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Climate change mitigation: how effective is green quantitative easing?

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Abstract

We develop a two-sector incomplete markets integrated assessment model to analyze the effectiveness of green quantitative easing (QE) in complementing fiscal policies for climate change mitigation. We model green QE through an outstanding stock of private assets held by a monetary authority and its portfolio allocation between a clean and a dirty sector of production. Green QE leads to a partial crowding out of private capital in the green sector and to a modest reduction of the global temperature by 0.04 degrees of Celsius until 2100. A moderate global carbon tax of 50 USD per tonne of carbon is 4 times more effective.

Keywords: Climate Change; Integrated Assessment Model; 2-Sector Model; Green Quantitative Easing; Carbon Taxation

J.E.L. Classification Codes: E51; E62; Q54

Non-technical Summary

This paper examines whether central banks - a *monetary authority* - can effectively contribute to mitigate global warming through green quantitative easing, i.e. through a shift of a monetary authorities' privately-issued financial asset holdings towards the green sector of the economy. As a secondary question we further investigate the effectiveness of this policy in combination with fiscal policies - set by a *fiscal authority* -, more precisely, a carbon tax.

In our setup, green quantitative easing refers to a change in the portfolio allocation of a given outstanding stock of private sector securities (bonds) held by the monetary authority, towards bonds issued by the green sector.

To answer the question on the effectiveness of green quantitative easing, we develop a quantitative integrated assessment model with green and dirty capital.[1] In our global model, aggregate output is produced employing intermediate goods that are in turn produced in the dirty sector and in the clean (green) sector. Intermediate goods are produced using capital, labour and energy as inputs, with the dirty sector using carbon-based energy and the green sector using clean (renewable) energy. Markets do not take future climate damages into account and therefore rely too much on the dirty sector for the production of intermediate goods in the absence of policy interventions. Over time, this negative production externality leads to a reduction of total output as the global temperature increases.

Capital and labour are supplied to the intermediate firms by households, which allocate their savings between bonds issued by the two intermediate sectors (clean and dirty). The return on the capital used in the firms is stochastic and imperfectly correlated across sectors, which presents an income risk for the households seeking to optimise their consumption over time. This feature of the model is of central importance as the imperfect correlation of returns calibrated according to the data realistically implies that, in response to the portfolio allocation decision by the monetary authority, private households will not perfectly reallocate their portfolios towards dirty assets. Therefore, in our model, the portfolio reallocation decision by the monetary authority is not neutralized by private household reactions.

In this model setting, without policy intervention, the global temperature increases by 3.5 degree of Celsius above pre-industrial levels by 2100. This is in line with IPCC scenarios of climate change and well above the Paris agreement target to mitigate global warming.

Next we simulate three policy experiments. First, we model the effect of carbon pricing by a fiscal authority, which increases the price of dirty energy through a carbon tax. Second, we consider the contribution of green quantitative easing, where the monetary authority changes the

[1] An integrated assessment model unites a macroeconomic perspective with the possible damages of climate change, which are modelled as future output losses due to an increase in temperature that is caused by the build-up of carbon emissions in the atmosphere.

composition of its private asset portfolio to only green bonds. Finally, we consider both policies in combination. Since our model is calibrated to the world, as global warming knows no borders, these simulated fiscal and monetary policies would require cooperation across countries that we abstract from.

The initial carbon tax is 50 USD per tonne of carbon in 2021 (equivalent to 13.6 USD per tonne of CO2).[2] We hold the implied tax rate of 6.6% constant along the transition and find that the global temperature increase could be reduced by 0.17 degrees of Celsius, compared to the baseline. This carbon tax is at the low end of many policy proposals to reduce carbon emissions, and chosen to facilitate the comparison with green quantitative easing. Green quantitative easing is modeled as a stylized scenario where the monetary authority's private capital portfolio, which is initially split across both sectors in proportion to total capital in the economy, is reallocated to clean capital only. This additional supply of capital to the clean intermediate sector reduces its return and, since clean and dirty capital returns are imperfectly correlated, households will find it optimal to partially reallocate their savings to dirty capital. As a net effect, the capital stock employed for production in the clean sector will increase relative to the capital stock in the dirty sector, which triggers a relative increase of labour demand in the clean sector and a relative expansion of its output. The monetary authority can thus influence the relative production across the two sectors in the economy through the allocation of its asset portfolio.

Our green quantitative easing simulation is set up to investigate its maximum possible effect. We assume a complete and immediate switch to green bonds and no uncertainty about the classification of green and dirty bonds. The share of private assets held by the monetary authority (10 percent of GDP) is calibrated based on the privately-issued securities holdings of central banks of advanced economies, i.e. including asset-backed securities as well as commercial bonds. We also assume in our baseline scenario a very high elasticity of substitution between clean and dirty intermediate goods. Despite this calibration tailored to achieve the maximum possible effect, the impact of green quantitative easing is rather modest compared to the carbon tax. We find that the emission reduction through the carbon tax is about 4-times larger than the maximum reduction that could be achieved through this green quantitative easing policy. Put differently, achieving the same effect as the maximum reduction through green quantitative easing would require a carbon tax of about 11 USD per tonne of carbon (equivalent to about 3 USD per tonne of CO2).

When combining both policies, we find that green quantitative easing complements fiscal policy, i.e. green quantitative easing on top of a carbon tax will reduce the increase of global temperature further. However, the whole is less than the sum of its parts: the marginal effect

[2] Since there is no aggregate uncertainty in our model, an equivalent fiscal policy would be to set a carbon price through an emission trading scheme.

of the two policies in combination is lower than in isolation. Within the dirty sector, a carbon tax increases the production costs of energy relative to the costs of capital and labour, which triggers a decline in dirty energy demand. In turn, green quantitative easing increases the costs of dirty capital, which partially leads to higher dirty energy demand. Thus, both policies partially counteract each other.

We conclude that green quantitative easing may be an effective complementary policy instrument, in particular if governments around the world fail to coordinate on introducing a sizeable carbon tax or equivalent carbon pricing through other fiscal policies.

1 Introduction

Climate change evoked by mankind will be one of the greatest global challenges in the next decades. Since pre-industrial times, global temperature has increased by approximately 1.1 degrees of Celsius as a result of carbon and other greenhouse gas emissions IPCC (2021). If this trend were to continue, extreme weather events would not only become more frequent—causing large macroeconomic costs—but the world would also observe irreversible global environmental damages. To effectively reverse this trend, ambitious policy measures need to be adopted as the window of opportunity to act is closing rapidly. While there is broad consensus on the effectiveness and usefulness of carbon pricing as a policy tool to combat climate change, recently a vivid debate emerged on whether and how central banks should play a role in addressing climate change.

This paper examines whether central banks—which we throughout the paper refer to as a monetary authority—can contribute to mitigate climate change through green quantitative easing (QE). Our key research question is whether a portfolio shift of the monetary authority towards the green sector of the economy can effectively reduce climate change and how this compares with fiscal mitigation policies, such as a global carbon tax. In our setup, green QE refers to portfolio reallocation of a given outstanding stock of bonds held by the monetary authority, which is initially market neutral, i.e. proportionally to private holdings split across the two sectors, towards bonds issued by the green sector of production.

To address our research question, we develop a two-sector quantitative integrated assessment model where aggregate output is produced employing clean and dirty intermediate goods. Markets are incomplete for two reasons. First, households face risky asset returns in the two intermediate goods sectors and can self insure against this risk by saving in risk-free bonds. Second, there exists a climate change externality leading to a damage to aggregate output, as frequently employed in the climate change literature. The model is calibrated to the world economy with one monetary and one fiscal authority, which comes along with the implicit assumption that both these authorities coordinate on the introduction of a global carbon tax and green QE.

Intermediate goods in the economy are produced using capital and labour, and either clean or dirty energy as inputs. Energy production itself takes place using a simple technology employing some exogenously growing technology level and labour as the only input. Dirty energy production leads to an accumulation of carbon in the atmosphere, which causes an increase of the global temperature leading to a damage to aggregate output.

Households live until infinity and maximize their expected discounted life-time utility over consumption streams. Every household runs two intermediate goods firms in the two sectors by employing its own household capital and by hiring labour and energy on the respective labour and energy market. Since the return processes on capital in the two firms is stochastic, households are heterogeneous, with their heterogeneity resulting from different (histories of) return realizations. This return risk is idiosyncratic, thus there is no aggregate risk in the economy. The shocks on the returns of the two capital stocks are imperfectly correlated across sectors. This is an important feature of the model as it implies that in response to a reallocation of capital by the monetary authority, private households will only partially offset its effect by reallocating their savings and thus their will be only a partial crowding out.

Households not only hire labour on the market for production, but also exogenously supply their own labour on the market and from this labour supply they earn a deterministic wage income. Given these income processes, households solve a consumption savings problem and choose to allocate their savings between the two capital stocks as well as a risk-free bond that is assumed to be in zero net supply across households.

We calibrate the model to standard targets in the climate change literature and to a zero correlation of asset returns across the two sectors.[3] We next simulate the transition of the economy over the next decades—from 2020 to 2100—and compute the resulting temperature increase. As a baseline scenario, we assume a carbon tax of zero and a constant ratio of assets held by the central bank of 4 percent of the value of the economy's capital stock which is split proportionally to private holdings across the two intermediate goods sectors. This implies that the balance sheet of the central bank grows with the economy. While we do not model the rationale for such a long-run QE policy, our assumption can be interpreted as approximating a real world economy in which QE policies take place with a certain regularity. Our ad-hoc approach is based on the insight that demographic and climate change processes will likely lead to a persistently low interest rate environment—which our simulations also show—and it is therefore reasonable that such unconventional monetary policies will be implemented again in future recessions. In this baseline scenario, the global temperature increases until 2100 to about 3.5 degrees of Celsius above pre-industrial levels. This is in line with the IPCC scenarios of climate change and well above the Paris agreement target of 1.5 degrees of Celsius.

Next, we consider three policy experiments. First, we model the effects of carbon pricing by a fiscal authority, which increases the price of dirty energy.[4] We introduce the carbon tax in the year 2020 at an initially low level of 50 USD per ton of carbon emissions, which corresponds to a tax of 13.6 USD per ton of carbon dioxide (CO_2) and to an ad valorem carbon tax of 6.6 percent. We hold this tax rate constant along the transition so that the absolute tax level increases to 70 USD

[3]The assumption of the zero calibration in our baseline calibration is based on the empirical findings in Broadstock and Cheng (2019).

[4]Since there is no aggregate uncertainty, setting the carbon price through an emission trading scheme would be equivalent to a carbon tax.

per ton of carbon in 2100. With this tax rate in place the global temperature increase would be reduced by 0.17 degrees of Celsius, compared to the baseline.

Second, we consider a stylised green QE policy set up to investigate its maximum possible effect. The share of private assets held by the monetary authority (10 percent of GDP) is calibrated based on the privately-issued securities holdings of central banks of advanced economies, i.e. including asset-backed securities as well as commercial bonds. We assume a complete and immediate switch to green bonds and no uncertainty about the classification of green and dirty bonds. We also calibrate the model with a very high elasticity of substitution between clean and dirty intermediate goods. The reallocation of the monetary authority's portfolio towards the clean sector increases the capital stock employed for production in that sector relative to the capital stock in the dirty sector. This triggers a relative increase of labour demand in the clean sector and a relative expansion of output. The monetary authority can thus influence the relative production across the two sectors in the economy. Despite our calibration tailored to achieve the maximum possible effect of green QE, its impact is rather modest compared to the global carbon tax. We find that the global temperature reduction achieved through the global carbon tax of initially 50 USD per ton of carbon is 4.3 times larger than what would be achieved through green QE. Put differently, to achieve the same reduction of the global temperature as through green QE, this would require a carbon tax of about 11 USD per ton of carbon (or equivalent to about 3 USD per ton of CO_2).

Third, we consider the two policies, a global carbon tax and green QE, in combination and examine whether they are substitutes or complements. We find that green QE complements fiscal policy, i.e., green QE on top of a carbon tax will help to reduce the increase of global temperature further. However, the whole is less than the sum of its parts: the effect of the two policies in combination is lower than in isolation. The reason is that the shifts of input factors in the intermediate goods sector induced by a carbon tax, away from dirty energy, are partly diminished by the impact of the increase in dirty capital costs through green QE, raising the demand for dirty energy.

We consider a number of alternative calibrations of the model for sensitivity analyses. Two assumptions turn out to be crucial. First, assuming that the level instead of the share of private assets held by the monetary authority will be kept constant along the transition implies that the share of assets held relative to the global capital stock will converge to zero. In this case, green QE is about 15 times less effective than the assumed initial carbon tax of 50 USD per ton of carbon. Second, a crucial parameter turns out to be the elasticity of the ratio of energy inputs with respect to the energy price ratio (short, energy elasticity), which in our baseline we calibrate to a value of 2 implying a final output substitution elasticity of intermediate goods (short, intermediate goods elasticity) of 26. In our sensitivity analysis we calibrate the model to an energy elasticity of 1 implying a much lower intermediate goods elasticity of 2.25. With this calibration, the effectiveness of the global carbon tax is of similar magnitudes but the effectiveness of green QE is strongly reduced so that the global carbon tax is about 31 times more effective. An in-between value of the intermediate goods elasticity of 4 (consistent with an energy elasticity of 1.12) in turn leads to a relative effectiveness of the global carbon tax by a factor of 14.

In conclusion, we find that a global carbon tax is considerably more effective in reducing the increase of the global temperature than green QE. However, green QE can usefully complement a carbon tax, in particular if governments only insufficiently coordinate on implementing green fiscal policies.

Relation to Existing Literature

Our infinitely lived agents integrated assessment model follows the tradition since William Nordhaus (cf. Nordhaus and Boyer (2000) for a detailed description) and borrows elements from Golosov, Hassler, Krusell, and Tsyvinski (2014) and van der Ploeg and Rezai (2021), in particular with respect to the calibration of the climate module. We add two central features to this existing literature. First, we extend this work by exogenously modeling green QE through the monetary authority. Second, output in the two sectors of the economy is plausibly stochastic and the returns to capital are imperfectly correlated.

The portfolio choice of private households is a crucial mechanism so that the green QE policy by the monetary authority is not perfectly neutralized on private markets. In this regard, our paper connects to the literature on the effectiveness of quantitative easing, in particular the so-called portfolio re-balancing channel of QE. Central bank asset purchases will not influence their price if in response private investors completely offset the impact by re-balancing their portfolios (Wallace 1981). Portfolio re-balancing can affect security prices when private investors are not indifferent with respect to the composition of their portfolios, for example when they have a preference for certain maturities (Vayanos and Vila 2021). Different other channels for the effectiveness of QE that are suggested in the literature, among which signalling the central banks' intentions or its impact on the balance sheet constraints of financial intermediaries (see, e.g., Krishnamurthy and Vissing-Jorgensen (2011) and Gertler and Karadi (2011)), are not considered in this paper.

We maintain the long-run focus of prototypical integrated assessment models and thus analyze a stylized long-run green QE policy. This perspective is shared in recent work by Ferrari and Landi (2022) who, as we, develop a two sector integrated assessment model with a monetary authority to study the effectiveness of green QE in the long-run. Similar to our main finding they report that the effect of green QE on the stock of pollution is relatively small. The main differences to our approach are their focus on the EU whereas our model is calibrated to the world economy and the motive for holding green assets by the private sector, which they model through a preference for green investments. Our model with idiosyncratic return risk in the two sectors features a standard endogenous portfolio choice instead of an explicit preference based explanation. Through this structure we do not directly calibrate a hard-wired motive for holding green bonds but rather achieve it through a calibration of the risk return structure of the asset classes, which we base on respective data moments.

Our long-run focus distinguishes us from work on green QE for the short-run. E.g., Ferrari and Landi (2020) and Benmir and Roman (2020) study climate policies along the business cycle by combining a climate model with a New Keynesian DSGE model with the financial accelerator framework of Gertler and Karadi (2011). As we do, they understand green QE as a tilting of the portfolio held by the central bank towards the green sector. Ferrari and Landi (2020) avoid a perfect crowding out by introducing costly portfolio rebalancing for private agents. They find that an aggressive expansion of green QE (i.e., selling dirty and buying clean assets) during expansions is welfare improving.[5] Related, Diluiso, Annicchiarico, Kalkuhl, and Minx (2021) study the interactions between climate change and monetary policies arguing that inflationary pressures caused by climate change policies may demand very strong monetary policy reactions.

Furthermore, our approach of modeling green QE is comparable to policies aiming at a preferential treatment of green corporate bonds in central banks' collateral frameworks. By increasing the number of corporate bonds associated with fewer green house gas emissions, central banks could steer the demand towards greener corporate bonds. Pelizzon, Riedel, Simon, and Subrahmanyam (2020) find that pledgeability as collateral affects the financing conditions and investment decisions of firms. Analyzing elegibility events in the Eurosystem Collateral Framework they report that upon receiving the eligibility status of their corporate bonds, firms increase leverage and expand their balance sheet. Giovanardi, Kaldorf, Radke, and Wicknig (2021) study different degrees of preferential treatment of green corporate bonds within a DSGE setup, thus focusing on business cycle frequencies. They find a very limited climate change mitigating effect of such preferential treatments, which also come at the cost of an increase in entrepreneurial risk-taking. The optimal green collateral policy is thus characterized by a very modest preferential treatment, with very low beneficial effects for the climate. Both, green QE and green collateral policies function through an increase in the demand for green corporate bonds. A major difference between the two instruments is that with green QE a central bank directly decides about the quantity and composition of green bond purchases, whereas in the case of a green collateral policy, the central bank incentivizes private banks to make these choices. These monetary policy instruments differ from a new type of facility, recently introduced by the central banks of Japan and China, by which low interest funds are provided to financial institutions to finance firms' green investment

[5] Related, Benmir, Jaccard, and Vermandel (2020) find that optimal carbon taxes should be pro-cyclical.

projects and their efforts to cut carbon emissions.[6] In our model setting such a facility would be comparable to providing subsidies to capital in the clean intermediate sector in order to support green investment.

By modeling idiosyncratic return risk our work also relates to the standard incomplete markets literature in quantitative macroeconomics pioneered in so-called Aiyagari-Bewley-Huggett-Imrohoglu models (Bewley 1986; Huggett 1993; Aiyagari 1994; Imrohoroglu 1989). More specifically, our model adopts the setup of Angeletos (2007) to a two sector economy with a climate module. Specifically, the (idiosyncratic) return risk in combination with a risk-free labor income gives rise to closed form solutions of the household decision functions[7], which is a convenient property of the model as it allows us to compute the solution over very long horizons in our rather complex model in limited time.

Finally, we relate to the literature on asset pricing and climate change, e.g., by Hambel, Kraft, and van der Ploeg (2020) who emphasize a trade-off between asset diversification and climate change mitigation. They further show that green assets feature higher risk premia than brown assets. The recent empirical literature indeed partially finds lower risk premia for green assets. Bolton and Kacperczyk (2021) and Bolton and Kacperczyk (2020) analyze the US, respectively the worldwide, stock markets and find a positive carbon premium that has been rising over the recent years. Kapraun, Latino, Scheins, and Schlag (2021) investigate a large dataset of government and corporate bonds. In the primary market, they find that green bonds have lower yields than non-green bonds. However, in the secondary market this reverses and they find green bonds featuring higher yields. Degryse, Goncharenko, Theunisz, and Vadazs (2020) investigate an international sample of syndicated loans and find that green firms borrow at significantly lower spreads. For sake of parsimony, we sidestep these aspects and are agnostic about any mechanisms that may lead to differential asset returns by calibrating our model to equal mean returns and equal return variances in both sectors. However, a crucial parameter in our model is the correlation of asset returns, which in our baseline scenario we calibrate according to the evidence in Broadstock and Cheng (2019) suggesting a zero correlation.

The remainder of the paper is organized as follows. Section 2 presents the model and Section 3 discusses the calibration. Section 4 presents our results including our extensive sensitivity analyses and Section 5 concludes the paper. Detailed derivations are contained in the appendix.

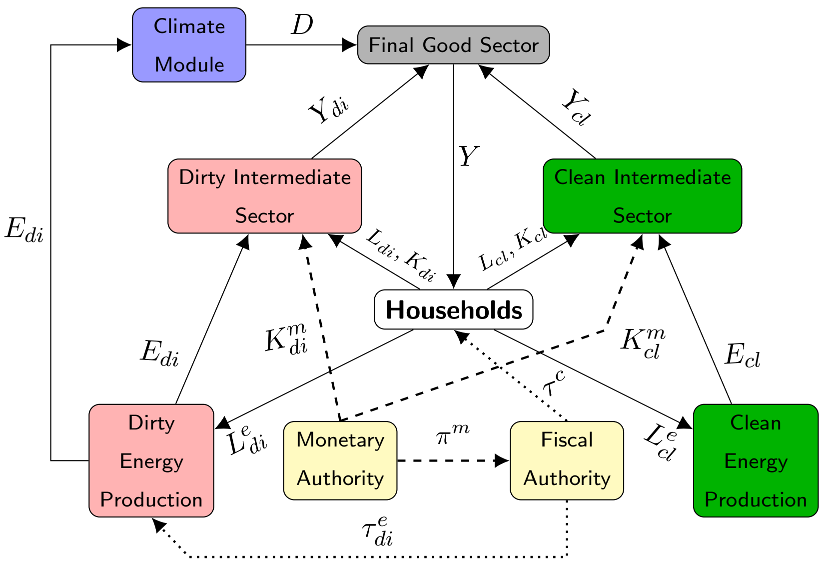
[6]See https://greencentralbanking.com/2021/11/10/pboc-launches-targeted-green-lending/ and https://www.boj.or.jp/en/announcements/release_2021/rel210716b.pdf, respectively.

[7]An insight relating back to Merton (1969) and Samuelson (1969).

2 A Two-Sector Integrated Assessment Model with Risky Returns

We develop a two sector world economy integrated assessment model with a monetary and a fiscal authority. Figure 1 provides an overview of the various sectors and entities in the economy, and Table 1 collects the main indices used throughout. The final consumption good is produced by a dirty and a clean intermediate goods sector, which itself uses capital, labour and energy as input. Labour is supplied by households and capital is supplied by households and a monetary authority. Energy is supplied by a dirty and a clean energy production sector, using labour supplied by households as input. We take the total capital stock of the monetary authority supplied to firms as given and thus the monetary authority solely faces a portfolio choice allocation problem and can thereby influence the production of clean and dirty intermediate inputs. Profits generated by the monetary authority flow to the fiscal authority which additionally raises revenue from dirty energy production by energy (carbon) taxes. Dirty energy production leads via its emissions to an accumulation of a carbon stock in the atmosphere which creates a temperature increase and with it causes a damage through a reduction of aggregate output. We now describe the main elements of the model in more detail.

Figure 1: Overview of the 2 Sector Integrated Assessment Model



2.1 Time, Risk and Population Structure

Time in the model is discrete and runs from $t = 0, \ldots, \infty$. At time $t = 0$ a continuous distribution of infinitely lived representative agents are born with total initial size $N_0 = 1$, which grows exogenously at time varying rate n_t. Each period a (heterogeneous) household i earns a deterministic labour income, stochastic returns on physical capital holdings from owning firms and risk-free returns from owning bonds.

Table 1: Indices

Index	Value	Interpretation
t	$t \in \{0, 1, \ldots, \infty\}$	Time
i	$i \in \{1, 2, \ldots, \infty\}$	Type
s	$s \in \{cl, di\}$	Sector (clean, dirty)
c	$c \in \{ra, sl\}$	Carbon Stocks (rapidly, slowly depreciating stock)

Notes: List of indices used in the integrated assessment model.

2.2 Production

2.2.1 Final Good Production

The final output good Y_t is composed of two intermediate goods produced in a clean and a dirty sector Y_{ts}, $s \in \{cl, di\}$, and augmented according to a CES aggregator with substitution elasticity ϵ, henceforth referred to as intermediate goods elasticity. At this outer layer of the production side we further assume an exogenous technology level Υ_t, which grows at the exogenous rate g. Additionally, there is a negative aggregate production externality D_t from air pollution which proportionally reduces aggregate output and thus

$$Y_t = (1 - D_t) \cdot \Upsilon_t \cdot \left(\sum_{s \in \{cl,di\}} \kappa_s Y_{ts}^{1-\frac{1}{\epsilon}} \right)^{\frac{1}{1-\frac{1}{\epsilon}}}, \tag{1}$$

where κ_s are the sectoral output shares with $\sum_{s \in \{cl,di\}} \kappa_s = 1$. The representative firm takes as given the final goods price p_t and the intermediate goods input prices p_{ts} and maximizes profits under perfect competition giving the intermediate goods demand

$$Y_{ts} = \left(\frac{\kappa_s}{p_{ts}/p_t} \right)^{\epsilon} ((1 - D_t) \cdot \Upsilon_t)^{\epsilon - 1} Y_t, \text{ for } s \in \{cl, di\}, \tag{2}$$

and the price index for the final good as

$$p_t = \frac{1}{(1 - D_t)\Upsilon_t} \left(\sum_{s \in \{cl,di\}} \kappa_s^{\epsilon} p_{ts}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}},$$

cf. Appendix A.1. We choose p_t as numeraire in our setup and set $p_t = 1$ for all t.

2.2.2 Intermediate Goods Production

Every household runs the two intermediate goods firms s by employing its own household capital k_{tis} and hiring labour ℓ_{tis} and energy e_{tis} on the respective labour and energy markets. Production of intermediate goods takes place according to a two-nests Cobb-Douglas technology with inner nest capital share parameter α and outer nest non-energy share parameter γ (so that the energy share is $1-\gamma$). The value of the capital employed in production is subject to an idiosyncratic sector specific shock ζ_{tis} so that gross output is

$$y_{tis} = \psi_s \left[(k_{tis})^{\alpha}(\ell_{tis})^{1-\alpha}\right]^{\gamma} \cdot e_{tis}^{1-\gamma} + \zeta_{tis} k_{tis}, \tag{3}$$

where ψ_s is a technology level parameter.

Let $\boldsymbol{\zeta}_{ti} = (\zeta_{ticl}, \zeta_{tidi})'$ be a vector containing the shocks in both sectors. We assume that $\boldsymbol{\zeta}_{ti}$ is i.i.d. with CDF $\boldsymbol{\Psi}\left((\mu^{\zeta}_{cl}, \mu^{\zeta}_{di})', \Sigma, \dots\right)$, where

$$\boldsymbol{\Sigma} = \begin{bmatrix} \left(\sigma^{\zeta}_{cl}\right)^2 & \rho^{\zeta}_{cl,di}\sigma^{\zeta}_{cl}\sigma^{\zeta}_{di} \\ \rho^{\zeta}_{cl,di}\sigma^{\zeta}_{cl}\sigma^{\zeta}_{di} & \left(\sigma^{\zeta}_{di}\right)^2 \end{bmatrix}.$$

$\boldsymbol{\Sigma}$ pins down the variances of the shocks and explicitly allows for them to be correlated across sectors when $\rho^{\zeta}_{cl,di} \neq 0$. The details on the shock distribution are described in Appendix A.2. Households take as given the intermediate goods prices p_{ts}, wages, respectively the return on labour, r^l_t, energy prices p^e_{ts} and an exogenous depreciation rate on capital δ_s so that profits are

$$\pi_{ts} = p_{ts} \cdot y_{tis} - r^l_t \ell_{tis} - p^e_{ts} e_{tis} - \delta_s k_{tis}. \tag{4}$$

Assuming free entry and exit, profit maximization yields the demand for energy and labour as

$$e_{tis} = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1-\gamma}{(1-\alpha)\gamma} \cdot \left(\frac{r^l_t}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p^e_{ts}}{p_{ts}}\right)^{-\frac{1-\gamma(1-\alpha)}{\alpha\gamma}} \cdot k_{tis} \tag{5a}$$

$$\ell_{tis} = \Gamma(\psi_s, \alpha, \gamma) \cdot \left(\frac{r^l_t}{p_{ts}}\right)^{-\frac{1}{\alpha}} \cdot \left(\frac{p^e_{ts}}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot k_{tis}, \tag{5b}$$

where the constant $\Gamma(\psi_s, \alpha, \gamma)$ is

$$\Gamma(\psi_s, \alpha, \gamma) = \left[\psi_s(1-\gamma)\right]^{\frac{1-\gamma}{\alpha\gamma}} \cdot \left[\psi_s(1-\alpha)\gamma\right]^{\frac{1}{\alpha}}. \tag{6}$$

Using (5) in (3) we can rewrite output as

$$y_{tis} = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1}{(1-\alpha)\gamma} \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot k_{tis} + \zeta_{tis} \tag{7}$$

which is linearly increasing in k_{tis} and, using this in (4) gives profits as

$$\pi_{tis} = \left[\Gamma(\psi_s, \alpha, \gamma) \cdot \frac{\alpha}{(1-\alpha)} \cdot p_{ts} \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s + p_{ts}\zeta_{tis}\right] k_{tis},$$

which are also proportional to k_{tis}. Defining the idiosyncratic return on capital as

$$r_{tis} = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{\alpha}{(1-\alpha)} \cdot p_{ts} \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s + p_{ts}\zeta_{tis} \tag{8}$$

we can thus rewrite profits as

$$\pi_{tis}(k_{tis}) = r_{tis} \cdot k_{tis}. \tag{9}$$

2.2.3 Energy Production

Energy employed for production in the two intermediate goods sectors s is produced in two perfectly separated (across the two sectors) energy producing firms that employ labour L_{ts}^e and a technology stock Υ_{ts}^e, which grows exogenously and deterministically at the sector specific rates g_s. The energy production technology is linear and accordingly

$$E_{ts} = \Upsilon_{ts}^e L_{ts}^e.$$

Dirty energy production is subject to proportional carbon taxes $\tau_{ts=di}^e \geq 0$, whereas energy in the clean sector is untaxed (or may be subsidized), $\tau_{ts=cl}^e \leq 0$, and thus profits in the two energy producing firms are

$$\pi_{ts}^e = p_{ts}^e \left(1 - \tau_{ts}^e\right) \Upsilon_{ts}^e L_{ts}^e - r_t^l L_{ts}^e.$$

Assuming free entry and exit drives profits in the energy sector to zero and thus energy prices are given by

$$p_{ts}^e = \frac{r_t^l}{\left(1 - \tau_{ts}^e\right) \Upsilon_{ts}^e}. \tag{10}$$

2.3 Carbon Stock Accumulation, Temperature and the Damage Function

As in Golosov, Hassler, Krusell, and Tsyvinski (2014) and Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2021), the total carbon stock S_t in the atmosphere is composed of two stocks, a rapidly and a slowly depreciating stock, S_{tc} for $c \in \{ra, sl\}$, thus

$$S_t = \sum_{c \in \{ra,sl\}} S_{tc}$$

which accumulate through dirty energy emissions and feature persistence parameters ρ_c, where $1 > \rho_{c=sl} > \rho_{c=ra} > 0$, thus

$$S_{tc} = \rho_c S_{t-1c} + \phi_c \xi E_{ts=di} \tag{11}$$

where $\xi > 0$ and $\phi_c > 0$ and $\sum_{c \in \{ra,sl\}} \phi_c = 1$. Each unit of S_t leads to an increase of the global temparature according to

$$T_t = \lambda \frac{\log(S_t / S_{pre})}{\log(2)}, \tag{12}$$

where S_{pre} is the pre-industrial area carbon stock in the atmosphere, and $\lambda > 0$. The temperature increase in turn leads to the negative externality on aggregate output through the damage function

$$D_t = 1 - \frac{1}{1 + \nu T_t^2}. \tag{13}$$

for $\nu > 0$.

2.4 Fiscal and Monetary Authorities

The model features a fiscal and a monetary authority. The fiscal authority levies Carbon taxes at rates $\tau^e_{ts=di} \geq 0$ and receives profits from the monetary authority π^m_t. These sources of income are distributed to households in the form of subsidies on consumption, $\tau^c_t \leq 0$ and thus each period the fiscal authority features a balanced budget of

$$\tau^e_{ts=di} E_{ts=di} + \pi^m_t + \tau^c_t C_t = 0. \tag{14}$$

The monetary authority in turn holds an exogenous amount of capital K^m_t in the economy which is growing at exogenous time varying rate $g^m_t \geq 0$. This capital is exogenously split across the two capital stocks in the intermediate goods production sectors, thus

$$K_t^m = \sum_{s\in\{cl,di\}} K_{ts}^m.$$

The monetary authority earns the average marginal products in the two sectors and its profits are thus

$$\pi_t^m = \sum_{s\in\{cl,di\}} \mathbb{E}[r_{ts}]K_{ts}^m.$$

2.5 Households

2.5.1 Preferences

Each household i at time t has Epstein-Zin-Weil (Epstein and Zin 1989; Epstein and Zin 1991; Weil 1989) recursive preferences u_{ti} over consumption c_{ti} and continuation utility u_{t+1i} which is discounted at factor $\beta \in (0,1)$ and features risk aversion parameterized by θ and resistance to intertemporal substitution υ. Thus, preferences are given by

$$u_{ti} = \left[c_{ti}^{1-\upsilon} + \beta \cdot \left(\mathbb{E}[u_{t+1i}^{1-\theta}]\right)^{\frac{1-\upsilon}{1-\theta}}\right]^{\frac{1}{1-\upsilon}}, \tag{15}$$

where $\mathbb{E}$ is an expectations operator with expectations taken with respect to the idiosyncratic shocks to the return on physical capital.

2.5.2 Endowments

Households operate the two intermediate goods firms. Accordingly, household i enters into model period t with capital stocks k_{tis} in the two firms and earns in the current period stochastic profits generated from production in those firms π_{tis}. Households also earn a deterministic labour income $r_t^l \ell_t$ where r_t^l denotes the wage rate on the exogenous labour endowment ℓ_t, which is the same for all households. Furthermore, households enter the period with bond holdings b_{ti}, which are in zero net supply across all households and earn a risk-free return r_t^f. The household spends its income from these sources on consumption of the final good c_{ti}—which has price p_t and is taxed, respectively subsidized, at rate τ_t^c—, on savings in the two capital goods k_{t+1is} as well as on risk free bond purchases b_{t+1i}. Thus the dynamic budget constraint of household i is

$$\sum_{s\in\{c,d\}} k_{t+1is} + b_{t+1i} + (1+\tau_t^c)p_t c_{ti} = \sum_{s\in\{c,d\}} k_{tis}\left(1+r_{tis}\right) + (1+r_t^f)b_{ti} + r_t^l \ell_t$$

where $r_{tjs} = \frac{\pi_{tis}}{k_{tis}}$ is the stochastic return on capital in sector s.

2.5.3 Analysis of the Household Problem

Conditional on the aggregate law of motion of the economy, i.e., for given prices, wages, interest rates and taxes, the household model permits a closed form solution. To derive it, first rewrite the budget constraint in terms of cash-on-hand

$$x_{ti} = \sum_{s\in\{c,d\}} k_{tis}\left(1 + r_{tis}\right) + (1 + r^f_t)b_{ti} + r^l_t \ell_t$$

to get

$$\sum_{s\in\{c,d\}} k_{t+1is} + b_{t+1i} = x_{ti} - (1 + \tau^c_t)p_t c_{ti}.$$

Next, define the portfolio shares as shares invested in the respective asset as a function of total savings $x_{ti} - (1 + \tau^c_t)c_{ti}$ as

$$\vartheta_{tis} = \frac{k_{t+1is}}{x_{ti} - (1 + \tau^c_t)p_t c_{ti}}, \qquad 1 - \sum_{s\in\{cl,di\}} \vartheta_{tis} = \frac{b_{t+1is}}{x_{ti} - (1 + \tau^c_t)p_t c_{ti}}$$

to note that

$$x_{t+1i} = \left(1 + r^f_{t+1} + \sum_{s\in\{cl,di\}} \vartheta_{tis}\left(r_{t+1is} - r^f_{t+1}\right)\right)\left(x_{ti} - (1 + \tau^c_t)p_t c_{ti}\right) + r^l_{t+1}\ell_{t+1}. \tag{16}$$

Next, denote by h_t the human capital wealth of a household at date t, which is the discounted sum of future labour income

$$h_t = \sum_{j=0}^{\infty} r^l_{t+1+j}\ell_{t+1+j} \prod_{k=0}^{j}\left(1 + r^f_{t+k+1}\right)^{-1}$$

which thus obeys the human capital wealth accumulation equation

$$h_{t+1} = h_t(1 + r^f_{t+1}) - r^l_{t+1}\ell_{t+1}. \tag{17}$$

Finally, define total wealth of the household as the sum of cash-on-hand and human capital wealth,

$$w_{ti} = x_{ti} + h_t,$$

and take the sum of (16) and (17) to get

$$w_{t+1i} = \left(w_{ti} - (1+\tau_t^c)p_t c_{ti}\right) R_{t+1i}^p \left(\{\hat{\vartheta}_{tis}\}_{s\in\{cl,di\}}\right), \tag{18}$$

where

$$R_{t+1i}^p \left(\{\hat{\vartheta}_{tis}\}_{s\in\{cl,di\}}\right) = 1 + r_{t+1}^f + \sum_{s\in\{cl,di\}} \hat{\vartheta}_{tis} \left(r_{t+1is} - r_{t+1}^f\right)$$

is a portfolio return on total savings $w_{ti} - (1-\tau_t^c)c_{ti}$ and where

$$\hat{\vartheta}_{tis} = \frac{k_{t+1is}}{w_{ti} - (1+\tau_t^c)p_t c_{ti}}, \qquad 1 - \sum_{s\in\{cl,di\}} \hat{\vartheta}_{tis} = \frac{b_{t+1is} + h_t}{w_{ti} - (1+\tau_t^c)p_t c_{ti}}$$

are the portfolio investments in the respective asset in relation to total savings.

Maximization of (15) subject to the resource constraint (18) gives rise to optimal decisions in terms of consumption policy functions and portfolio allocation decisions as stated in the next proposition, which we formally prove in Appendix A.3:

Proposition 1. • *Consumption policy functions are linear functions of total wealth*

$$c_{ti} = m_t w_{ti}$$

where the marginal propensities to consume are

$$m_t = \frac{\Theta\left(p_t, p_{t+1}, \tau_t^c, \tau_{t+1}^c, R_{t+1}^p\left(\{\hat{\vartheta}_{ts}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1}}{1 + (1+\tau_t^c)\Theta\left(p_t, p_{t+1}, \tau_t^c, \tau_{t+1}^c, R_{t+1}^p\left(\{\hat{\vartheta}_{ts}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1}}, \tag{19}$$

where

$$\Theta\left(p_t, p_{t+1}, \tau_t^c, \tau_{t+1}^c, R_{t+1}^p\left(\{\hat{\vartheta}_{ts}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) = \left(\beta \frac{p_t(1+\tau_t^c)}{p_{t+1}(1+\tau_{t+1}^c)} \left(\mathbb{E}_t\left[R_{t+1}^p\left(\{\hat{\vartheta}_{ts}^*\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}\right)^{-\frac{1}{\upsilon}}$$

- *The optimal portfolio shares are given by*

$$\begin{pmatrix} \hat{\vartheta}_{tcl}^* \\ \hat{\vartheta}_{tdi}^* \end{pmatrix} \approx \frac{1}{\theta}\mathbf{\Sigma}^{-1} \begin{pmatrix} \ln(1+\mathbb{E}\left[r_{t+1cl}\right]) - \ln(1+r_{t+1}^f) \\ \ln(1+\mathbb{E}\left[r_{t+1di}\right]) - \ln(1+r_{t+1}^f) \end{pmatrix}, \tag{20}$$

which in case of a zero correlation of return shocks across sectors, i.e. for $\rho^{\zeta}_{cl,di} = 0$, *simplifies to*

$$\hat{\vartheta}^*_{ts} \approx \frac{\ln(1 + \mathbb{E}\left[r_{t+1s}\right]) - \ln(1 + r^f_{t+1})}{\theta \cdot Var(\ln(1 + r_{t+1s}))}, \tag{21}$$

$s \in \{cl, di\}$.

Thus, the marginal propensities to consume out of total wealth and the optimal portfolio shares in t, s are the same for all i, $m_{ti} = m_t$, $\hat{\vartheta}_{tis} = \hat{\vartheta}_{ts}$. Linearity of policy functions in total wealth and identical marginal propensities to consume in any t, s across all households is a very convenient property of the model as it simplifies the aggregation to the effect that we only need to keep track of the average policies and not their distribution.

2.6 Definition of Equilibrium

We define the equilibrium in this economy sequentially. By the result in Proposition 1 we do not need to keep track of the distribution of heterogenous households and thus household specific variables are not indexed by i and it is understood that the household variables in the formal equilibrium definition indexed by t, respectively by t and s, refer to average allocations.

Definition 1. *Given an initial total wealth level* w_0, *initial carbon stocks* $\{S_{0c}\}_{c\in\{ra,sl\}}$, *a sequence of technology levels and of the population* $\{\Upsilon_t, \{\Upsilon^e_{ts}\}_{s\in\{cl,di\}}, N_t\}^{\infty}_{t=0}$ *and a sequence of policy parameters* $\{\tau^c_t, \tau^e_{ts=di}, \{K^m_{ts}\}_{s\in\{cl,di\}}\}^{\infty}_{t=0}$, *a competitive equilibrium is an allocation* $\{\{E_{ts}, K_{ts}, L_{ts}, Y_{ts}, \hat{\vartheta}_{ts}\}_{s\in\{cl,di\}}, x_{t+1}, h_{t+1}, w_{t+1}, S_t, T_t, D_t\}^{\infty}_{t=0}$, *a sequence of prices* $\{\{p_{ts}, p^e_{ts}, r_{ts}\}_{s\in\{cl,di\}}, r^f_t, r^l_t\}^{\infty}_{t=0}$ *and a sequence of profits* $\{\{\pi_{ts}\}_{s\in\{cl,di\}}, \pi^m_t\}^{\infty}_{t=0}$ *such that*

1. *given prices* $\{\{p_{ts}, p^e_{ts}, r_{ts}\}_{s\in\{cl,di\}}, r^f_t, r^l_t\}^{\infty}_{t=0}$ *and policies* $\{\tau^c_t, \tau^e_{ts=di}, \{K^m_{ts}\}_{s\in\{cl,di\}}\}^{\infty}_{t=0}$ *households behave optimally with resulting optimal policy functions for* $c_t, \hat{\vartheta}_{ts}, w_{t+1}$ *as characterized in Proposition 1.*

2. *prices satisfy (5),(10) and*

$$r_{ts} = \int r_{tis} di$$

where r_{tis} *is given in (8);*

3. *the government budget constraint (14) holds in all* $t \geq 0$;

4. *the sequence of carbon stocks, global temperature and global damage* $\{\{S_{tc}\}_{c\in\{ra,sl\}}, T_t, D_t\}^{\infty}_{t=0}$ *evolve according to (11)–(13);*

5. *markets clear:*

$$L_t = N_t \ell_t \tag{22a}$$

$$K_{ts} = N_t \int k_{tsi} di, \text{ for } s \in \{cl, di\} \tag{22b}$$

$$B_t = N_t \int b_{ti} di = 0 \tag{22c}$$

$$L_{ts} = N_t \int \ell_{tsi} di = N_t \cdot \Gamma(\psi_s, \alpha, \gamma) \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot K_{ts}, \text{ for } s \in \{cl, di\} \tag{22d}$$

$$E_{ts} = N_t \int e_{tsi} di = N_t \cdot \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1-\gamma}{(1-\alpha)\gamma} \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma(1-\alpha)}{\alpha\gamma}} \cdot K_{ts}, \text{ for } s \in \{cl, di\} \tag{22e}$$

$$Y_{ts} = N_t \int y_{t,j,i}^X di = N_t \cdot \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{1}{(1-\alpha)\gamma} \cdot \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} \cdot K_{ts}, \text{ for } s \in \{cl, di\} \tag{22f}$$

where $\Gamma(\psi_s, \alpha, \gamma)$ *is given in (6).*

3 Calibration and Policy Experiments

3.1 Overview of Calibration

We calibrate the model by fixing some parameters exogenously (first stage parameters) and by calibrating others (second stage parameters) to match selected moments in an initial steady state year, which we pick to be year 2010. While the latter set of parameters are calibrated jointly, for clarity of identification of the parameter values we relate each parameter with a specific target. Tables 2 and 3 provide an overview of all first- and second-stage parameters and the subsequent sections provide the details of the calibration by sector in the economy.

3.2 Population and Labour Supply

The exogenous initial size of the population N_0 is normalized to one. The population growth rate n_t and the working age population ratio (WAPR) ω_t are calibrated from the population growth rate information (and projections) provided by the World Population Prospects of United Nations (UN) (United Nations 2020). The initial year 2010 population growth rate is $n_0 = 0.0121$ and the population growth rate shrinks gradually to reach zero growth by year 2100 thus $n_t = 0$, for all $t > 90$. Aggregate labour in the model is $L_t = \omega_t N_t$. In our baseline scenario, we abstract from time variation in the working age population ratio by letting $\omega_t = \omega_0$ (which we normalize

Table 2: Calibration: First Stage Parameters

Parameter	Value	Target (Source)
Population and labour supply		
Initial population size N_0	1	Data moment (United Nations)
Initial population growth rate n_0	0.0121	Data moment (United Nations)
Initial working age population ratio ω_0	1	Constant (baseline)
Final good technology		
Intermediate Goods Elasticity ϵ	26	Energy Elasticity $\eta = 2$
Intermediate good technology		
Non-energy share: γ	0.96	Kotlikoff et al. (2019)
Capital share: α	0.33	Standard value
Corr. of depreciation shocks: $\rho^{\zeta}_{cl,di}$	0	Zero correlation
Climate Module		
Initial carbon stock: S_0	802 GtC	
Pre-industrial carbon stock: S_{pre}	581 GtC	
Stock 1 share: ϕ_s	[0.5,0.5]	
Emission share in atmosphere: ξ	0.4	
Carbon stock persistence: ρ_c	$\rho_c = [0.996, 0.999], c \in \{ra, sl\}$	
Temp. increase with S: λ	3	
Temperature to damage: ν	0.0028388	
Preferences		
Elasticity inter-temp. substit., $1/\upsilon$	0.5	Standard value

Notes: Calibration in the baseline model. First stage parameters calibrated with reference to other studies or without using the model. Steady state year is year 2010.

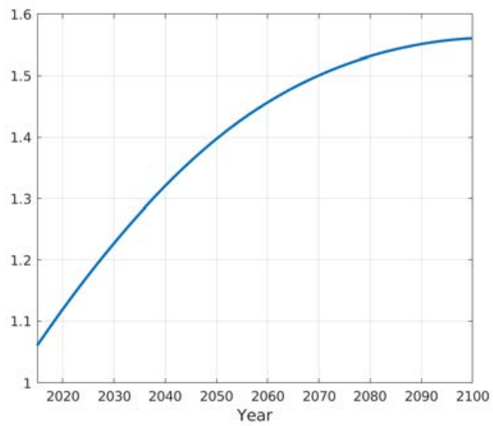
Table 3: Calibration: Second Stage Parameters

Parameter	Value	Moment
Final good technology		
Interm. good weight $\kappa_s, s \in \{cl, di\}$	$[0.45, 0.55]$	$E_{0s=di}/E_{0s=cl} = 4$
Growth rate final good TFP, g	0.0098	$(\frac{Y_{2100}}{L_{2100}}/\frac{Y_{2020}}{L_{2020}})^{\frac{1}{80}} - 1 = 1.50\%$
Intermediate good technology		
Iterm. productivity factor: $\psi_{s=cl} = \psi_{s=di}$	4811	$E_{0s=di} = 30$ GtCO2
Expected depreciation rate: $\delta_s, s \in \{cl, di\}$	$[0.015, 0.087]$	$\mathbb{E}[r_{0s}] = 6.94\%, s \in \{cl, di\}$
Std. of depreciation shock: $\sigma^{\zeta}_s, s \in \{cl, di\}$	$[0.030, 0.021]$	$\sigma^{r_{0s}} = 8.4\%, s \in \{cl, di\}$ (std. of capital returns)
Energy production technology		
Clean productivity factor, $\Upsilon^e_{0s=cl}$	128	$p^e_{0s=cl} = 810$ USD/tC
Dirty productivity factor, $\Upsilon^e_{0s=di}$	192	$p^e_{0s=di} = 540$ USD/tCe
Growth rate clean prod. fact., $g^e_{s=cl}$	0.020	$(p^e_{2100s=cl}/p^e_{2020s=cl})^{\frac{1}{70}} - 1 = -0.50\%$
Growth rate dirty prod. fact., $g^e_{s=di}$	0.011	$(\frac{E_{2035s=di}}{Y_{2035}}/\frac{E_{2020s=di}}{Y_{2020}})^{\frac{1}{15}} - 1 = -0.50\%$
Preferences		
Time discount factor: β	0.997	$K/Y = 2.5$
Relative risk aversion: θ	63.9	$r^f = 2.9\%$
Central bank portfolio		
Capital holdings $K^m_{0s}, s \in \{cl, di\}$	$[6244, 8930]$	$\frac{K^m_{0cl}+K^m_{0cl}}{Y_0} = 10\%$ & $K^m_{0cl}/K_{0cl} = K^m_{0di}/K_{0di}$

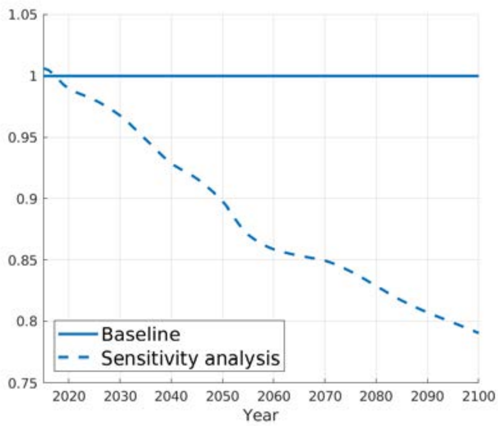
Notes: Calibration in the baseline model. Second stage parameters calibrated endogenously by matching of moments in steady state year 2010.

to 1 in the base year) and thus the aggregate of labour grows at the same rate as the population. As a sensitivity analysis, we feed into the model a time varying working age population ratio. Figure 2 displays the evolution of the aggregate population size in panel (a) and the working age population ratio in panel (b), from year 2015 to year 2100. Population features a gradually decreasing growth rate and is thus hump-shaped over the next 80 years. This reflects the increase in the world population from 7.8 Billion in year 2020 to about 10.9 Billion people in year 2100 according to the median variant of the UN projections.

Figure 2: Population and Working age population ratio



(a) Population size



(b) WAPR

Notes: Aggregate population size in panel (a) and working age population ratio (WAPR) in panel (b). Population size in panel (a) normalized such that is equal to one in the year 2010. Panel (b) shows WAPR in the baseline setup (held constant at one) and as used in sensitivity analysis WAPR. Population size in baseline and sensitivity setup and WAPR in case of the sensitivity setup correspond to the median variant of the UN projections.
Source: United Nations (2020).

3.3 Production

Final Good Production We take an indirect approach to the calibration of the parameter governing the elasticity of final output in the two goods, Y_{ts}, $s \in \{cl, di\}$ in equation (1). In our model with the two separate firms for energy production there is no direct parameter that would govern the energy demand elasticity (short energy elasticity), which is the percent change in the ratio of dirty to clean energy demand $\frac{E_{ts=d}}{E_{ts=c}}$ in response to a percent change of relative prices $\frac{p^e_{ts=d}}{p^e_{ts=c}}$ denoted as $\eta_{\frac{E_{ts=d}}{E_{ts=c}}, \frac{p^e_{ts=d}}{p^e_{ts=c}}}$. According to Papageorgiou, Saam, and Schulte (2017) the energy demand elasticity is about 2-3, where the lower value refers to the electricity-generating sector and values close to 3 are in nonenergy industries. We take the lower value as target for the intermediate goods elasticity ϵ. In appendix B.1 we derive that locally—i.e., holding constant the (expected) marginal remuneration of capital $\mathbb{E}r_{ts}$, $s \in \{cl, di\}$ and labour r_t^l—the energy elasticity is given by

$$\eta_{\frac{E_{ts=di}}{E_{ts=cl}}, \frac{p^e_{ts=di}}{p^e_{ts=cl}}} = \epsilon \cdot (1-\gamma) + \gamma,$$

which we invert to calibrate ϵ for given target $\eta_{\frac{E_{ts=di}}{E_{ts=cl}}, \frac{p^e_{ts=di}}{p^e_{ts=cl}}}$ and a given non-energy share parameter γ.[8] For our calibrated value of $\gamma = 0.96$ (see below), this gives a calibrated value of the intermediate goods elasticity of $\epsilon = 26$ and thus, as a consequence of the low energy share of $1-\gamma = 0.04$, our model requires a high intermediate goods elasticity to match a conservative energy demand elasticity of 2. It turns out that the intermediate goods elasticity ϵ is a crucial parameter for our quantitative findings on the relative effectiveness of green QE. In our sensitivity analysis, motivated by Golosov, Hassler, Krusell, and Tsyvinski (2014) and Hassler, Krusell, and Olovsson (2021), we also calibrate the model with lower values of the intermediate goods elasticity. Specifically, we assume an energy elasticity of $\eta \approx 1$ implying an intermediate goods elasticity of $\epsilon = 2.25$. Since elasticity estimates are downward biased due to adjustment costs (Caballero 1994), we also consider an in-between value of $\epsilon = 4$ (implying $\eta = 1.12$) as reasonable in light of the long-run focus of our analysis.

The relative weights on the two goods in (1), $\kappa_s, s \in \{cl, di\}$ are calibrated such that (i) we normalize $\kappa_{s=di} = 1 - \kappa_{s=cl}$ and (ii) match the ratio of energy output in the two sectors of $\frac{E_{0s=di}}{E_{0s=cl}} = 4$ giving $\kappa_{s=cl} = 0.45$ and thus $\kappa_{s=di} = 0.55$. The final good output growth rate g is calibrated to generate a total per capita annual output growth of 1.5% in the period from year 2020 to year 2100, giving $g = 0.0098$.

Intermediate Goods Production We set the capital share, cf. equation (3), exogenously to $\alpha = 0.33$, corresponding to standard estimates of capital shares in production. The non-energy share parameter γ is set to 0.96, following Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2021). The technology levels in both sectors are normalized such that $\psi_{cl} = \psi_{di}$ and calibrated to generate dirty energy production of 30 gigatons of CO2 in the initial steady state equilibrium.

The average depreciation rates $\delta_s, s \in \{cl, di\}$ and the standard deviation of depreciation shocks $\sigma_s^\zeta, s \in \{c, d\}$ are calibrated to yield expected average returns of 6.94% in both sectors and a standard deviation of expected returns of 8.4%, based on empirical estimates of Piazzesi, Schneider, and Tuzel (2007). This gives $\delta_s = [0.015, 0.087], s \in \{cl, di\}$. For our baseline results, based on Broadstock and Cheng (2019)[9] we assume a zero correlation in the returns

[8] Recall that $1-\gamma$ is the energy elasticity in intermediate goods production, cf. equation (3).

[9] For the US bond market, Broadstock and Cheng (2019) report a negative correlation before mid-2013, and a positive correlation thereafter so that an on average zero correlation is an appropriate assumption for the long-run.

in both sectors, i.e. we set $\rho^{\zeta}_{cl,di} = 0$. We further consider a positive correlation for sensitivity analysis, based on Reboredo, Ugolini, and Aiube (2020).[10]

Energy Production Recall from equation (10) that energy prices are inversely proportional to the technology level in the energy sector. Based on this relationship we calibrate the technology parameters $\Upsilon^e_{0s}, s \in \{cl, di\}$ to match the absolute price levels[11] in the two sectors per ton of carbon emission (tCe) of USD 810, respectively 540, which requires $\Upsilon^e_{0s=cl} = 128$ and $\Upsilon^e_{0s=di} = 192$. We denote by g^e_s the time constant growth rates in the two sectors, i.e., $\Upsilon^e_{ts} = \Upsilon^e_{t-1s}(1+g^e_s)$. We endogenously determine the growth rate in the clean energy sector $g^e_{s=cl}$ such that energy prices fall by 0.5% on average over the 80 years between year 2020 and 2100. This calibration is based on Nordhaus (2017), also see Kotlikoff, Kubler, Polbin, Sachs, and Scheidegger (2021). We endogenously determine the growth rate in the dirty energy sector $g^e_{s=di}$ such that CO2 emissions relative to GDP reduce at a rate of -0.5% annually over the period from year 2020 to 2035, which corresponds to the average value of the share of CO2 emissions relative to world GDP over the period 1995 to 2018 as measured in PPP units at constant prices, which we compute from World Bank (2021). Our calibration gives $g^e = [0.020, 0.011]$ for the two clean and dirty energy sector growth rates, respectively.

3.4 Household Preferences

The elasticity of inter-temporal substitution $1/\upsilon = 0.5$, corresponding to the standard estimate in the literature. The remaining household preference parameters are calibrated endogenously to match a capital output ratio of 2.5 by choice of the discount factor, which gives $\beta = 0.997$, and a risk-free rate of return of 2.9% by choice of the coefficient of risk aversion which requires $\theta = 63.9$. This high value is not surprising because shocks in our model are assumed to be distributed as log-normal (thus, there are no extreme events), and there are no additional income shocks (no background risk) for households.

3.5 Carbon Stock Accumulation, Temperature and the Damage Function

The calibration of the climate module closely follows Golosov, Hassler, Krusell, and Tsyvinski (2014), where we draw on van der Ploeg and Rezai (2021) to adjust for the yearly periodicity of our calibration approach.[12]

Carbon Stock The initial carbon stock in the atmosphere is set to $S_0 = 802$ gigatons of carbon, where $S_{0c=sl} = 684$ GtC and $S_{0c=ra} = 118$ GtC. As to the dynamics of the two carbon

[10] Reboredo, Ugolini, and Aiube (2020) identify network connectedness as a source of a potential positive correlation across asset classes.

[11] Recall that final consumption is the numeraire good in the economy so that these absolute price levels are equal to the relative prices in units of the final consumption good.

[12] See Kotlikoff, Kubler, Polbin, and Scheidegger (2021) for an adoption to a quinquennial frequency.

stocks in equation (11) we assume that 40% of dirty energy output leads to an accumulation of the carbon stocks and thus $\xi = 0.4$, which is split up equally across the two stocks, thus $\phi_c = 0.5, c \in \{sl, ra\}$. The slow decumulating carbon stock features a persistence of $\rho_{c=sl} = 0.999$, and the rapidly decumulating of $\rho_{c=ra} = 0.995$.

Temperature and Damage Function We calibrate the temperature function in (12) by setting $\lambda = 3$ and $S_{pre} = 581$, and the damage function in (13) by letting $\nu = 0.0028388$.

3.6 Fiscal and Monetary Authorities

Monetary Authority The monetary authority's portfolio is calibrated such that in the initial steady state capital held by the monetary authority (both clean and dirty) equals 10% of GDP, i.e. $\frac{K^m_{0cl}+K^m_{0cl}}{Y_0} = 0.1$. This is a rough estimate of the overall privately-issued security holdings of advanced-economy central banks at the beginning of the year 2021.[13] Furthermore, capital holdings in clean and dirty assets by the monetary authority are set such that they are proportional to private capital holdings, i.e. $\frac{K^m_{0cl}}{K_{0cl}} = \frac{K^m_{0di}}{K_{0di}}$. Since the capital-to-output ratio is 2.5, this results in 4% of capital of both sectors held directly by the central bank. Given the endogenously determined sizes of the two sectors in the economy, this requires $K^m_{0s=cl} = 6244$ and $K^m_{0s=di} = 8930$. In our baseline experiment, we hold the shares constant, i.e., $\frac{K^m_{ts}}{K_s} = 0.04$ for $s \in \{cl, di\}$ for all $t > 0$, so that the wealth holdings of the monetary authority grow with the capital stock of the economy. As sensitivity analyses we consider two alternative scenarios regarding the stock of capital held by the central bank. First, we assume a constant absolute size of the outstanding capital held by the central bank, which implies that the relative size diminishes to zero over time. Second, we recalibrate the model to an initially higher share of assets held by the monetary authority in the dirty sector of the economy.

Fiscal Authority In the initial steady state equilibrium, the fiscal authority does not levy carbon taxes on emissions, thus $\tau^e_{ts} = 0$. Since revenues from asset holdings of the monetary authority are paid back to households in the form of consumption subsidies, the consumption tax rate is negative, $\tau^c_t < 0$. With -0.54% in the initial steady state this subsidy is small.

3.7 Thought Experiments

Taking as given the exogenous dynamics of population and technology, we compute transitions under alternative fiscal and monetary policy scenarios over 200 model periods, starting in year 2010 with an initial steady state.[14] We treat the first 10 years as a phase-in period and show results until 2100, that is overall we focus on the evolution of key model outcome variables for the next 80 years from 2020-2100.

[13] Our estimates are based on balance sheet data publicly available by July 2021 of the European Central Bank and the central banks of Canada, Japan, Sweden, United Kingdom and the United States.

[14] The model is closed by setting the final period equal to the final steady state.

First, we conduct a *baseline experiment*, where all policy parameters are held constant at their respective 2010 values, that is the initial carbon tax is zero and the capital allocation of the monetary authority relative to the total capital stock is held constant and proportional to private asset holdings across the two sectors. Consequently, the claims on private capital held by the monetary authority grow with the time varying growth rate of the aggregate capital stock. This assumption can be interpreted as approximating a real world economy in which asset purchases by the monetary authority take place with a certain regularity. Our economy does not feature aggregate risk and thus there are no recessions, which would endogenously lead to repeated non-standard monetary policy interventions (QE) if a zero lower bound on interest rates becomes binding. Since two of the worldwide secular economic mega-trends—demographic change and climate change—will likely lead to a persistently low interest rate environment, we regard it as plausible that such unconventional monetary policies will be implemented again in future recessions and our assumption of a constant share is therefore a reasonable approximation.[15]

Next, we consider a *carbon tax* policy reform scenario where a carbon tax is introduced, with the tax rate being held constant. The initial carbon tax in year 2020 is set to 50 USD per ton of carbon. The implied carbon tax rate is $\tau^e_{ts=di} = 0.066$ which we hold constant for all $t \geq 10$. Revenues from carbon taxation are redistributed to households through consumption subsidies.[16]

In our second policy reform scenario, the *green QE* scenario, the portfolio composition of the monetary authority changes such that it reshuffles all of its privately-issued capital holdings towards the green sector. In the green QE policy scenario we assume that, first, the monetary authority's private asset portfolio holdings are relatively large, second, that the private asset stock held by the monetary authority is growing in line with the world capital stock, third, all countries in the world execute QE policies, fourth, clean and dirty asset returns are uncorrelated so that the QE policy will not lead to a full crowding out of private clean capital investments and fifth, that the elasticity of substitution between clean and dirty energy production is relatively high so that the price changes induced by green QE lead to relatively large shifts in demand. We thus evaluate the maximum climate change mitigating potency of such policies. We acknowledge that we model green QE as a stylised scenario, in particular by abstracting from modelling financial

[15] Our simulations support this argument. We show in Appendix C that the risk-free interest rate is flat over the projection period, which is a result of the accumulating climate change damages suppressing aggregate productivity. This is reinforced in our sensitivity analysis, where we model population aging through a gradual reduction of the working age population ratio leading to a relative shortage of labor. As is well established from the macroeconomics of demographic change literature, this leads to a reduction of marginal productivities thus decreasing returns, cf., e.g., Krueger and Ludwig (2007) and, for a model with differential asset returns, Geppert, Ludwig, and Abiry (2016).

[16] An alternative use of revenues from carbon taxation would be to subsidize clean energy production, which would support the green transition further. Likewise, profits of the monetary authority could be used for such purpose.

intermediaries. We also do not consider the role of frictions in the monetary policy transmission mechanism, which would possibly amplify the effects of green QE.[17]

Finally, as a *full policy scenario* we consider both instruments jointly and thereby investigate whether both policies are substitutes or complements in mitigating the adverse effects of climate change.

4 How Effective are Green Quantitative Easing and Carbon Taxation?

Before we address our main research question on the effectiveness of green QE in comparison to carbon taxation in mitigating climate change damages, we compute a baseline path of the economy along which we hold constant all policy instruments.

4.1 Climate Change with Constant Policies

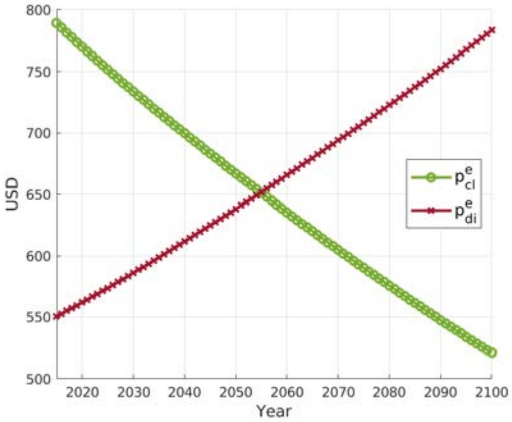
Intermediate Goods Production Figure 3 displays prices in the clean and the dirty energy sector in panel (a). The increasing prices of dirty energy and the falling price of clean energy is a consequence of the calibrated increase of relative productivity in the clean energy sector. With regard to the ensuing dirty energy production and thus dirty emissions, two mechanisms are at work in the model. On the one hand, demand for goods through population growth and technological progress in the final goods sector will lead to an increase of harmful emissions, $E_{ts=di}$. On the other hand, the technological progress in the clean sector $\Upsilon_{ts=cl}$ by increasing the relative price of dirty intermediate goods leads to a substitution of intermediate goods production towards the clean sector. Two forces lead to this substitution. The one is a reduction of demand for dirty energy in the intermediate goods sector. The second is a substitution towards clean intermediate goods in the production of the final good. Over our projection period, the first mechanism dominates. Consequently, dirty energy emissions are increasing over the entire period, but at a decreasing rate and clean energy emissions gain relative importance. Since by this gradual substitution the clean intermediate goods sector expands relative to the dirty sector, the aggregate input factors capital and labor in the economy are increasingly employed in the production of clean intermediate goods, cf. panels (c) and (d) of the figure.

Overall, these dynamic adjustments lead to an increase of the relative price of dirty intermediate inputs $\frac{p_{ts=di}}{p_{ts=cl}}$ by 1% and a reduction of relative output $\frac{y_{ts=di}}{y_{ts=cl}}$ by about -27%, cf. Figure 4.

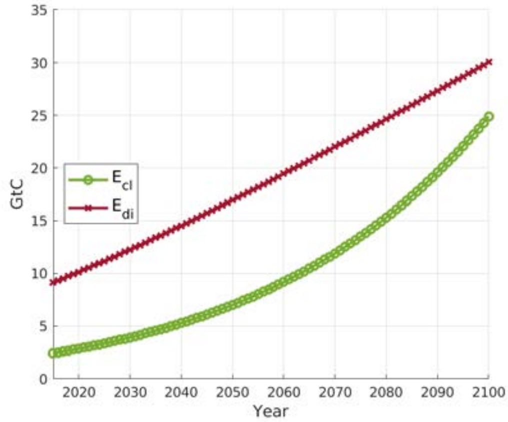
Climate Implications The implications of the above shown gradual substitution towards cleaner intermediate goods production for the global climate are shown in Figure 5, where Panel (a) shows the level of the emissions of the dirty sector $E_{ts=di}$. Panel (b) displays the resulting time paths of the carbon stocks that accumulate as a consequence of these emissions according to the

[17] Within the structure of our model there is no role for a triggering mechanism of a form that private investors may follow the example of the monetary authority. If such a mechanism were at work, then we would underestimate the role of green QE policies.

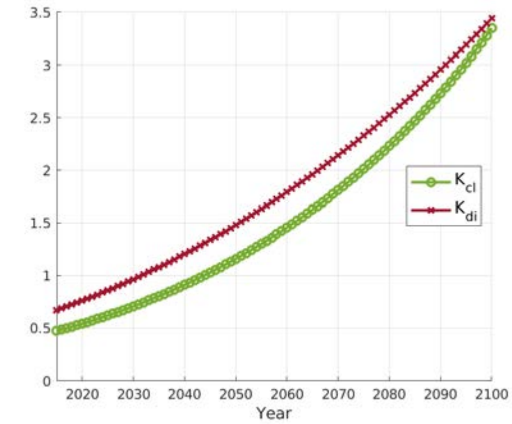
Figure 3: Baseline: Intermediate Production Inputs



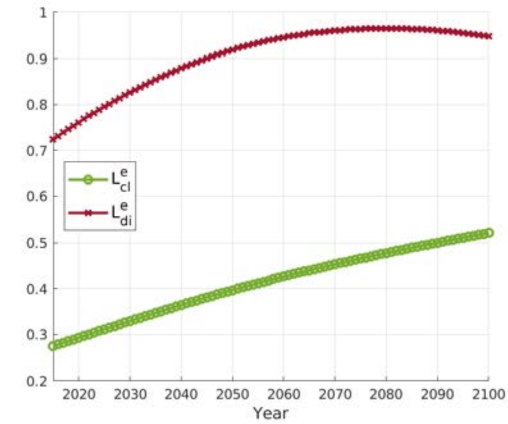
(a) Energy price



(b) Energy input



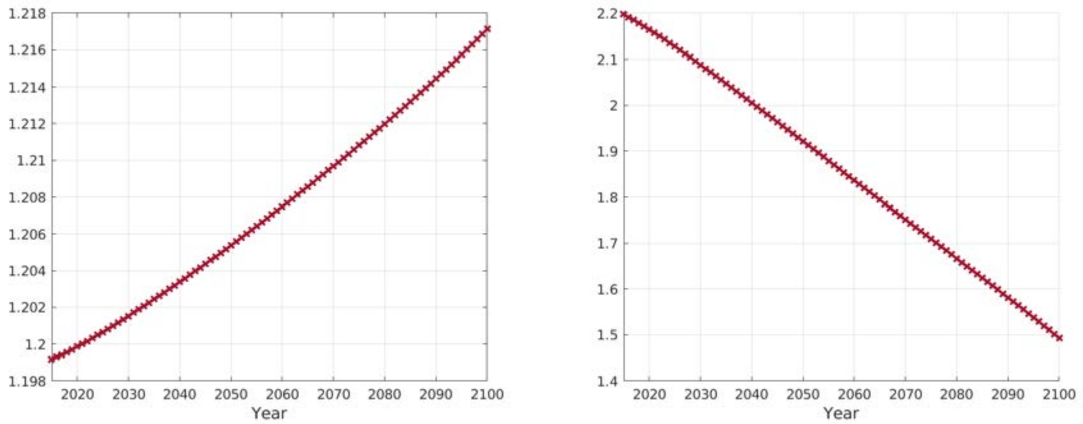
(c) Capital input



(d) Labor input

Notes: Energy prices p^e_{ts} in panel (a), energy inputs E_{ts} in panel (b), capital stocks K_{ts} in panel (c), and labor input L_{ts} in panel (d), for $s \in \{cl, di\}$. Capital is normalized such that clean and dirty capital sum to one in the year 2010. Labor is normalized to one in the year 2010.

Figure 4: Baseline: Final Production Inputs



(a) Relative price dirty to clean $\left(\frac{p_{t,di}}{p_{t,cl}}\right)$ (b) Relative good input dirty to clean $\left(\frac{Y_{t,di}}{Y_{t,cl}}\right)$

Notes: Intermediate production: relative price of dirty to clean goods, $\frac{p_{ts=di}}{p_{ts=cl}}$ in panel (a), and relative intermediate goods input, $\frac{Y_{ts=di}}{Y_{ts=cl}}$ in panel (b).

calibrated process described in (11). By year 2100 the total carbon stock will have increased by about 63% relative to its year 2020 level. This leads to an increase of global temperature as shown in Panel (c). According to our model, the year 2020 temperature level is about 1.5 degrees Celsius above the pre-industrial level. Observe that the initial level exceeds the current range of estimates by the IPCC (2021) of $0.9 - 1.3$ degrees slightly.[18] According to our model, without policy intervention the global temperature will increase to about 3.5 degrees, an increase over 80 years by 2 degrees, or 0.025 degrees per year. The resulting damage in terms of a percent output loss, shown in Panel (d) of the figure, increases from 0.6% in 2020 to 3.5% in 2100, a factor of 5.

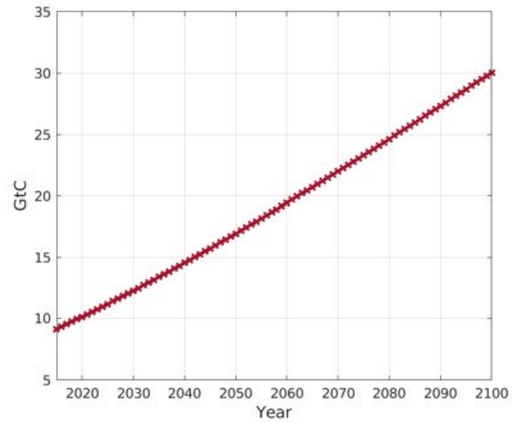
4.2 Climate Change with Policy Intervention

We now analyze the two policy reform scenarios, the introduction of a carbon tax and a portfolio shift of the capital holdings by the monetary authority. We first study both policies in isolation before turning to a joint analysis.

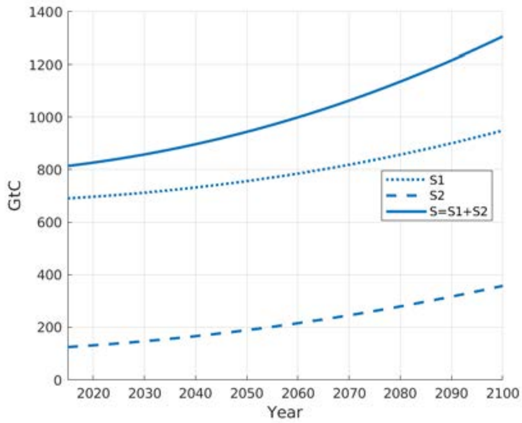
Policies in Isolation As our first policy scenario, we consider the introduction of a global carbon tax in year 2020 at a level of 50 USD per ton of carbon and hold constant the allocation of capital by the monetary authority. As the year 2020 price of dirty energy in our model is

[18]This upward bias is a consequence of the climate module we adopt from Golosov, Hassler, Krusell, and Tsyvinski (2014) (GHKT). The annual variant of the GHKT model calibrated in van der Ploeg and Rezai (2021) also features a 1.5 degree increase in their baseline year 2010.

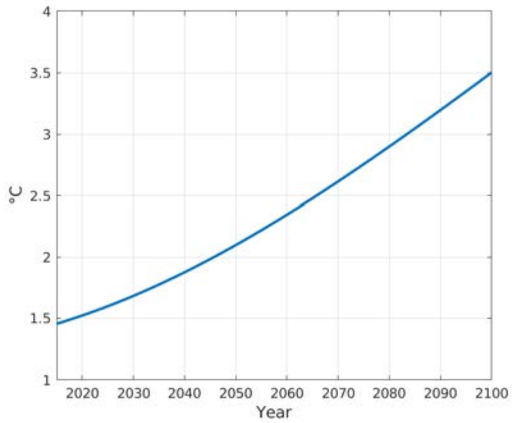
Figure 5: Baseline: Climate Variables



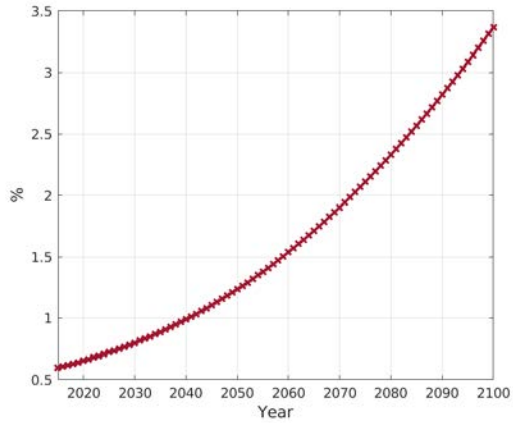
(a) CO2 emissions



(b) Carbon stock



(c) Temperature



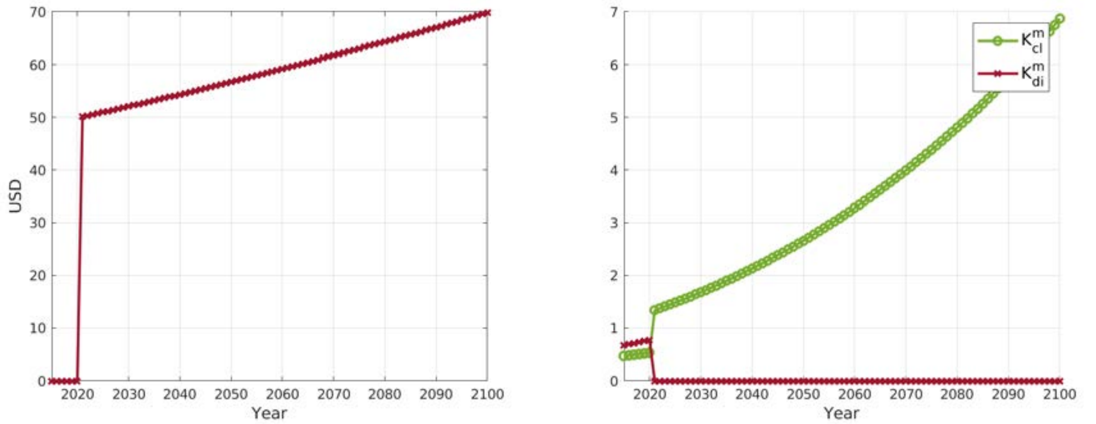
(d) Damage

Notes: Climate variables: CO2 emissions $E_{ts=di}$ in panel (a), carbon stocks $S_t, \{S_{tc}\}_{c\in\{ra,sl\}}$ in panel (b), world temperature T_t in degree Celsius compared to pre-industrial times in panel (c), and aggregate damage D_t (in percent) in panel (d).

at 750 USD this corresponds to a tax rate of 6.6%. We hold this tax rate constant along the transition, $\tau_{ts=di} = 0.066$, which implies that the absolute amount of carbon taxation increases at the growth rate of the dirty energy price $p^e_{ts=di}$. By 2100 the absolute carbon tax reaches almost 70 USD per ton of carbon. Panel (a) of figure 6 shows the time path of the absolute amount of the carbon tax expressed in USD per ton of carbon.

As our second policy scenario, we assume that in year 2020 there is a full shift towards capital holdings in the clean intermediate goods sector but we abstract from an introduction of a carbon tax. Panel (b) of the same figure shows the according capital holdings of the monetary authority in the two sectors. While this complete portfolio allocation is, of course, an extreme assumption, it enables us to investigate the effects of QE on climate change assuming a (hypothetical) situation where QE is at its maximum potency.

Figure 6: Reforms: Carbon Taxation and Portfolio Reallocation



(a) Carbon tax

(b) Capital holdings by the monetary authority

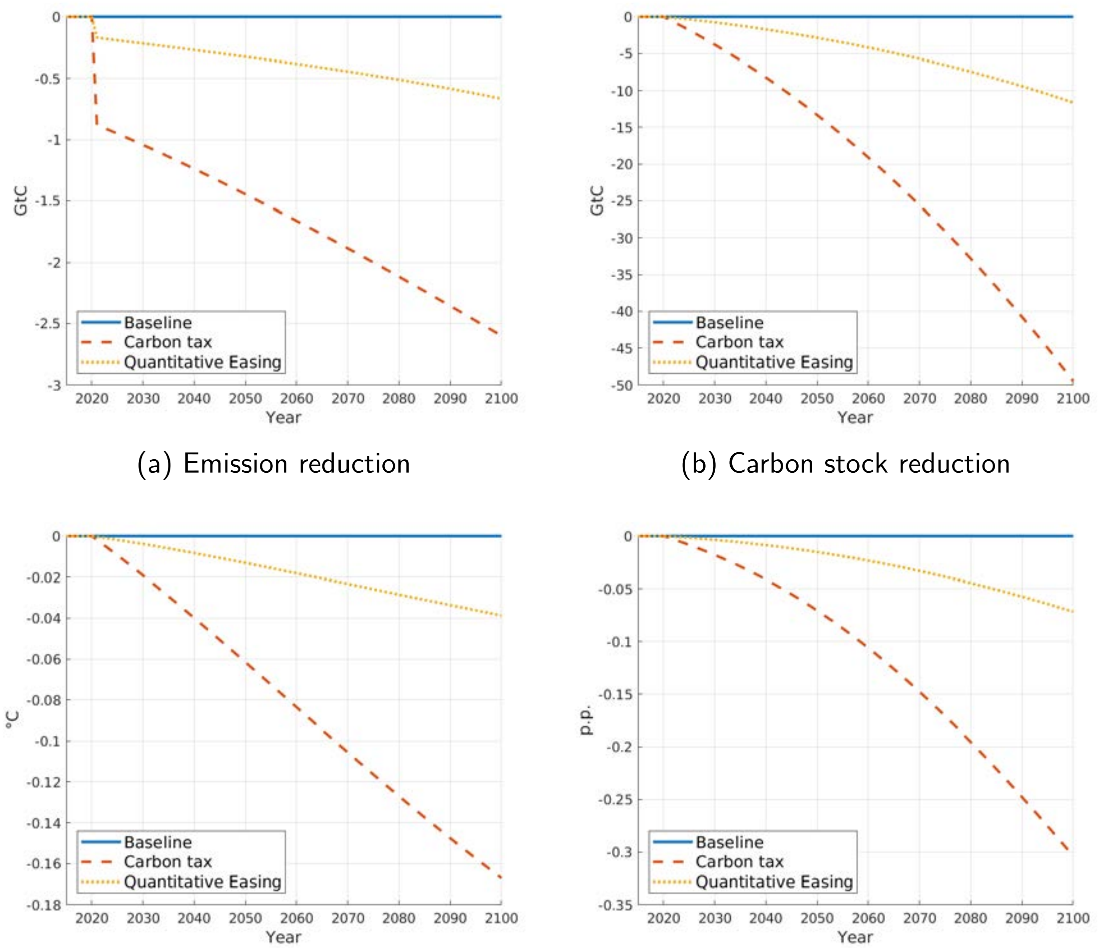
Notes: Policy reforms: carbon tax (in US dollars) in panel (a) and portfolio allocation of monetary authority in panel (b). Capital held by the monetary authority is normalized such that clean and dirty capital holdings sum to one in the year 2010.

Figure 7 shows the key outcome variables of our experiments, in terms of changes relative to the baseline path. Turning to the reduction of global temperature we observe from panel (c) that the global temperature reduction in the carbon tax experiment is about $\Delta T\text{-}\tau = -0.167$ degrees of Celsius. In the green QE scenario, it is only about $\Delta T\text{-}QE = -0.039$. Thus, in terms of *relative effectiveness* $\frac{\Delta T\text{-}\tau}{\Delta T\text{-}QE}$, the effect the carbon tax is about 4.3 times larger. Carbon taxes through changing the relative price of dirty energy lead to a reduction of dirty energy production and thus a reduction of the increase in the global temperature through two mechanisms. First, the price increase of dirty energy leads to a substitution of energy through capital and labour in the production of the dirty intermediate good, thus reducing the use of dirty energy in the production process (supply side mechanism). Second, the price increase of dirty energy $p^e_{ts=di}$ increases the price of the dirty intermediate good $p_{ts=di}$ which leads to a substitution in the production of the final output away from dirty intermediate towards clean intermediate goods (demand side mechanism).

The portfolio reallocation of the monetary authority, in contrast, has a theoretically ambiguous effect on dirty energy demand. First, on impact, i.e., holding factor prices constant, a reduction of capital employed for production in the dirty intermediate goods sector and a simultaneous increase of capital in the clean intermediate goods sector increases the marginal return on capital in the dirty and decreases it in the clean energy sector. This leads to an adjustment of private capital, which is reallocated from clean to dirty intermediate goods production and thus the portfolio reallocation by the monetary authority leads to a partial crowding out of private capital in the clean intermediate goods production. Also, the increased rate of return on capital in the dirty intermediate goods production increases capital costs for the intermediate goods firms leading to a substitution from capital towards energy and labour employed in production. This supply side mechanism increases the use of dirty energy. On the other hand, the increased capital costs of the firm leads to an increase of the intermediate goods price $p_{ts=di}$ which induces a substitution in the production of the final good towards the clean intermediate input and through this demand side channel reduces the demand for energy. Quantitatively, it turns out that the reduction of dirty energy due to the demand side mechanism dominates the increase in dirty energy due to the supply side mechanism. In effect, dirty energy use drops.

One key feature of the calibration of our two sector two physical assets model is the assumed zero correlation of the idiosyncratic returns across the two sectors. It implies that financial investors will hold a diversified portfolio of wealth across the two sectors. This explains why QE in our model is not neutral: a portfolio reallocation by the monetary authority towards the clean sector does not induce a perfect crowding out of private capital in the clean sector, but leads to a partial crowding out only. To illustrate the extent of this partial crowding out in our model, Figure 8 shows the allocation of capital in both sectors, by the monetary authority as dashed lines and by the private sector as solid lines. As a consequence of the portfolio reallocation, the monetary authority shifts its capital holding towards the clean sector. In response to this, private investors hold less capital in the clean sector, but this crowding out effect is much smaller than the additional capital held by the monetary authority in the clean sector. Likewise, the substitution of private investors into dirty capital holdings is smaller than the reduction of dirty capital holdings by the monetary authority. Thus the net effect on capital holdings is positive in the clean sector and negative in the dirty sector.

Figure 7: Reforms: Climate Variables



(a) Emission reduction

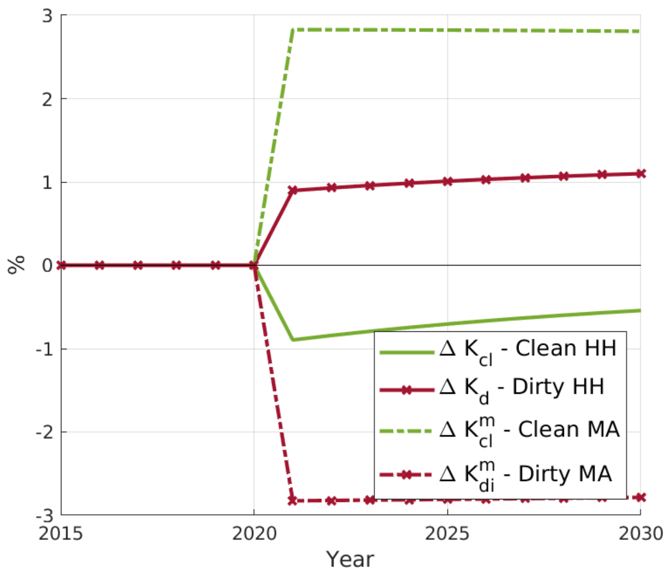
(b) Carbon stock reduction

(c) Temperature reduction

(d) Damage reduction

Notes: Policy reforms: emission reduction relative to baseline in gigatons of carbon in panel (a), carbon stock reduction relative to baseline in gigatons of carbon in panel (b), temperature reduction in degrees of Celsius in panel (c), and damage reduction in percentage points in panel (d).

Figure 8: Quantitative Easing: Why no crowding out?

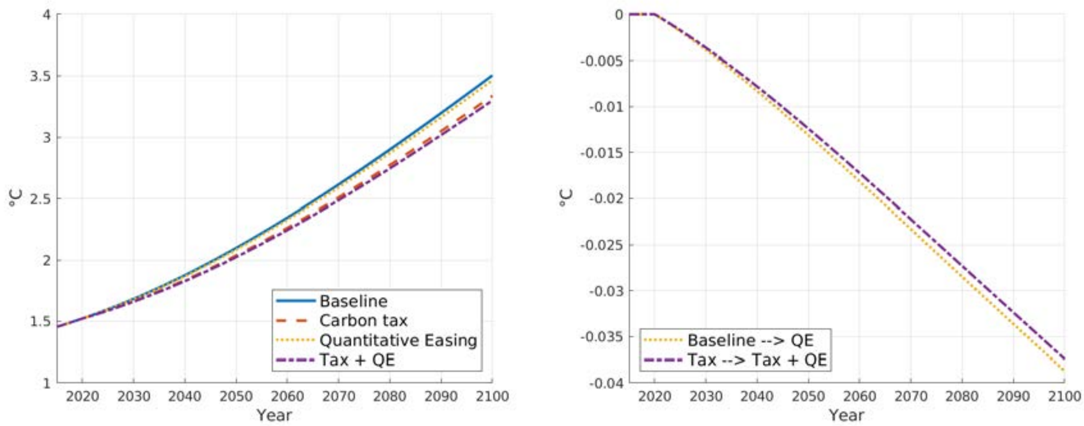


Notes: Policy reforms: Difference in dirty and clean sector capital holdings by private households and the monetary authority. HH denotes holdings of the households and MA of the monetary authority. The holdings are reported as the percentage share of total outstanding capital in the baseline scenario.

Equivalent Carbon Tax From the above analysis we observe that QE has a much milder effect on key climate variables than carbon taxation. We can thus conclude that relative to carbon taxation green QE is a less efficient instrument to mitigate the adverse societal effects of climate change. To look at this finding from a different perspective we next compute the carbon tax it would take to achieve the same effect as for the green QE policy. The corresponding carbon tax schedule is introduced in 2021 at some time constant carbon tax rate. The resulting equivalent carbon tax required to achieve in 2100 the same global temperature reduction as for the green QE policy in levels is only 11.06 USD per ton of carbon (22% of the tax in our baseline scenario of 50 USD per ton of carbon), corresponding to about 3 USD per ton of CO2.

Joint Policies A closely related question is whether green QE can be used complementary to carbon taxes. We therefore next consider both instruments jointly with results displayed in Figure 9. This shows that green QE has an additional climate change mitigating effect when it comes on top of a carbon tax policy, see panel (a) of figure 9, and thus green QE complements the green fiscal policy. Yet, the model results do not support a positive interaction of both policy instruments, see panel (b) of Figure 9. Thus, the climate change mitigating impact of carbon taxes would not be magnified when green QE is simultaneously at work. On the one hand, green QE alone leads to a reduction of the global temperature by 0.039 degrees Celsius. On the other hand, the joint effect of green QE and a carbon tax relative to a scenario where carbon taxation is used in isolation implies a global temperature reduction of 0.037 degrees. The reason for the negative interaction effect is the substitution of input factors due to changes in the costs structure in production. Specifically, on the one hand, the carbon tax increases the cost of dirty energy, leading dirty firms to partially substitute energy with labour and capital. On the other hand, green QE by increasing the cost of dirty capital leads to a partial substitution of capital with labour and energy. In combination these effects partially offset each other.

Figure 9: Temperature Reduction with Both Policy Instruments



(a) Temperature (b) Temperature reduction

Notes: Policy reforms: Temperature compared to pre-industrial in degrees of Celsius in panel (a) for baseline, carbon tax, QE and carbon tax plus QE; panel (b) shows the temperature reduction from baseline to QE and from the carbon tax to the carbon tax plus QE.

4.3 Sensitivity Analyses

We examine the sensitivity of our main findings with respect to some key model parameters and assumptions of the policy analyses. First, rather than assuming that the stock of private assets held by the monetary authority is constant in *relative* amounts as in our main analysis, we assume in the sensitivity exercise that it is constant in *absolute* amounts so that over time the relative size of private assets held by the monetary authority convergences to zero. As shown in column 2 of table 4 (scenario "Flat QE"), the carbon tax is then substantially more effective than green QE (the relative effectiveness $\frac{\Delta T\text{-}\tau}{\Delta T\text{-}QE}$ increases to 15.2, compared to 4.3 in the baseline calibration).

Second, following the notion of market neutrality we assume in our main analysis that the private asset holdings of the monetary authority in clean and dirty capital are proportional to the market shares. However, Papoutsi, Piazzesi, and Schneider (2021) show that the corporate bond portfolio of the European Central Bank has a carbon bias, i.e. sectors associated with

Table 4: Sensitivity Analyses

	Baseline	Flat QE	CO2 Bias	Pos. Corr.	Low SE	WAPR	CO2 Re.
T in 2100	3.505	3.503	3.505	3.408	3.539	3.190	2.566
ΔT-τ	-0.167	-0.167	-0.167	-0.170	-0.155	-0.149	-0.107
ΔT-QE	-0.039	-0.011	-0.056	-0.032	-0.005	-0.036	-0.029
ΔT-$\tau/\Delta T$-QE	4.3	15.2	3	5.3	31	4.1	3.7

Notes: Different calibrations for sensitivity analyses. "Flat QE": Size of monetary authority's balance sheet held constant over time. "CO2 Bias": Size of dirty assets on monetary authority's balance sheet 43% larger, i.e. $K^m_{0s=di} = 12770$. "Pos. Corr.": Correlation between clean and dirty returns set to $\rho^{\zeta}_{cl,di} = 0.4$. "Low SE": Low energy elasticity $\eta_{\frac{E_{ts=di}}{E_{ts=cl}}, \frac{p^e_{ts=di}}{p^e_{ts=cl}}} \approx 1$ so that intermediate goods elasticity is $\varepsilon = 2.25$. "WAPR": time varying working age population ratio ω_t. "CO2 Re.": strong CO2 reduction in baseline such that share of CO2 in GDP decreases at -1.5% annually. $\frac{\Delta T\text{-}\tau}{\Delta T\text{-}QE}$: relative effectiveness of the carbon tax.

higher emissions have a larger weight. Column 3 of table 4 (scenario "CO2 Bias") reports results where we replicate the carbon bias detailed in Papoutsi, Piazzesi, and Schneider (2021) by increasing the monetary authority's holdings of dirty capital by 43%.[19] The shift in the monetary authority's portfolio towards clean capital is thus larger in absolute size, giving green QE more power. The additional temperature reduction equals about 43%, and is thus approximately equal to the additional size of dirty capital held by the central bank relative to our baseline scenario. Consequently, the relative effectiveness of the carbon tax decreases to 3.

The two preceding sensitivity analyses (constant absolute size of the monetary authority's balance sheet and carbon bias of its assets holdings) illustrate the importance of the size of the dirty assets held by the monetary authority. The more dirty assets it holds, the more effective is green QE.

Third, as discussed in section 4.2 (see also figure 8) the impact of green QE hinges on the imperfect correlation between the returns in the clean and dirty intermediate production sectors. In our main analysis we assumed a zero correlation. Column 4 of table 4 reports results with a positive correlation of $\rho^{\zeta}_{cl,di} = 0.4$ between the shocks return (scenario "Pos. Corr."), which is based on the empirical findings in Reboredo, Ugolini, and Aiube (2020).[20] Central bank intervention through green QE thus triggers a stronger reaction by private investors and we observe a larger crowding out of private capital. This lowers the efficacy of green QE compared

[19] We thank Melina Papoutsi for sharing the data on the aggregate sectoral shares of the ECB's portfolio holdings.

[20] Reboredo, Ugolini, and Aiube (2020) report in table 1 correlations between indices of green (corporate and government) bonds and investment grade corporate bonds of 0.89 and 0.91 in the EU and US markets, respectively. Correlations between indices of green bonds and (large and mid-cap) equities are at -0.10 and -0.26 in the EU and US markets, respectively. An equal weighted average of these correlations stands at 0.36.

to a carbon tax. However, the effect is not that large; the relative effectiveness factor increases to 5.3.

Fourth, a key parameter in our model is the intermediate goods elasticity ϵ. For our main analysis we determine this parameter such that the resulting energy elasticity is $\eta = 2$, corresponding to empirical estimates ranging from 2 to 3. Column 5 of Table 4 (scenario "Low SE") reports results for an energy elasticity of about one, $\eta \approx 1$, which is consistent with an intermediate goods elasticity of $\epsilon = 2.25$, cf. Appendix B.1. The effects of green QE are then substantially smaller, so that the relative effectiveness is now at 31. To understand this finding note that green QE can only lower emissions by increasing the price of the dirty intermediate good. A lower intermediate goods elasticity ϵ implies that demand for the dirty intermediate good does not react much to its price changes. Since this is the only lever of green QE, it becomes much less effective. Assuming instead an in-between calibration with an intermediate goods elasticity of $\epsilon = 4$ (consistent with an energy elasticity of $\eta = 1.12$), which we also regard as reasonable given the long-run focus of our analysis, leads to a relative effectiveness of 14.4.

Fifth, we feed into the model a time varying working age population ratio as described in our calibration section 3 (scenario "WAPR"). In the baseline scenario without any adjustment of policy instruments, a shrinking income per capita (because of decreasing productive labor force relative to total population) implies lower dirty emissions so that the global temperature increases by less until 2100. Consequently, also in the policy experiments both policy instruments are less potent in reducing emissions and thereby in reducing the trend increase of the global temperature so that also the relative effectiveness is not affected much.

Sixth, we follow Nordhaus (2017) and assume a faster rate of reduction of CO2 emissions of 1.5% annually rather than 0.5% (scenario "CO2 Re."). This leads to a stronger price increase of dirty relative to clean energy prices so that the gradual substitution towards clean intermediate goods is faster in the baseline analysis and the global temperature thus increases by less. Both instruments turn out to be less potent, while the effectiveness of the carbon tax decreases more strongly so that the relative effectiveness is reduced to 3.7.

5 Concluding Discussion

We develop and calibrate a two-sector (clean and dirty) integrated assessment model to study the roles of green quantitative easing (QE) and carbon taxation for mitigating global warming. Green QE is modelled through an exogenous portfolio reallocation by the monetary authority. As a key element of our model we assume imperfectly correlated risky returns in the two sectors of production, which we calibrate in accordance with empirical findings. Consequently, the assumed exogenous reallocation of capital by the monetary authority does not lead to a perfect crowding out of private capital employed for production in the green sector and therefore green QE has real effects.

We consider an ambitious green QE policy by assuming a complete reallocation of capital towards the clean sector instead of a proportional split between the clean and dirty sector. We compare the climate change mitigating effects of the green QE policy to those of a global carbon tax, which is introduced at a moderate tax of 50 USD per ton of carbon and grows exogenously such that the ad valorem tax stays constant. We find that the effects of the moderate global carbon tax on climate change mitigation are substantially larger—by a relative effectiveness factor of about 4.3—than what is achieved through green QE. Put differently, it would only require a carbon tax of initially 11 USD per ton of carbon (or equivalently of 3 USD per CO2) to achieve the same global temperature reduction with green quantitative easing. We show that our quantitative results crucially depend on two assumptions, first, a calibration of a high intermediate goods elasticity in final output and second, a constant share (instead of a constant level) of the capital stock held by the monetary authority.

We also find that pursuing a green QE policy on top of the introduction of the carbon tax leads to an additional climate change mitigation. Thus, while the effects of green QE are rather mild, they can make a positive contribution in a world where fiscal policy instruments are in place. However, we do not find positive interaction effects. In fact, green QE has a larger effect if used in isolation than in combination with a carbon tax.

We view our model with an explicit portfolio choice between assets as an important first step to incorporate a finance perspective into the evaluation of long-run climate change mitigating policies. On the policy side, we treat the amount of assets held by the monetary authority as given and assume that it grows with the size of the economy. We thus assume that in a persistent low interest rate environment—which is an endogenous outcome of our model—the monetary authority will repetitively resort to asset purchases, which we do not explicitly model. Among various other avenues, we leave for future research an extension of our model towards endogenous quantitative easing policies, which requires extending our model by adding aggregate shocks and an explicit role for (non-)conventional monetary policy. This would allow us to address the trade-off between on the one hand undoing QE policies during economic booms and on the other hand pursuing green QE to combat climate change.

Furthermore, financial frictions in the model reflecting that the clean sector potentially faces stronger credit frictions than the dirty sector may lead to larger effects of green QE, at least in the short-run. An additional—complementary—extension of our model would be to add directed technical change (cf., e.g. Acemoglu, Aghion, Bursztyn, and Hemous (2012) and Acemoglu, Akcigit, Hanley, and Kerr (2016)), which may increase the effectiveness of green QE if through the use of the instrument the initial setup costs of the green sector are mitigated and technical change is directed more strongly towards the clean sector of the economy. In such a set-up, it would also be interesting to study the impact of fiscal subsidies to the clean sector of the economy—financed from the proceeds of untilted asset purchases or other fiscal revenue sources—as a policy experiment, also given the real world governance problems of implementing green QE, which we sidestep in our analysis.

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A Analytical Derivations and Proofs

A.1 Intermediate Goods Demand

The final representative firm operates under perfect competition maximizing

$$\max_{\{Y_{ts}\}_{s\in\{cl,di\}}} \left\{ p_t Y_t - \sum_{s\in\{cl,di\}} p_{ts} Y_{ts} \right\}$$
$$= \max_{\{Y_{ts}\}_{s\in\{cl,di\}}} \left\{ p_t \cdot (1-D_t) \cdot \Upsilon_t \cdot \left(\sum_{s\in\{cl,di\}} \kappa_s Y_{ts}^{1-\frac{1}{\epsilon}} \right)^{\frac{1}{1-\frac{1}{\epsilon}}} - \sum_{s\in\{cl,di\}} p_{ts} Y_{ts} \right\}$$

which gives the price of intermediate good s as

$$\frac{p_{ts}}{p_t} = \kappa_s \left((1-D_t)\cdot \Upsilon_t\right)^{\epsilon-1} \left(\frac{Y_t}{Y_{ts}}\right)^{\epsilon}, \text{ for } s \in \{cl, di\}.$$

and thus the intermediate goods demand

$$Y_{ts} = \left(\frac{\kappa_s}{\frac{p_{ts}}{p_t}}\right)^{\epsilon} \left((1-D_t)\cdot \Upsilon_t\right)^{\epsilon-1} Y_t, \text{ for } s \in \{cl, di\}.$$

and the price of the final good as

$$p_t = \frac{1}{(1-D_t)\Upsilon_t} \left(\sum_{s\in\{cl,di\}} \kappa_s^{\epsilon} p_{ts}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}.$$

A.2 The Shock Distribution

The distribution of $\boldsymbol{\zeta_{ti}} = (\zeta_{ticl}, \zeta_{tidi})'$, $\boldsymbol{\Psi}$ is defined implicitly via the distribution of the gross returns on capital. The gross return on capital is assumed to follow a multivariate log-normal distribution with

$$\begin{pmatrix} \log(1+r_{ticl}) \\ \log(1+r_{tidi}) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \log(1+\mathbb{E}r_{tcl}) - \frac{\left(\sigma_{cl}^{\zeta}\right)^2}{2} \\ \log(1+\mathbb{E}r_{tdi}) - \frac{\left(\sigma_{di}^{\zeta}\right)^2}{2} \end{pmatrix}, \boldsymbol{\Sigma} \right)$$

where

$$\mathbb{E}r_{ts} = \int \mathbb{E}r_{tis} di = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{\alpha}{(1-\alpha)} \cdot p_{ts} \left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s$$

is the average marginal profit from an additional unit of capital in sector s. Given this distributional assumption we get $Var\left(r_{tis}\right) = (1+\mathbb{E}r_{ts})^2 \cdot \left(\exp\left(\sigma_s^{\zeta}\right)^2 - 1\right)$ and $Cov\left(r_{tis=cl}, r_{tis=di}\right) = (1+\mathbb{E}r_{ts=cl})(1+\mathbb{E}r_{ts=di}) \cdot \left(\exp\left(\rho_{cl,di}^{\zeta}\sigma_{cl}^{\zeta}\sigma_{di}^{\zeta}\right) - 1\right)$. So that the correlation is $Corr\left(r_{tis=cl}, r_{tis=di}\right) = \frac{\exp\left(\rho_{cl,di}^{\zeta}\sigma_{cl}^{\zeta}\sigma_{di}^{\zeta}\right)}{\sqrt{\prod_{s\in\{cl,di\}}\left[\exp\left(\sigma_s^{\zeta}\right)^2 - 1\right]}}$. Using $\exp(x) \approx 1+x$, the correlation is $Corr\left(r_{tis=cl}, r_{tis=di}\right) \approx \rho_{cl,di}^{\zeta}$.

A.3 Proof of Proposition 1

The proof is by guess and verify using the method of undetermined coefficients. We start by showing linearity of policy functions in total wealth, which differs across all i through optimal portfolio shares $\hat{\vartheta}_{tis}^*$. In a second step we show that $\hat{\vartheta}_{tis}^* = \hat{\vartheta}_{ts}^*$ for all i and thereby that $m_{tis}^* = m_{ts}^*$ for all i.

Proof. 1. Claims: The consumption policy function in each period t for household i is

$$c(w_{ti}) = m_{ti} w_{ti}$$

for some m_{ti} and the associated value function is

$$U(w_{ti}) = \varrho_{ti} w_{ti}$$

for some ϱ_{ti}.

2. Induction step: In any period t we get under the induction claim, writing $U(w_{ti}) = \varrho_{ti} w_{ti}$

$$U(w_{ti}) = \max_{c_{ti}, \hat{\vartheta}_{ti}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} w_{t+1i} \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}} \right)^{\frac{1}{1-\upsilon}} \right\}.$$

Using the resource constraint we get

$$\begin{aligned} & U_{ti}(w_{ti}) \\ &= \max_{c_{ti}, \hat{\vartheta}_{ti}, w_{t+1i}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right) R_{t+1i}^p \left(\{\hat{\vartheta}_{tis}^*\}_{s\in\{cl,di\}} \right) \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}} \right)^{\frac{1}{1-\upsilon}} \right\} \\ &= \max_{c_{ti}, \hat{\vartheta}_{ti}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right)^{1-\upsilon} \left(\mathbb{E}_t \left[\left(\varrho_{t+1i} R_{t+1i}^p \left(\{\hat{\vartheta}_{tis}^*\}_{s\in\{cl,di\}} \right) \right)^{1-\theta} \right] \right)^{\frac{1-\upsilon}{1-\theta}} \right)^{\frac{1}{1-\upsilon}} \right\} \\ &= \max_{c_{ti}, \hat{\vartheta}_{ti}} \left\{ \left(c_{ti}^{1-\upsilon} + \beta \left(w_{ti} - (1+\tau_t^c) p_t c_{ti} \right)^{1-\upsilon} \Lambda_{t+1i} \right)^{\frac{1}{1-\upsilon}} \right\} \end{aligned}$$

where $\Lambda_{t+1i} \equiv \left(\mathbb{E}_t\left[\left(\varrho_{t+1i}R^p_{t+1}\left(\{\hat{\vartheta}^*_{tis}\}_{s\in\{cl,di\}}\right)\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}$.

Take the first-order condition w.r.t c_{ti} to obtain

$$\begin{aligned} c_{ti}^{-\upsilon} &= \beta\left(w_{ti}-(1+\tau^c_t)p_t c_{ti}\right)^{-\upsilon}(1+\tau^c_t)p_t\Lambda_{t+1i} \\ \Leftrightarrow \qquad c_{ti} &= \left(w_{ti}-(1+\tau^c_t)p_t c_{ti}\right)\Xi_{t+1i} \end{aligned}$$

for

$$\Xi_{t+1i} = \left(\beta(1+\tau^c_t)p_t\Lambda_{t+1i}\right)^{-\frac{1}{\upsilon}},$$

and thus

$$c_{ti} = m_{ti}w_{ti}$$

where

$$m_{ti} = \frac{\Xi_{t+1i}}{1+(1+\tau^c_t)p_t\Xi_{t+1i}}$$

Use this back in the objective to get

$$
\begin{aligned}
&U(w_{ti})\\
&=\left((m_{ti}w_{ti})^{1-\upsilon}+\beta\left(\mathbb{E}_t\left[\left(\varrho_{t+1i}(1-(1+\tau_t^c)p_t m_{ti})w_{ti}R^p_{t+1i}\left(\{\hat{\vartheta}^*_{tis}\}_{s\in\{cl,di\}}\right)\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}\right)^{\frac{1}{1-\upsilon}}\\
&=\left((m_{ti})^{1-\upsilon}+\beta(1-(1+\tau_t^c)p_t m_{ti})^{1-\upsilon}\Lambda_{t+1i}\right)^{\frac{1}{1-\upsilon}}w_t\\
&=\left(\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-\upsilon}+\frac{\Xi_{t+1i}^{-\upsilon}}{(1+\tau_t^c)p_t}\left(\frac{1}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-\upsilon}\right)^{\frac{1}{1-\upsilon}}w_t\\
&=\left(\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-\upsilon}\left(1+\frac{1}{(1+\tau_t^c)p_t\Xi_{t+1i}}\right)\right)^{\frac{1}{1-\upsilon}}w_t\\
&=\left(\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-\upsilon}\frac{1+(1+\tau_t^c)p_t\Xi_{t+1i}}{(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{\frac{1}{1-\upsilon}}w_t\\
&=\left(\frac{1}{(1+\tau_t^c)p_t}\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{1-\upsilon}\frac{1+(1+\tau_t^c)p_t\Xi_{t+1i}}{\Xi_{t+1i}}\right)^{\frac{1}{1-\upsilon}}w_t\\
&=\left(\frac{1}{(1+\tau_t^c)p_t}\left(\frac{\Xi_{t+1i}}{1+(1+\tau_t^c)p_t\Xi_{t+1i}}\right)^{-\upsilon}\right)^{\frac{1}{1-\upsilon}}w_t\\
&=\left(\frac{1}{(1+\tau_t^c)p_t}m_{ti}^{-\upsilon}\right)^{\frac{1}{1-\upsilon}}w_t.
\end{aligned}
$$

We therefore get

$$
\varrho_{ti}=\left(\frac{1}{(1+\tau_t^c)p_t}m_{ti}^{-\upsilon}\right)^{\frac{1}{1-\upsilon}},
$$

which is non-stochastic, and we can accordingly rewrite Λ_{t+1i} as

$$
\Lambda_{t+1i}\equiv\frac{1}{(1+\tau_{t+1}^c)p_{t+1}}m_{t+1i}^{-\upsilon}\left(\mathbb{E}_t\left[R^p_{t+1}\left(\{\hat{\vartheta}^*_{tis}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}
$$

and thus

$$
\begin{aligned}
\Xi_{t+1i}&=\left(\beta\frac{(1+\tau_t^c)p_t}{(1+\tau_{t+1}^c)p_{t+1}}\left(\mathbb{E}_t\left[R^p_{t+1}\left(\{\hat{\vartheta}^*_{tis}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]\right)^{\frac{1-\upsilon}{1-\theta}}\right)^{-\frac{1}{\upsilon}}m_{t+1i}\\
&=\Theta\left(p_t,p_{t+1},\tau_t^c,\tau_{t+1}^c,R^p_{t+1}\left(\{\hat{\vartheta}^*_{tis}\}_{s\in\{cl,di\}}\right),\beta,\upsilon,\theta,\Psi\right)m_{t+1i}
\end{aligned}
$$

and thus

$$m_{ti} = \frac{\Theta\left(p_t, p_{t+1}, \tau_t^c, \tau_{t+1}^c, R_{t+1}^p\left(\{\hat{\vartheta}_{tis}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}}{1 + (1+\tau_t^c)\Theta\left(p_t, p_{t+1}, \tau_t^c, \tau_{t+1}^c, R_{t+1}^p\left(\{\hat{\vartheta}_{tis}\}_{s\in\{cl,di\}}\right), \beta, \upsilon, \theta, \Psi\right) m_{t+1i}}.$$

3. From the FOC w.r.t. $\hat{\vartheta}_{tis}$ we get

$$\frac{\partial \mathbb{E}_t\left[R_{t+1}^p\left(\{\hat{\vartheta}_{tis}\}_{s\in\{cl,di\}}\right)^{1-\theta}\right]}{\partial \hat{\vartheta}_{tis}} = 0$$

and we thus get $\hat{\vartheta}_{tis}^* = \hat{\vartheta}_{ts}^*$ for all i, which implies that $m_{tis} = m_{ts}$ for all i. Assuming that $R_{t+1}^p\left(\{\hat{\vartheta}_{tis}\}_{s\in\{cl,di\}}\right)$ is distributed as log-normal we get as an approximation applying results in Campbell and Viceira (2002) that under the assumed cross-sectional independence of the returns

$$\hat{\vartheta}_{ts}^* \approx \frac{\ln(1+\mathbb{E}\left[r_{t+1s}\right]) - \ln(1+r_{t+1}^f)}{\theta \cdot Var(\ln(1+r_{t+1s}))},$$

□

B Calibration Appendix

B.1 Intermediate Goods Elasticity ϵ and Energy Elasticity η

Start from equation (8) and integrate out across all i to get using $\mathbb{E}[\zeta_{tis}] = 0$ that

$$\mathbb{E}r_{ts} = \Gamma(\psi_s, \alpha, \gamma) \cdot \frac{\alpha}{(1-\alpha)} \cdot p_{ts}\left(\frac{r_t^l}{p_{ts}}\right)^{-\frac{1-\alpha}{\alpha}} \cdot \left(\frac{p_{ts}^e}{p_{ts}}\right)^{-\frac{1-\gamma}{\alpha\gamma}} - \delta_s$$

from which we get

$$p_{ts} = \left(\frac{1-\alpha}{\alpha\Gamma(\psi_s, \alpha, \gamma)}\right)^{\alpha\gamma} \cdot (\mathbb{E}r_{ts} + \delta_s)^{\alpha\gamma} r_t^{l\,(1-\alpha)\gamma} p_{ts}^{e\;1-\gamma} \tag{23}$$

and thus

$$\frac{p_{ts=cl}}{p_{ts=di}} = \left(\frac{\mathbb{E}r_{ts=cl} + \delta_{s=cl}}{\mathbb{E}r_{ts=di} + \delta_{s=di}}\right)^{\alpha\gamma} \left(\frac{p_{ts=cl}^e}{p_{ts=di}^e}\right)^{1-\gamma}. \tag{24}$$

From the demand for intermediate goods by the final firm (2) we get the intermediate goods demand ratio

$$\frac{Y_{ts=di}}{Y_{ts=cl}} = \left(\frac{\kappa_{s=di} p_{ts=cl}}{\kappa_{s=cl} p_{ts=di}}\right)^{\epsilon}. \tag{25}$$

Using (24) in the above we obtain

$$\frac{Y_{ts=di}}{Y_{ts=cl}} = \Xi\left(\{\mathbb{E}r_{ts}, \delta_s, \kappa_s\}_{s\in\{cl,di\}}\right)\left(\frac{p^e_{ts=cl}}{p^e_{ts=di}}\right)^{\epsilon(1-\gamma)} \tag{26}$$

for some time varying $\Xi\left(\{\mathbb{E}r_{ts}, \delta_s, \kappa_s\}_{s\in\{cl,di\}}\right)$.

Next, on the supply side for intermediate goods, we get from (22e) and (22f)

$$Y_{ts} = \frac{1}{1-\gamma}\frac{p^e_{ts}}{p_{ts}}E_{ts}$$

and using (23) in the above we obtain

$$\frac{Y_{ts=di}}{Y_{ts=cl}} = \Lambda\left(\alpha, \gamma, \{\mathbb{E}r_{ts}, \delta_s, \Gamma(\psi_s, \alpha, \gamma)\}_{s\in\{cl,di\}}, r^l_t\right)\frac{p^e_{ts=cl}}{p^e_{ts=di}}^{-\gamma}\frac{E_{ts=di}}{E_{ts=cl}} \tag{27}$$

for some time varying $\Lambda\left(\alpha, \gamma, \{\mathbb{E}r_{ts}, \delta_s, \Gamma(\psi_s, \alpha, \gamma)\}_{s\in\{cl,di\}}, r^l_t\right)$.

Combining the intermediate goods demand and supply side, i.e., equations (26) and (27), we thus get

$$\frac{E_{ts=di}}{E_{ts=cl}} = \frac{\Lambda\left(\alpha, \gamma, \{\mathbb{E}r_{ts}, \delta_s, \Gamma(\psi_s, \alpha, \gamma)\}_{s\in\{cl,di\}}, r^l_t\right)}{\Xi\left(\{\mathbb{E}r_{ts}, \delta_s, \kappa_s\}_{s\in\{cl,di\}}\right)}\left(\frac{p^e_{ts=cl}}{p^e_{ts=di}}\right)^{\epsilon(1-\gamma)+\gamma}. \tag{28}$$

Holding constant the (expected) returns $\{\mathbb{E}r_{ts}\}_{s\in\{cl,di\}}, r^l_t$ we thus find that the energy demand elasticity is given by

$$\eta_{\frac{E_{ts=di}}{E_{ts=cl}}, \frac{p^e_{ts=di}}{p^e_{ts=cl}}} = \epsilon\cdot(1-\gamma)+\gamma.$$

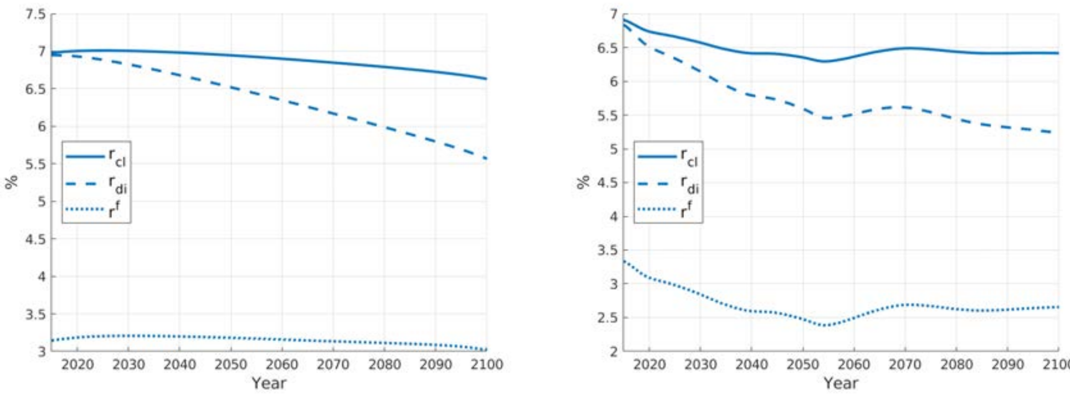
Observe that the energy elasticity is thus bounded from below by γ if the final output production features perfect complements ($\epsilon = 0$). Also note that it is equal to 1 if we assume Cobb-Douglas production of final output ($\epsilon = 1$).

C Additional Results

Figure 10 shows the average rates of return in the two intermediate goods production sectors, $r_{ts}, s \in \{cl, di\}$ as well as the risk-free rate r_t^f in the baseline economy of the main setup with a fixed working age population ratio in the left panel and that of the baseline economy of the sensitivity analysis with a time-varying working working age population ratio. Both panels display decreasing risky asset returns and an eventually decreasing risk-free rate r_t^f. The risk-free rate is weakly hump shaped for the case of the main setup, while with a time-varying working age population ratio it decreases from the beginning until 2055, from where on it stays roughly constant. The decrease in rates in the left panel is a consequence of increasing climate change damages. The decrease in the right panel is stronger as on top of climate damages also the demographic change suppresses rates.

While the level of our calibrated model risk-free rate exceeds current market interest rates (for numerical instability reasons), we argue on the basis of this finding that it is appropriate to assume that the world economy will continue to be in a low interest rate environment over the projection period. In a world with cyclical fluctuations, this will likely lead to repeated application of non-conventional monetary policy through quantitative easing so that our assumption of a constant share of total assets held by the central bank is a reasonable approximation.

Figure 10: Baseline: Climate Variables



(a) Fixed WAPR - Main setup (b) Time-varying WAPR - Sensitivity setup

Notes: Financial returns in the main setup with a constant working age population ratio in panel (a) and with a time-varying working age popoulation ratio as in sensitivity analysis WAPR in panel (b). Each panel shows the risky returns $r_{ts}, s \in \{cl, di\}$ and the risk-free return r_t^f.

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