)	0.00 (–)
y	**−1.70** (0.04)	**−1.72** (0.04)	**−0.84** (0.01)	**−0.85** (0.01)
Δwhh	4.11 (–)	4.49 (–)	1.00 (–)	1.00 (–)
$\Delta_4 p$	**−4.11** (0.24)	**−4.49** (0.24)	1.00 (–)	1.00 (–)
RS	0.00 (–)	0.00 (–)	1.37 (–)	1.35 (–)
RO	0.00 (–)	0.00 (–)	**−1.37** (0.08)	**−1.35** (0.09)
t	0.00 (–)	0.00 (–)	0.005 (–)	0.005 (–)
$t_{2008:4}$	n.a.	**0.004** (0.002)	n.a.	**−0.003** (0.001)
$t_{2014:2}$	n.a.	**−0.011** (0.002)	n.a.	0.00 (–)
Restrictions on individual CIs	$\chi^2(3)$=2.94 [0.401]	$\chi^2(3)$=1.22 [0.748]	$\chi^2(3)$=3.17 [0.365]	$\chi^2(4)$=5.98 [0.200]
Joint restrictions on two CIs	$\chi^2(6)$=5.02 [0.540]	$\chi^2(7)$=10.42 [0.166]		

Trend restricted to lie in the cointegrating space. Restricted coefficients without standard errors, significant coefficients in bold, standard errors in round brackets and p-values in squared brackets. All p-values are asymptotically χ^2 distributed. Trend coefficient in CI_2 restricted to 0.005; normalisation by $\beta_{2y} = 0.85$ (0.005/0.85=0.0059) yields average quarterly GDP growth, corresponding to average annual GDP growth of 2.3%.

Table 8: Loading coefficients α of the two cointegrating relationships

	$\Delta(m-p)$	Δy	$\Delta\Delta whh$	$\Delta\Delta_4 p$	ΔRS	ΔRO
CI_1 2007	-0.04 [0.03]	**0.12** **[0.03]**	0.01 [0.01]	**0.04** **[0.02]**	**0.10** **[0.02]**	0.01 [0.01]
CI_1 2017	-0.03 [0.02]	**0.08** **[0.02]**	0.01 [0.01]	**0.05** **[0.02]**	**0.09** **[0.02]**	0.01 [0.01]
CI_2 2007	**0.25** **[0.11]**	$\mathbf{-0.27}$ **[0.11]**	-0.04 [0.03]	-0.08 [0.08]	$\mathbf{-0.34}$ **[0.09]**	-0.02 [0.03]
CI_2 2017	**0.22** **[0.08]**	$\mathbf{-0.16}$ **[0.08]**	-0.04 [0.03]	-0.11 [0.06]	$\mathbf{-0.25}$ **[0.07]**	-0.01 [0.02]

β restricted as above, standard errors in brackets; significant coefficients in bold.

Testing variables for weak exogeneity

$r=2$, $\chi^2(2)$	15.67 $(0.00)^{**}$	26.96 $(0.00)^{**}$	**1.89** **(0.39)**	15.18 $(0.00)^{**}$	23.73 $(0.00)^{**}$	6.96 $(0.03)^{*}$

β restricted as above, p-values in parentheses; * and ** denote significance on the 1% and 5% level, respectively.

Two alternative velocity specifications can be derived from the estimated cointegration relationships. The first variant reformulates the first money demand cointegration relation, solving for $-(m-p-y)$:

$$v_{CI,1} = 0.72y_t + 4.50(\Delta whh - \Delta_4 p)_t + 0.004t_{2008:4} - 0.011t_{2014:2} \tag{10}$$

The second variant is a linear combination of the two cointegrating relationships, again expressed with regard to $-(m-p-y)$:

$$v_{CI,2} = 3.65(\Delta whh - \Delta_4 p)_t - 1.15(RS - RO) \tag{11}$$

$$+0.007t_{2008:4} - 0.011t_{2014:2} - 0.005t \tag{12}$$

Figure 3 shows that the two empirical velocity variables remain broadly constant over long time spans. These sub-periods are related to the pre-European Monetary Union (EMU) period in the 1990s, the introduction of the euro with a common monetary policy from 1999 onwards, and the period of the ECB's non-standard monetary policy measures in response to the break in velocity induced by the GFC after 2008.

In terms of the scenario analysis discussed in Section 2.3, the case that velocity $-(m-p-y)_t \sim I(0)$ would require that $d_{11} - d_{21} - d_{31} = 0$ and $d_{12} - d_{22} - d_{33} = 0$ and that $g_1 - g_2 - g_3 = 0$. Stationarity of velocity implies common stochastic movements in money, prices and income. However, Figure 3 shows that this does not seem to be the case for euro area data. Nevertheless, non-stationary velocity cointegrates with interest rates or with real wealth growth, as derived from the estimated model above.

3.4 Stability, robustness and recursive estimation

Next we present formal tests for parameter stability of the estimated model, using Anders Warne's (2009) software "*Structural VAR*", in which these tests are implemented (see also Bruggeman et al. (2003) for a detailed discussion and applications of these tests). Table 9 shows misspecification tests for our cointegrated VAR model under the imposed restrictions on β (see Table 7). The presence of ARCH effects, a key driver for misspecification, is broadly rejected. Like in Table 3, standard χ^2 tests strongly reject normality, which is caused by some large outliers particularly in the GDP and interest rate equations, see the discussion in Section 3.1. No serial correlation can be rejected

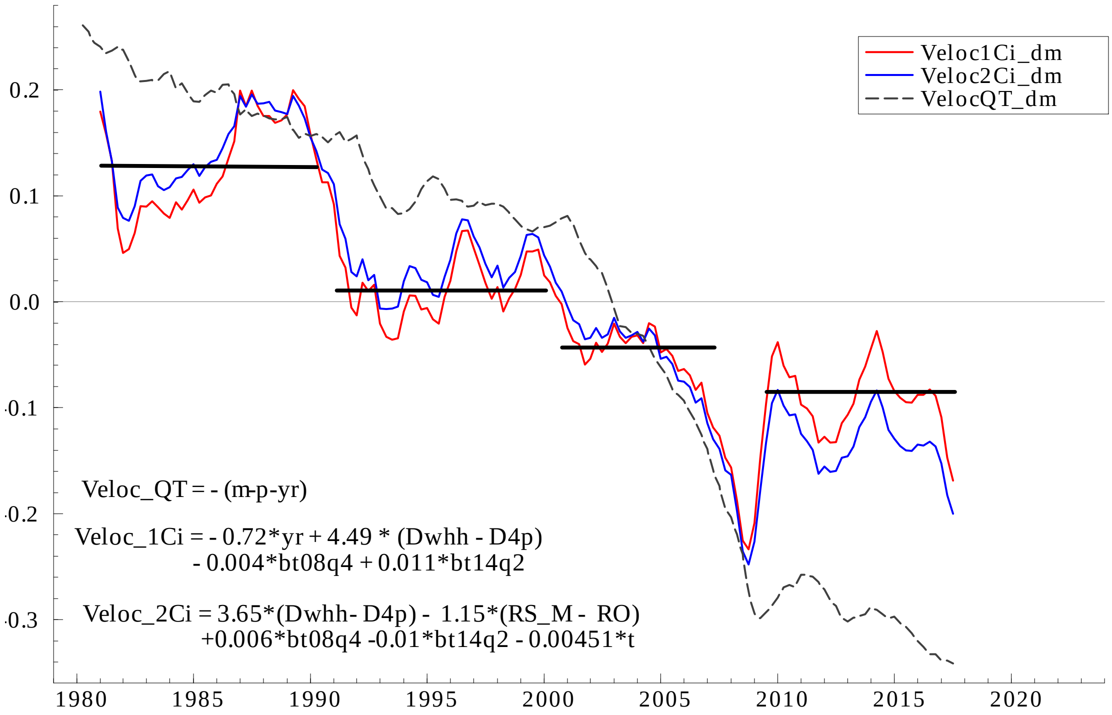


Figure 3: Naïve and model based empirical velocity

as well, though for the wealth growth, inflation and the own rate equations only at a significance level of 1%. Again, adding more impulse dummies to account for outliers would improve the test results but we refrain for the reasons explained above.

Figure 4 shows the recursive estimates of the β coefficients which remain remarkably stable over the entire sample throughout the GFC. The same applies for the recursive estimates of the two non-zero eigenvalues shown in Figure 5 which remain stable and clearly above zero over the entire estimation sample. This is consistent with Figure 6 which shows the results of the Hansen and Johansen (1999) fluctuation test for the sum of the two recursive eigenvalues when both eigenvalues are estimated under the imposed restrictions on β.

Finally, we present the results of the Nyblom (1989) tests for parameter constancy of β. We apply two different versions of the test, i.e. we test constancy when β is either restricted or unrestricted. The results presented in Table 10 show that constancy of β is not rejected.

Overall, the results from the re-estimation of Beyer (2009) with a data set that extends the previous sample ending in 2007 to 2017 and thus includes the aftermath of the GFC show that the model has kept its empirical properties. To account for the effects of the crisis, we have included two broken trends that mark the start of the crisis with the failure of Lehman Brothers in 2008:4 and the beginning of the ECB's unconventional monetary policy measures in 2014:2. Together with two shift dummies and an impulse dummy for 2008:4 the estimated key parameters remain remarkably stable also throughout the financial crisis.

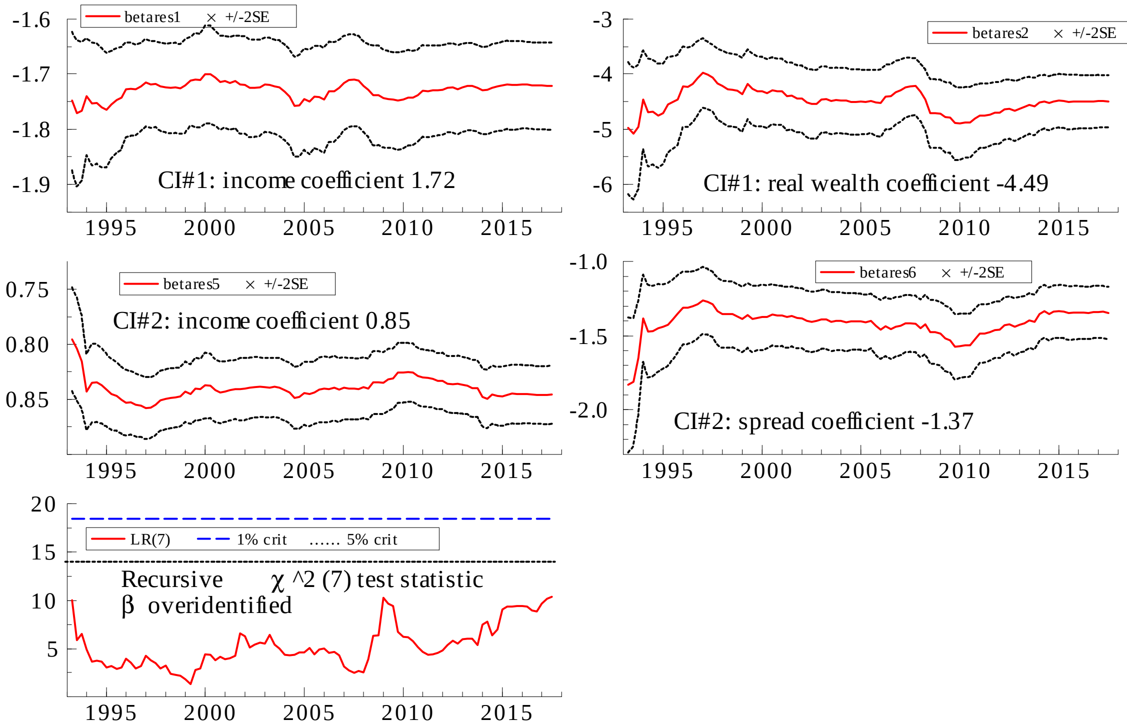
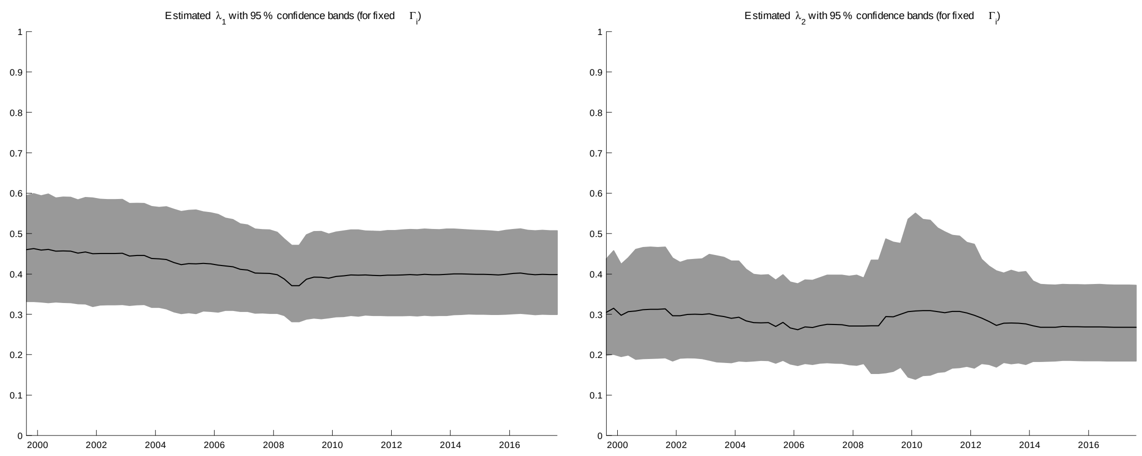


Figure 4: Recursive estimates



Recursive eigenvalue 1

Recursive eigenvalue 2

Figure 5: Recursive eigenvalues and 95 per cent confidence bands

Table 9: Misspecification tests

	$(m-p)$	y	Δwhh	$\Delta_4 p$	RS	RO
LM test (ARCH)	1.04	**4.93**	0.98	2.62	0.10	0.39
$\chi^2(1)$ [p value]	[0.30]	[0.03]	[0.32]	[0.11]	[0.75]	[0.52]
LM test (ARCH)	4.12	6.89	5.79	**11.33**	2.10	3.47
$\chi^2(4)$ [p value]	[0.39]	[0.14]	[0.21]	[0.03]	[0.71]	[0.48]
Normality	1.81	**14.15**	**8.02**	**10.12**	**32.12**	**34.46**
$\chi^2(2)$ [p value]	[0.40]	[0.00]	[0.03]	[0.01]	[0.00]	[0.00]
AR(12) Portmanteau	13.84	10.62	**23.78**	**26.05**	11.86	**19.07**
$\chi^2(9)$ [p value]	[0.14]	[0.30]	[0.01]	[0.01]	[0.22]	[0.02]

LM tests for first and fourth order ARCH effects; non-normality and autocorrelation of residuals for each equation of the restricted VAR. Null hypothesis: no ARCH in equation p, normality is a Wald-type test with a null hypothesis of no non-normality; null hypothesis for the Portmanteau test is no autocorrelation. Significant test statistics at the 5% level in bold.

Table 10: Nyblom tests for parameter constancy of β

β unrestricted			
α: 12 free parameters; β: 12 free parameters			
sup_Q	103.72	Mean_Q	19.83
p-value bootstrap	[0.18]	p-value bootstrap	[0.16]
β restricted			
α: 12 free parameters; β: 4 free parameters			
sup_Q	0.54	Mean_Q	0.10
p-value bootstrap	[0.99]	p-value bootstrap	[0.98]

Γ_i fixed over full sample. Null hypothesis: β is constant, test statistics: sup_Q, Mean_Q{t=1992:1,...,2017:3}, Q($t|T$=2017:3; $p=6, r=2, k=3$). Bootstrap simulation of Nyblom tests with 1000 replications, 144 draws.

4 An $I(2)$ model for euro area money demand

We now turn to the analysis of money demand in an $I(2)$ system, which will allow us to formally test the nominal-to-real transformation. There are several reasons why nominal money or nominal wealth and the price level might not cointegrate empirically despite we would expect them to do so in theory. First, it is not clear which price index should be used to measure the price level. We use here the GDP deflator, which represents a comprehensive measure of prices in the economy, also covering price developments for investment and export goods. If households use money predominantly for purchasing consumer goods, the consumer price index might be a better price index to deflate money holdings. A similar argument can be made in the context of nominal housing wealth, where the appropriate deflator could be a house price index. In the long run, all these different price indices should move together so that differences between them would be stationary and not affect long-run price homogeneity.[13] However, data for the euro area are relatively short, which complicates inference about long-run trends. Second, like

[13]See Juselius (1997) for a study of the long-run relations between different price indices for the United States.

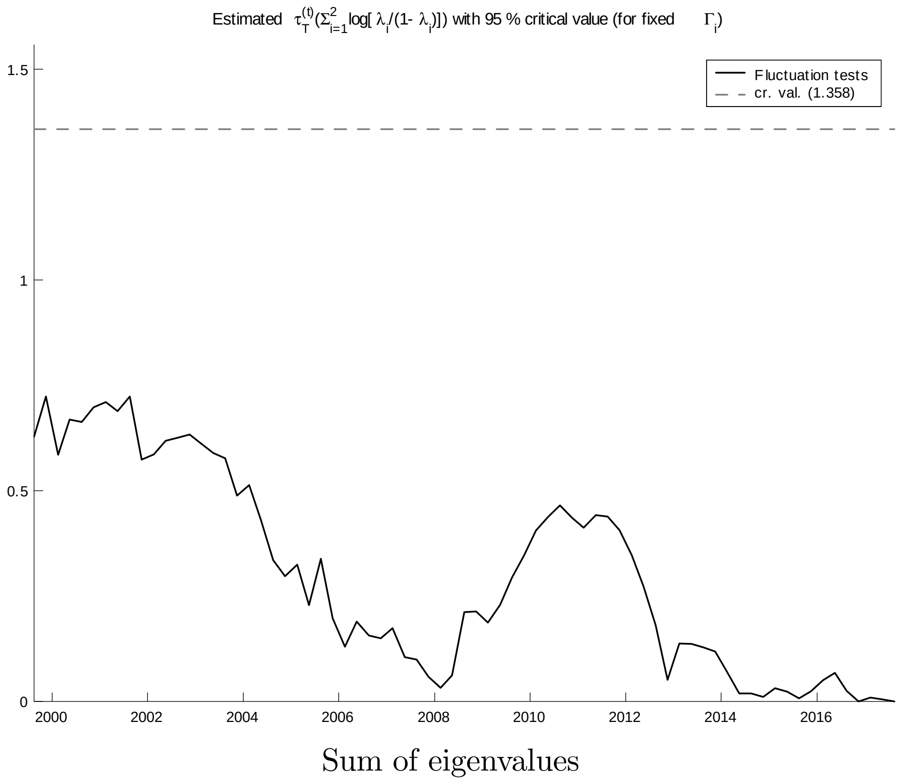


Figure 6: Fluctuation test and 95 per cent critical value

other studies of euro-area money demand, we use aggregated national data prior to the introduction of the euro in 1999 which may lead to aggregation issues (see Beyer et al. 2001) and complicate inference. Third, the recent decade was characterised by unusually large financial shocks that may not be fully captured by the deterministic variables that we include in the model.

For $I(2)$ variables, different forms of cointegration can occur, see Johansen (1992*b*). First, linear combinations of $I(2)$ variables may be $I(1)$ or even $I(0)$. If money demand is homogeneous in the price level, we would expect that nominal money cointegrates with the price level such that real money is $I(1)$. A similar relationship may exist for real wealth. Second, a linear $I(1)$ cointegrating relationship may cointegrate with $I(1)$ variables, such as interest rates, or with differences in $I(2)$ variables, which are again $I(1)$, like, for instance inflation. One example of a money demand model would be velocity cointegrating with interest rates or some other measure of opportunity cost for holding money, resulting in a stationary relationship.

The $I(2)$ CVAR model can be formulated in acceleration rates, changes and levels (see Juselius 2006). Based on the VAR model in equation (1) and assuming three lags, i.e. $k = 3$, the $I(2)$ model can be written as

$$\begin{aligned} \Delta^2 X_t &= \Gamma_2 \Delta^2 X_{t-1} + \Gamma \Delta X_{t-1} + \Pi X_{t-1} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \\ \varepsilon_t &\sim N_p(0, \Omega), \Gamma = -\left(I - \Gamma_1 - \Gamma_2\right). \end{aligned} \tag{13}$$

The p-dimensional data vector X_t is different from the $I(1)$ model above and is now given by $X_t = [m_t, p_t, y_t, whh_t, RS_t, RO_t]$, as defined in Section 2.3. While the $I(1)$ model is formulated as a reduced rank hypothesis on $\Pi = \alpha\beta'$ where α is a $p \times r$ matrix of adjustment coefficients and β is a $p \times r$ matrix describing long-run relationships among the variables, the $I(2)$ model embodies an additional reduced rank hypothesis, $\alpha'_{\perp}\Gamma\beta_{\perp} = \xi\eta'$, where ξ, η are $(p-r) \times s_1$ matrices and $\alpha_{\perp}\beta_{\perp}$ are the orthogonal complements of α, β.

The maximum likelihood parametrisation first suggested by Johansen (1997) shows the stationary relations in the $I(2)$ model,

$$\Delta^2 X_t = \alpha(\beta' X_{t-1} + \delta'\Delta X_{t-1}) + \zeta\tau'\Delta X_{t-1} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \qquad (14)$$

where $\delta = -((\alpha'\Omega^{-1}\alpha)^{-1}\alpha'\Omega^{-1}\Gamma)\tau_{\perp}(\tau'_{\perp}\tau_{\perp})^{-1}\tau'_{\perp}$ is a $p \times r$ matrix of coefficients determined such that $(\beta' X_{t\ 1} + \delta'\Delta X_{t\ 1}) \sim I(0)$; $\tau = [\beta, \beta_{\perp 1}]$ is a $(p+1) \times (r+s_1)$ matrix describing stationary relationships among the differenced variables with $\beta_{\perp 1}$ being the orthogonal complement of $[\beta, \beta_{\perp 2}]$ and $\tau_{\perp}$ the orthogonal complement of τ, and ζ is a $p \times (p - s_2)$ matrix of restricted medium-run adjustment coefficients. The number of polynomially cointegrating relations[14] is denoted by r, s_1 is the number of cointegration relations that become stationary by differencing, s_2 is the number of $I(2)$ trends, and $p = r + s_1 + s_2$. While the trend, t, and the broken trends are restricted to lie in $\beta' X_{t\ 1}$, the impulse dummy does not need such a restriction as it cumulates to zero, see the discussion in Section 2.1. The constant is restricted to be in $\delta'\Delta X_{t-1}$.

The determination of the reduced rank indices is based on the maximum likelihood trace test procedure in Nielsen and Rahbek (2007). As the standard p-values reported by CATS 3 correspond to a model without dummies, we bootstrap the rank test statistics. The resulting p-values are reported in Table 11. To choose the $I(1)$ and $I(2)$ rank, we start with the most restricted model $(r = 0, s_1 = 0, s_2 = 6)$ and then move on through the rows of the Table 11 until the first non-rejection at the 5% level, which is at $(r = 1, s_1 = 1, s_2 = 4)$, implying one stationary relation, one $I(1)$ trend and four $I(2)$ trends. When we look at the largest changes in the p-values as an indication which restrictions match well with the data, we would pick $(r = 2, s_1 = 1, s_2 = 3)$ which implies the same number of cointegrating relations that we find for the $I(1)$ model. With respect to our scenario analysis in equation (3), we would expect one nominal $I(2)$ trend, consistent with a choice of $(r = 2, s_1 = 3, s_2 = 1)$. By choosing $(r = 2, s_1 = 3, s_2 = 1)$, five of the $pk = 18$ roots of the VAR are restricted to unity.[15] This specification, however, would still contain a lot of persistence, as can be seen from the largest unrestricted root with a modulus of 0.92, which is reported at the bottom of Table 11. For the specification indicated by the bootstrapped test, $(r = 1, s_1 = 1, s_2 = 4)$ and nine unit roots, a largest unrestricted root of 0.81 would result.

If we assume that the nominal trends entering the GDP deflator and the house price index are driven by different sources, we could choose a specification with $(r = 2, s_1 = 2, s_2 = 2)$ and impose six unit roots. However, in this case the modulus of the largest unrestricted root is still quite large with 0.81. By contrast, our preferred choice $(r = 2, s_1 = 1, s_2 = 3)$ implies seven unit roots and a largest unrestricted root of 0.76. Hence, all large roots in the model are eliminated. We therefore proceed with a specification of $(r = 2, s_1 = 1, s_2 = 3)$, which implies two polynomially cointegrating relations $\beta' X_t + \delta'\Delta x_t$, one relation, $\beta'_{\perp 1}\Delta X_t$, that becomes stationary only by differencing, and three $I(2)$ trends.

[14] In the $I(2)$ model, $\beta' x_{t\ 1}$ cointegrates with $\delta'\Delta X_{t\ 1}$ to become stationary. For this reason, the r stationary relations are called polynomially cointegrating relations.

[15] In the $I(2)$ model, $s_1 + 2s_2$ of the roots of the VAR are restricted to unity.

Table 11: $I(2)$ trace tests

$p-r$	r	$s_2=6$	$s_2=5$	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$
6	0	546.0 [0.00]	423.3 [0.00]	350.9 [0.00]	299.9 [0.00]	270.0 [0.00]	248.2 [0.00]	238.8 [0.00]
5	1		347.4 [0.01]	278.4 [0.08]	226.1 [0.12]	193.6 [0.21]	173.8 [0.18]	165.7 [0.12]
4	2			211.6 [0.17]	**163.2** **[0.41]**	134.0 [0.42]	116.3 [0.42]	106.7 [0.36]
3	3				116.6 [0.56]	94.5 [0.51]	71.2 [0.60]	63.8 [0.43]
2	4					52.1 [0.87]	42.7 [0.73]	35.4 [0.62]
1	5						15.4 [0.95]	12.8 [0.71]
Modulus of largest unrestricted root								
$CI(2,1,3)$			0.76		$CI(2,3,1)$			0.92
$CI(2,2,2)$			0.81		$CI(1,1,4)$			0.81

P-values based on bootstrapped rank tests. Preferred rank choice in bold.

4.1 Testing the nominal-to-real transformation

In the $I(2)$ model we can test whether homogeneity in the price level is satisfied in the data. If this hypothesis is accepted, the analysis of the $I(2)$ model and the corresponding $I(1)$ model would yield identical results and the cointegration relations between the variables would be retained. Kongsted (2005) discusses different dimensions in which the nominal-to-real transformation can fail. If this is the case, e.g. because a nominal variable does not evolve proportionally to the chosen price index, some $I(2)$ components remain in the model, which will have an influence on inference in the transformed $I(1)$ model.[16]

We first test some integration and cointegration hypotheses in the $I(2)$ model by the likelihood ratio tests described in Johansen et al. (2010), i.e. whether one of the τ vectors corresponds to real money or real wealth. This corresponds to a test of whether the $I(2)$ trends cancel and the variable in the system can be regarded as $I(1)$. The results of these tests are shown in Table 12. It turns out that neither real money nor real wealth can be regarded as $I(1)$. In addition, we test whether (naïve) velocity $-(m-p-y)$ cointegrates to an $I(1)$ variable which is rejected as well. Also interest rates as well as the interest rate spread seem to contain some $I(2)$ component.[17] This means that to be considered as $I(1)$ in the context of this system, the variables would need to cointegrate with additional variables than those tested in Table 12.

While the tests of a known τ vector strongly reject that real money and real wealth are $I(1)$, homogeneity in the price level can also be investigated by testing whether all τ vectors can be restricted in a way that the transformation of the system as in equation (3) is not rejected (see Johansen et al. 2010). Testing whether money and wealth are homogeneous in the price level yields a test statistic of $\chi^2(6) = 23.35$ with a p-value of zero. Price homogeneity of only money (without any restriction on real wealth), cannot be rejected with a test statistic of $\chi^2(3) = 5.99$ and a p-value of 0.11.

[16] Kongsted and Nielsen (2004) conclude that the misspecification resulting from an invalid nominal-to-real transformation is often not severe.

[17] Note that applying a near unit root correction to these tests is likely to increase the p-values to some extent. Such a correction has not been applied here as the correct size is unknown. While this may lead to an acceptance of interest rates and the interest rate spread being $I(1)$, the test statistics for real money and real wealth are much larger and it is therefore less likely that our conclusions would be changed.

Table 12: Testing hypotheses of $I(1)$ versus $I(2)$

	m	p	y	whh	RS	RO	$t_{2009:1}$	$t_{2014:3}$	t	$\chi^2(6)$	p-value
$\mathcal{H}_1$	1	-1	0	0	0	0	0	0	0	44.55	0.00
$\mathcal{H}_2$	0	-1	0	1	0	0	0	0	0	39.62	0.00
$\mathcal{H}_3$	1	-1	-1	0	0	0	0	0	0	26.19	0.00
$\mathcal{H}_4$	0	0	0	0	1	0	0	0	0	14.27	0.03
$\mathcal{H}_5$	0	0	0	0	0	1	0	0	0	13.89	0.03
$\mathcal{H}_6$	0	0	0	0	1	-1	0	0	0	14.93	0.02

Null hypothesis: the respective relation is $I(1)$, β is unrestricted.

By contrast, price homogeneity of only wealth yields a test statistic of $\chi^2(3) = 20.31$ and a p-value of zero. Transforming the $I(2)$ system into an $I(1)$ system is thus possible for real money, but is rejected with regard to real wealth. The reason might be that the appropriate price indexes for these transformations differ, i.e. whereas the GDP deflator can be used to deflate the monetary aggregate M3, a house price index would be better suited to deflate nominal housing wealth. Put differently, our results would suggest that house price developments and the GDP deflator were driven by different stochastic shocks.

Though one would not expect a double unit root in relations like real money or real wealth, there are a number of reasons why we might not be able to empirically reject this hypothesis. The fact that the empirically preferred specification indicates three $I(2)$ trends suggests that the variables in the system are driven by stochastic shocks coming from three different sources. First, there might be measurement issues with regard to the appropriate index for the price level. Second, though we include several dummies to account for the shifts that have taken place in the wake of the financial crisis, there might be still some $I(2)$ component left that we have not appropriately modelled. Third, we have already mentioned convergence of euro area member states and data aggregation issues that might as well lead to some stochastic trends that are difficult to capture.[18] We will look into this question further when we discuss the identified system.

4.2 An identified long-run structure for the $I(2)$ model

We next impose overidentifying restrictions on the two β vectors in order to interpret them in a structural way. The results are shown in Table 13. These restrictions are guided by our results for the $I(1)$ model. They are not rejected with a test statistic of $\chi^2(6) = 8.99$ and a bootstrapped p-value of 0.57.

We interpret the first β vector as a money demand relation where velocity reacts to wealth and the two interest rates. Neither the trend nor the two broken trends enter the cointegrating relation, indicating that this equilibrium relationship remained stable over the sample period, once changes in the dynamic coefficients are taken into account by including appropriate deterministic variables as specified in Section 2.2. The second β vector is more difficult to interpret. As no clear pattern in the coefficients is visible, we impose only three restrictions, namely that money and income do not enter this vector, plus a restriction on the two broken trends, i.e. the coefficients are restricted to be equal but with opposite signs. This restriction is motivated by the idea that the trend in the

[18] Beyer and Juselius (2010) indeed find that different data aggregation methods give rise to a small but highly persistent difference in the data series. The euro area money demand model estimated by Coenen and Vega (2001), however, is reasonably robust to the choice of the aggregation method.

Table 13: β', δ' and α' (normalized)

	m	p	y	whh	RS	RO	$t_{2009:1}$	$t_{2014:3}$	t
β_1	1.00 [–]	−1.00 [–]	−1.00 [–]	**−0.17** [0.03]	**−10.9** [1.21]	**25.7** [2.66]	0.00 [–]	0.00 [–]	0.00 [–]
δ_1	**−7.49** [1.38]	**−18.3** [2.76]	**−4.13** [0.46]	**9.05** [0.82]	−0.09 [0.62]	**−0.56** [0.15]	**−0.17** [0.02]	0.01 [0.03]	**7.58** [0.43]
α_1	**0.02** [0.01]	**−0.01** [0.01]	**0.03** [0.01]	−0.01 [0.00]	**0.00** [0.00]	0.00 [0.00]			
β_2	0.00 [–]	**7.06** [1.07]	0.00 [–]	1.00 [–]	**72.3** [9.24]	**−227** [21.3]	0.06 [0.01]	**−0.06** [0.01]	**−0.09** [0.01]
δ_2	0.43 [11.2]	**249** [23.2]	**41.7** [4.16]	**−53.4** [6.59]	**−32** [4.71]	**−2.67** [1.16]	0.03 [1.38]	0.08 [1.38]	**−40.5** [1.38]
α_2	**0.00** [0.00]	**−0.00** [0.00]	0.00 [0.00]	**0.00** [0.00]	**0.03** [0.01]	−0.00 [0.00]			

Standard errors in brackets. Test of overidentifying restrictions on β in the $I(2)$ model: $\chi^2(6) = 8.99$, p-value: 0.57. Significant coefficients are shown in bold.

equilibrium relation shifted in the wake of the financial crisis but moved back following the ECB's nonstandard monetary policy measures. We can interpret this relation as a wealth relation, though price homogeneity for wealth is rejected, probably because the GDP deflator and house price developments deviated significantly in the run-up and the aftermath of the financial crisis. Though the nominal-to-real transformation was rejected, we can impose price homogeneity on one of the β vectors, but not on the other.

4.3 Dynamic adjustment in the $I(2)$ model

When data are $I(2)$, $\beta' X_t$ is generally $I(1)$ and can be interpreted as an equilibrium error that exhibits pronounced persistence. The polynomially cointegrated relations incorporate a medium-term dynamic adjustment to $\beta' X_t$ through the term $\delta' \Delta X_t$ and thus can be seen as dynamic equilibrium relations. Long and persistent swings away from the long-run equilibrium values suggest the presence of some self-reinforcing feedback mechanisms in the system. We can distinguish between positive feedback, which increases the deviations from the equilibrium relation, and negative feedback, which drives the system back to the equilibrium relation. Such positive and negative feedback can take place either in the adjustment to the two polynomially cointegrating relations, $\alpha(\beta' X_t + \delta' \Delta X_t)$, or in the adjustment to the changes in the β relations, $\zeta \beta' \Delta X_t$.

The coefficients α and δ characterise two levels of equilibrium correction, as can be seen from equation (14): The α adjustment describes how the acceleration rates, $\Delta^2 X_t$, adjust to the dynamic equilibrium relations, $\beta' X_t + \delta' \Delta X_t$ and the δ adjustment describes how the growth rates, ΔX_t, adjust to the long-run equilibrium errors, $\beta' X_t$.[19]

The signs of α, β and δ determine whether the variable $X_{i,t}$ is error increasing or error correcting in the medium and the long run. If $\alpha_{ij}\beta_{mj} < 0$ the acceleration rate, $\Delta X_{i,t}^2$, adjusts to the dynamic equilibrium relation ($\beta_j' X_t + \delta_j' \Delta X_t$). This is the case for income and the policy rate in the money relation, and for prices in the wealth relation. An adjustment to ($\beta_j' X_t + \delta_j' \Delta X_t$) can also take place through $\alpha_{ij}\delta_{mj} < 0$. Looking at Table 13, we see that money exhibits this kind of adjustment in the first cointegrating relations whereas for the second relation, prices, wealth and the short-term interest rate adjust. Thus, in the send relation, the adjustment through this channel seems to be more important than via $\alpha_{ij}\beta_{mj}$.

[19] The interpretation of δ as a medium-run adjustment is conditional on $\alpha \neq 0$.

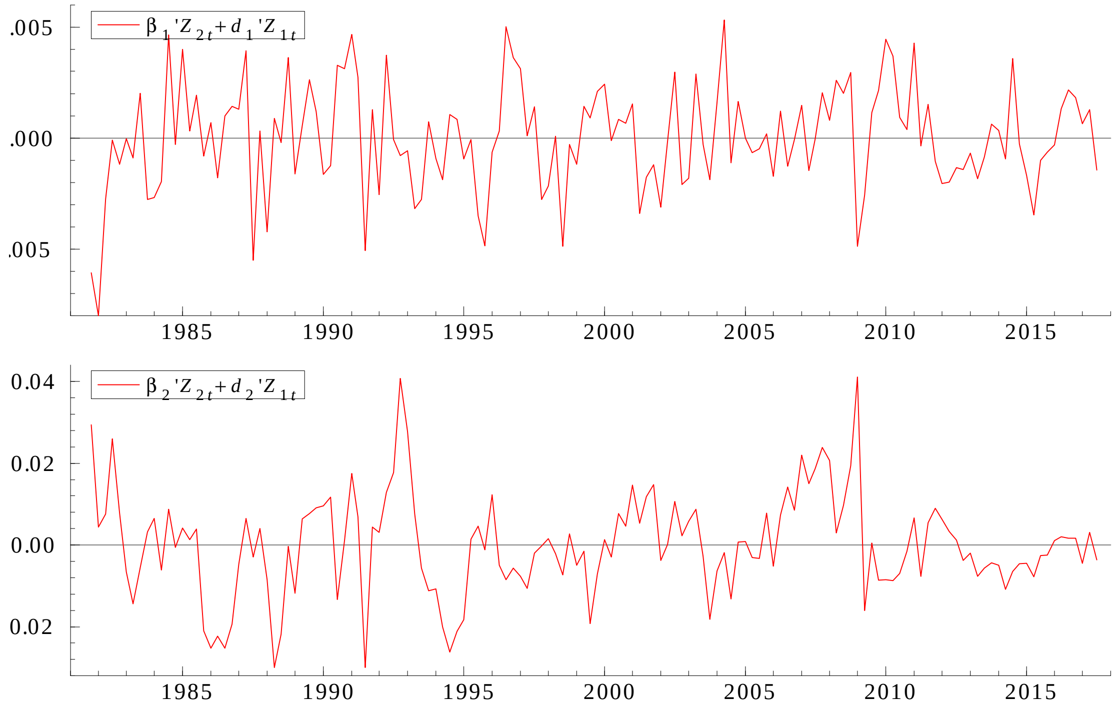


Figure 7: Multi-cointegrating relations

If $\delta_{mj}\beta_{mj} > 0$ (given $\alpha_{ij} \neq 0$) then $\Delta X_{i,t}$, is equilibrium error correcting to $\beta_j^{'}X_t$. An adjustment to $\beta_j^{'}X_t$ through this channel takes place for prices and income in the money relation and for prices in the wealth relation. In general, equilibrium error-increasing behaviour by one variable, $X_{i,t}$, can be compensated by error-correcting behaviour by another variable, $X_{k,t}$, to the same equilibrium error $\beta_j^{'}X_t + \delta_j^{\prime}\Delta X_t$. As long as all characteristic roots are either inside or on the unit circle[20] the system is stable and any error-increasing behaviour is sooner or later compensated by error correcting behaviour. For example, a variable can be equilibrium error increasing in the medium run ($\delta_{mj}\beta_{mj} < 0$) but error correcting in the long-run ($\alpha_{ij}\beta_{mj} < 0$). While a variable can move away from its long-run equilibrium for extended periods of time, as long as the system is stable it will sooner or later be subject to its equilibrating forces.

Overall, a visual inspection of the two overidentified multi-cointegrating relations shown in Figure 7 indicate that they are stable throughout our sample period and show little persistence.

5 Conclusions

In this paper, we analysed the long-run relations between money, prices, interest rates, income and wealth in an $I(1)$ and $I(2)$ framework.

In the $I(1)$ analysis, we found that the money demand relation estimated by Beyer (2009) remaind remarkably stable when the sample period is extended to 2017:3, once we allow for two breaks in the trend that account for the outbreak of the Global Financial Crisis and the introduction of the ECB's non-standard monetary policy measures that

[20] Roots inside the unit disk imply non-explosive behaviour as they are calculated as eigenvalues of the characteristic polynomials (Juselius, 2006).

helped to restore the working of the monetary transmission mechanism in the euro area (see European Central Bank 2015, 2017). We find two cointegrating relations that we interpret as a real money relation and a real wealth relation.

We then move on to expand the system into an $I(2)$ model in order to analyse in more detail the relation between nominal and real variables as well as the dynamic adjustment in the system. While for real money the nominal-to-real transformation is not rejected, the wealth relation cannot be expressed in real terms. We identify one of the cointegrating relations as money velocity, which is related to wealth and interest rates, but also needs the differenced processes to become stationary. Regarding the dynamic adjustment of the system, we find that much of the adjustment is coming from the growth rates of the variables that enter the polynomially contegrating relations.

Comparing the two complementary modelling approaches, our $I(1)$ analysis appears robust and provides empirical results that allow for meaningful economic interpretations. The $I(2)$ analysis provides evidence for an alternative economic interpretation of the underlying shocks in the model. It helps to understand in particular data-related issues and invites a more detailed analysis of aggregation bias and a further investigating of the most suitable price deflator for housing wealth.

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