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by Miguel A. León-Ledesma[2], Peter McAdam[3] and Alpo Willman[4]

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CONTENTS

Abstract

We examine the two-level nested Constant Elasticity of Substitution production function where both capital and labor are disaggregated in two classes. We propose a normalized system estimation method to retrieve estimates of the inter- and intra-class elasticities of substitution and factor-augmenting technical progress coefficients. The system is estimated for US data for the 1963-2006 period. Our findings reveal that skilled and unskilled labor classes are gross substitutes, capital structures and equipment are gross complements, and aggregate capital and aggregate labor are gross complements with an elasticity of substitution close to 0.5. We discuss the implications of our findings and methodology for the analysis of the causes of the increase in the skill premium and, by implication, inequality in a growing economy.
JEL Classification: E25, J23, J24, O40.
Keywords: Two-level CES production function, Factor-Augmenting Technical Progress, Factor Substitution, Aggregation, Skill-premium.

Non-technical Summary

The workhorse production function in almost all theoretical and empirical models in growth theory and, more generally, in micro- and macroeconomics has been the single-level production function. The single-level production function relates aggregate output to, commonly, two aggregate inputs: labor and capital (or land). That is to say, input types are aggregated into a single index.

One particular problem with this approach is that it obscures the interactions between and within different factor categories. Heterogeneity and aggregation problems have been a concern in economic theory for long. But implementation within a context amenable to empirical analysis implies that, in practical terms, researchers can only use a limited number of heterogeneous components in factor inputs. For example, there are high and low-skill types of labor and different strata of capital such as equipments, software, and buildings and infrastructure. All of these within-factor categories may be expected to contain highly specific characteristics with important consequences for economic outcomes. They may display different depreciation rates, different rates of productivity growth, different cyclical co-movements, different cross elasticities of substitution etc. By aggregating them, we may thus miss these important aspects. One such aspect of interest is the effect of capital accumulation and technical progress on income distribution. Much of the recent changes in income distribution in the world can be associated with changes in the return to different labor input skills leading to wage inequality.

An important and, in practice implementable, departure from the aggregative framework was made in the seminal contributions of Kazuo Sato and Zvi Griliches. Sato (1967) generalized the CES production function by nesting the CES at two levels and augmenting the list of possible inputs.

A popular focus for work on the two-level CES function is on explanations of the increase in the skill premium observed in western economies during the last three decades. Such phenomena clearly has implications for inequality, long-run growth and labor-market policies.

One approach, initiated by Griliches (1969), showed that – for US manufacturing – capital and skilled labor were more complementary than capital and unskilled labor. This spawned a considerable literature examining the so-called "capital-skill complementarity" hypothesis, e.g., Greenwood et al. (1997), Krussel et al. (2000), Duffy et al. (2004). The hypothesis gained particular currency given the sharp decline in the constant-quality relative price of equipment, e.g., Gordon (1990), particularly for information and communication technologies. This decline natu-

rally lead to an uptake in usage of such capital. Given complementarity between capital and skilled labor, the faster usage of such capital increased the relative demand for skilled labor and – despite the apparent increase in the supply of such labor – the skill or wage premium relative to unskilled labor increased in a dramatic and persistent manner (see Acemoglu (2009) for a textbook discussion). On the other hand, authors such as Katz and Murphy (1992), Acemoglu (2002b), and Autor et al. (2008), claimed that the skill premium can be attributed to technical change that was biased in favor of skilled workers. Given that skilled and unskilled workers are gross substitutes, an increase in skilled labor efficiency led to an increase in the relative wages (and factor shares) of skilled workers. Both approaches rely on particular nestings and estimated values for the elasticities of substitution between different categories of factors of production and their associated factor-biased technical progress parameters.

We make three contributions to the empirical literature on two-level CES production functions. First, we re-examine the Sato exercises using disaggregation in *both* capital and labor factors, rather than just in capital. Second, we estimate several specifications of the two-level CES function within a "normalized" system approach following Klump et al. (2007) and León-Ledesma et al. (2010a). This has several advantages over previous approaches to recover deep supply side parameters as it estimates jointly the production function and first order conditions accounting for cross-equation restrictions. Finally, our specification pays particular attention to the role of factor augmenting technical progress, also allowing us to be informative about the role of technical change in driving factor prices. We illustrate our results paying special attention to the evolution of the US skill premium.

1 Introduction

The workhorse production function in almost all theoretical and empirical models in growth theory and, more generally, in micro- and macroeconomics has been the single-level production function – whether Constant Elasticity of Substitution (CES) or Cobb Douglas – starting from Solow (1956, 1957) and Arrow et al. (1961). The single-level production function relates aggregate output to, commonly, two aggregate inputs: labor and capital (or land). That is to say, input types are aggregated into a single index.

One particular problem with this approach is that it obscures the interactions between and within different factor categories. Heterogeneity and aggregation problems have been a concern in economic theory for long. But implementation within a context amenable to empirical analysis implies that, in practical terms, researchers can only use a limited number of heterogeneous components in factor inputs. For example, there are high and low-skill types of labor and different strata of capital such as equipments, software, and buildings and infrastructure. All of these within-factor categories may be expected to contain highly specific characteristics with important consequences for economic outcomes. They may display different depreciation rates, different rates of productivity growth, different cyclical co-movements, different cross elasticities of substitution etc. By aggregating them, we may thus miss these important aspects. One such aspect of interest is the effect of capital accumulation and technical progress on income distribution. Much of the recent changes in income distribution in the world can be associated with changes in the return to different labor input skills leading to wage inequality.

An important and, in practice implementable, departure from the aggregative framework was made in the seminal contributions of Kazuo Sato and Zvi Griliches. Sato (1967) generalized the CES production function by nesting the CES at two levels and augmenting the list of possible inputs. Abstracting from technical progress (although this will become important later), this typical two-level CES production function can be written as,

$$Y = \left[\beta\{\alpha_1 X_1^{\chi_1} + (1-\alpha_1) X_2^{\chi_1}\}^{\chi/\chi_1} + (1-\beta)\left\{\alpha_2 Z_1^{\chi_2} + (1-\alpha_2) Z_2^{\chi_2}\right\}^{\chi/\chi_2}\right]^{1/\chi} \quad (1)$$

where X_i and Z_i, are different aggregate factor inputs (i.e. labor and capital), and $i = 1, 2$ are different factor categories. Defining $\gamma = 1/\chi$ this can be written more compactly as:

$$Y = \left[\beta CES_1^{1/\gamma} + (1-\beta)\, CES_2^{1/\gamma}\right]^{\gamma} \tag{2}$$

This nested two-level CES specification allows for different substitution possibilities between factors of production and categories within them. Thus, for instance, CES_1 may be Leontief, while CES_2 may be of the Cobb-Douglas type. The impact of factor accumulation and technical progress will crucially depend on the relative values of the elasticities of substitution determined by χ, χ_1 and χ_2.

A popular focus for work on the two-level CES function is on explanations of the increase in the skill premium observed in western economies during the last three decades as reported in Acemoglu (2002b).[1] Such phenomena clearly has implications for inequality, long-run growth and labor-market policies (e.g., Piva et al. (2005)).

One approach, initiated by Griliches (1969), showed that – for US manufacturing – capital and skilled labor were more complementary than capital and unskilled labor. This spawned a considerable literature examining the so-called "capital-skill complementarity" hypothesis, e.g., Greenwood et al. (1997), Krussel et al. (2000), Duffy et al. (2004). The hypothesis gained particular currency given the sharp decline in the constant-quality relative price of equipment, e.g., Gordon (1990), particularly for information and communication technologies. This decline naturally lead to an uptake in usage of such capital. Given complementarity between capital and skilled labor, the faster usage of such capital increased the relative demand for skilled labor and – despite the apparent increase in the supply of such labor – the skill or wage premium relative to unskilled labor increased in a dramatic and persistent manner (see Acemoglu (2009) for a textbook discussion). On the other hand, authors such as Katz and Murphy (1992), Acemoglu (2002b), and Autor et al. (2008), claimed that the skill premium can be attributed to technical change that was biased in favor of skilled workers. Given that skilled and unskilled workers are gross substitutes, an increase in skilled labor efficiency led to an increase in the relative wages (and factor shares) of skilled workers. Both approaches rely on particular nestings and estimated values for the elasticities of substitution between different categories of factors of production and their associated factor-biased technical progress parameters.

In this paper, we make three contributions to the empirical literature on two-level CES production functions. First, we re-examine the Sato exercises using disaggregation in *both* capital and labor factors, rather than just in capital. Second,

[1] In our data section, we graph some of these trends.

we estimate several specifications of the two-level CES function within a "normalized" system approach following Klump et al. (2007) and León-Ledesma et al. (2010a). This has several advantages over previous approaches to recover deep supply side parameters as it estimates jointly the production function and first order conditions accounting for cross-equation restrictions. Finally, our specification pays particular attention to the role of factor augmenting technical progress, also allowing us to be informative about the role of technical change in driving factor prices. We illustrate our results paying special attention to the evolution of the US skill premium.

The paper is organized as follows. In section 2, we describe the two-level production function form and the first order conditions that constitute the "normalized" supply system. In section (3), we discuss the underlying US macro data, its properties and transformations. In section (4) we show empirical estimates of a variety of production-technology system estimates based on pairwise factor combinations. Finally, we conclude.

2 The normalized two-step 4-factor CES production function

Assume that production Y is defined by the two-level CES production function with four separate inputs V_i $(i = 1, .., 4)$ with factor augmenting technical progress The inner part of the production function (in the normalized form) and the corresponding income identity are therefore defined as:[2]

$$X_{1,t} = \left[(1-\beta) \left(e^{\gamma_1 \widetilde{t}} \frac{V_{1,t}}{V_{1,0}} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \widetilde{t}} \frac{V_{2,t}}{V_{2,0}} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{3}$$

$$X_{2,t} = \left[(1-\pi) \left(e^{\gamma_3 \widetilde{t}} \frac{V_{3,t}}{V_{3,0}} \right)^{\frac{\zeta-1}{\zeta}} + \pi \left(e^{\gamma_3 \widetilde{t}} \frac{V_{4,t}}{V_{4,0}} \right)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1}} \tag{4}$$

where $\widetilde{t} = (t - t_0)$, η is the elasticity of substitution between V_1 and V_2 and ζ is the elasticity of substitution between V_3 and V_4. The term $e^{\gamma_i \widetilde{t}}$ denotes linear

[2]Normalization essentially implies representing the production and technology side of the model (i.e., production function and factor demands) in consistent indexed number form. Although a necessary expression of the CES function, it turns out also to be useful for econometric identification and for comparative static exercises. Our Appendix provides a re-fresher.

technical progress that increases the efficiency of factor i with constant growth rate γ_i (factor augmenting technical progress). Subscripts zero indicate values at the point of normalization. It is straightforward to see that (3)-(4) imply that $X_{1,0} = X_{2,0} = 1$.

Denoting factor prices by w_i the normalization implies that the distribution parameters β and π in (3)-(4) are defined by factor incomes at the normalization point as follows,

$$\beta = \frac{w_{2,0} \cdot V_{2,0}}{w_{1,0} \cdot V_{1,0} + w_{2,0} \cdot V_{3,0}} \tag{5}$$

$$\pi = \frac{w_{4,0} \cdot V_{4,0}}{w_{3,0} \cdot V_{3,0} + w_{4,0} \cdot V_{4,0}} \tag{6}$$

Now the two step-CES function for production Y is:

$$\begin{aligned} Y_t &= Y_0 \left[\alpha X_{1,t}^{\frac{\sigma-1}{\sigma}} + (1-\alpha) X_{2,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= Y_0 \left\{ \begin{array}{c} \alpha \left[(1-\beta) \left(e^{\gamma_1 \tilde{t}} \frac{V_{1,t}}{V_{1,0}} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \tilde{t}} \frac{V_{2,t}}{V_{2,0}} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1} \frac{\sigma-1}{\sigma}} + \\ (1-\alpha) \left[(1-\pi) \left(e^{\gamma_3 \tilde{t}} \frac{V_{3,t}}{V_{3,0}} \right)^{\frac{\zeta-1}{\zeta}} + \pi \left(e^{\gamma_4 \tilde{t}} \frac{V_{4,t}}{V_{4,0}} \right)^{\frac{\zeta-1}{\zeta}} \right]^{\frac{\zeta}{\zeta-1} \frac{\sigma-1}{\sigma}} \end{array} \right\}^{\frac{\sigma}{\sigma-1}} \end{aligned} \tag{7}$$

where σ is the elasticity of substitution between compound inputs X_1 and X_2 and the distribution parameter α is defined by factor incomes of the normalization point as follows,

$$\alpha = \frac{w_{1,0} \cdot V_{1,0} + w_{2,0} \cdot V_{2,0}}{w_{1,0} \cdot V_{1,0} + w_{2,0} \cdot V_{2,0} + w_{3,0} \cdot V_{3,0} + w_{4,0} \cdot V_{4,0}} \tag{8}$$

Assume a firm faces the demand function $Y_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\varepsilon} Y_t$. Profit maximization results in 4 first order conditions which, together with the (log) production function, results in the following 5-equation normalized supply side system:

$$\begin{aligned} \log w_{1,t} = \log \left[\frac{\alpha (1-\beta)}{(1+\mu)} \frac{Y_0}{V_{1,0}} \right] + \frac{(\eta - 1) \gamma_1}{\eta} \tilde{t} + \frac{1}{\sigma} \log \left(\frac{Y_t}{Y_0} \right) - \frac{1}{\eta} \log \left(\frac{V_{1,t}}{V_{1,0}} \right) \\ + \frac{\sigma - \eta}{\sigma (\eta - 1)} \log \left[(1-\beta) \left(e^{\gamma_1 \tilde{t}} \frac{V_{1,t}}{V_{1,0}} \right)^{\frac{\eta-1}{\eta}} + \beta \left(e^{\gamma_2 \tilde{t}} \frac{V_{2,t}}{V_{2,0}} \right)^{\frac{\eta-1}{\eta}} \right] \end{aligned} \tag{9}$$

$$\log w_{2,t} = \log\left[\frac{\alpha\beta}{(1+\mu)}\frac{Y_0}{V_{2,0}}\right] + \frac{(\eta-1)\gamma_2}{\eta}\tilde{t} + \frac{1}{\sigma}\log\left(\frac{Y_t}{Y_0}\right) - \frac{1}{\eta}\log\left(\frac{V_{2,t}}{V_{2,0}}\right)$$
$$+ \frac{\sigma-\eta}{\sigma(\eta-1)}\log\left[(1-\beta)\left(e^{\gamma_1\tilde{t}}\frac{V_{1,t}}{V_{1,0}}\right)^{\frac{\eta-1}{\eta}} + \beta\left(e^{\gamma_2\tilde{t}}\frac{V_{2,t}}{V_{2,0}}\right)^{\frac{\eta-1}{\eta}}\right] \quad (10)$$

$$\log w_{3,t} = \log\left[\frac{(1-\alpha)(1-\pi)}{(1+\mu)}\frac{Y_0}{V_{3,0}}\right] + \frac{(\zeta-1)\gamma_3}{\zeta}\tilde{t} + \frac{1}{\sigma}\log\left(\frac{Y_t}{Y_0}\right) - \frac{1}{\zeta}\log\left(\frac{V_{3,t}}{V_{3,0}}\right)$$
$$+ \frac{\sigma-\zeta}{\sigma(\zeta-1)}\log\left[(1-\pi)\left(e^{\gamma_3\tilde{t}}\frac{V_{3,t}}{V_{3,0}}\right)^{\frac{\zeta-1}{\zeta}} + \pi\left(e^{\gamma_4\tilde{t}}\frac{V_{4,t}}{V_{4,0}}\right)^{\frac{\zeta-1}{\zeta}}\right] \quad (11)$$

$$\log w_{4,t} = \log\left[\frac{(1-\alpha)\pi}{(1+\mu)}\frac{Y_0}{V_{4,0}}\right] + \frac{(\zeta-1)\gamma_4}{\zeta}\tilde{t} + \frac{1}{\sigma}\log\left(\frac{Y_t}{Y_0}\right) - \frac{1}{\zeta}\log\left(\frac{V_{4,t}}{V_{4,0}}\right)$$
$$+ \frac{\sigma-\zeta}{\sigma(\zeta-1)}\log\left[(1-\pi)\left(e^{\gamma_3\tilde{t}}\frac{V_{3,t}}{V_{3,0}}\right)^{\frac{\zeta-1}{\zeta}} + \pi\left(e^{\gamma_4\tilde{t}}\frac{V_{4,t}}{V_{4,0}}\right)^{\frac{\zeta-1}{\zeta}}\right] \quad (12)$$

$$\log(Y_t/Y_0) = \frac{\sigma}{\sigma-1}\log\left\{\begin{array}{c} \alpha\left[(1-\beta)\left(e^{\gamma_1\tilde{t}}\frac{V_{1,t}}{V_{1,0}}\right)^{\frac{\eta-1}{\eta}} + \beta\left(e^{\gamma_2\tilde{t}}\frac{V_{2,t}}{V_{2,0}}\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}\frac{\sigma-1}{\sigma}} \\ +(1-\alpha)\left[(1-\pi)\left(e^{\gamma_3\tilde{t}}\frac{V_{3,t}}{V_{3,0}}\right)^{\frac{\zeta-1}{\zeta}} + \pi\left(e^{\gamma_4\tilde{t}}\frac{V_{4,t}}{V_{4,0}}\right)^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}\frac{\sigma-1}{\sigma}} \end{array}\right. \quad (13)$$

where $1+\mu = \frac{\varepsilon}{\varepsilon-1}$ represents the equilibrium mark-up and the income identity of the firm can be written as,

$$Y_t = (1+\mu)(w_{1,t}\cdot V_{1,t} + w_{2,t}\cdot V_{2,t} + w_{3,t}\cdot V_{3,t} + w_{4,t}V_{4,t}) \quad (14)$$

Estimation of equations (9) to (13) yield the parameters: σ, ζ, η, and the various γ's. Parameters α, π and β are imposed reflecting their values in the data. This is because using the normalized form, these parameters can be directly interpreted as income shares at the point of normalization. This is an added advantage of normalization, as it reduces the parameter space to estimate.

In the next sections, we estimate the system for the US economy distinguishing between skilled and unskilled workers on the one hand, and structures and equipment capital on the other hand. We use a nesting such that both categories of capital and labor appear always within the same nested CES, implying the same substitution elasticity across different kinds of capital and different kinds of labor. These are, of course, not the only possible ways of nesting the two-level CES function. Our aim here is to show how the estimated system can track output and factor prices (notably the skill premium). We leave for future research an in-depth investigation into different kinds of CES nestings that could have consequences for the theoretical models of the skill premium discussed in the introduction. Here we focus mostly on the consequences of aggregating categories of labor and capital for production function estimation under different assumptions about technical progress.

3 Data

Annual data were obtained from various sources for the US economy for the 1963-2006 period. The annual frequency is determined by the availability of skilled/unskilled hours and wages. Data for output, capital, total employment, and labor compensation are for the US private non-residential sector. Most of the data come from NIPA series available from the Bureau of Economic Analysis. The output series are thus calculated as total output minus net indirect tax revenues, public-sector, and residential output. After these adjustments, the output concept used is compatible with that of the capital stock series used which is the quantity index of net stock of non-residential private capital from NIPA tables. We also pay special attention to the construction of the hours and wage series by skill level, and the user cost of capital.

Data by skill levels were obtained from Autor et al. (2008).[3] Skilled workers are defined as those with (some) college education and above. Unskilled workers are defined as those with education levels up to (and including) high school. Autor et al. (2008) provide relative supply and relative wages (the skill premium) for both categories. Relative supply is defined in terms of hours worked.[4] Because the coverage of these data coming from the Current Population Survey is different from our coverage for the non-residential private sector, we combined these data with Bureau of Labor Statistics (BLS) data. While preserving relative wages and relative labor supply, we correct both so as to be compatible with the evolution of total private employment and labor compensation. Hence, we proceed as follows.

[3] We thank David Autor for providing the files for annual data by skill levels.

[4] See Autor et al. (2008) for further detail on data construction. We chose to use relative supply in terms of hours rather than the 'efficiency units' measure also provided by the authors.

We define unskilled workers' wages (WU) as,

$$WU = \frac{W}{NU/N + (NS/N) * \tilde{W}}$$

where W are wages of all workers, NU number of unskilled workers, N is total private sector workers, NS is number of skilled workers and, finally, $\tilde{W}$ is the skilled/unskilled wage ratio. Then WS, skilled wages, is simply defined as $W \times \tilde{W}$. We now need to define how some of these variables are obtained. We define W as labor income ($NINC$) over total private sector employment. A problem in calculating labor-income is that it is unclear how the income of proprietors (self-employed) should be categorized in the labor-capital dichotomy. Some of the income earned by self-employed workers clearly represents labor income, while some represents a return on investment or economic profit. Following Klump et al. (2007), we use compensation per employee as a shadow price of labor of self-employed workers:

$$NINC = \left(1 + \frac{\text{self-employed}}{\text{total private employment}}\right) \cdot \text{Comp}$$

where $Comp$ = private sector compensation of employees.

We then define $W = \frac{NINC}{\text{total private sector employment}}$. Finally, we define NS as total private sector employment times relative skilled/unskilled hours worked, and $NU = N - NS$. These transformations preserve relative quantities but correct the levels in order to comply with our previous definitions and the self-employment transformation. This assumes, of course, that relative wages and relative labor supply in the private sector evolve in a similar fashion to those in the (wider) definition provided by Autor et al. (2008).

Our capital stock concept is private non-housing capital disaggregated into structures and equipment capital. As NIPA presents these data as the end-of-year levels, in our estimation we use the two year averages of these end of year stocks. The user cost of aggregate capital K was obtained using a residual method.[5] In order to do so, we first need to make an assumption about the share of income belonging to a pure mark-up. The mark-up share can be estimated directly within the normalized system. However, because of the relatively short sample and demands imposed by the system with three factors, we imposed an average mark-up

[5]Direct measures such as those used in León-Ledesma et al. (2010b) did not change the results substantially.

of 10%, $\mu = 0.1$. This is consistent with estimates of the system using two factors (capital and aggregate labor). Under this assumption, the real user cost of capital, r, is defined as:

$$r = \frac{Y/(1+\mu) - NINC}{K}$$

Similarly, in calculating the user cost measures also for the two disaggregates of the total capital stock, i.e. non-residential structures and equipment capital, we first decomposed the total capital income $Y/(1+\mu) - NINC$ into components associated to structures and equipment capital and then proportioned them to the stocks. These capital income shares were based on capital income estimates obtained by multiplying - for scaling purposes - current dollar capital stocks by the relevant real user cost term of each type of capital.

To calculate the real user cost term, the real interest component was defined as the difference of the sample averages of the ten year government bond rate and inflation in terms of the net investment deflators. As inflation of structures investment was higher than equipment investment, depreciation rates in turn were calculated on the basis of current dollar values of depreciations and current dollar value capital stocks and were markedly higher for structures than for equipment capital.

Figure 1 plots some relevant ratios related to capital and labor inputs. The top panel of 1 shows that the equipment capital (KQ) to output (Y) ratio displays a positive trend and the structures capital (KB) to output ratio has a negative trend over the sample and, hence, as the middle panel shows, the size of equipment capital relative to the structures capital rises reflecting the downward trend in their relative user prices (UCQ and UCB). As these opposite trends largely compensate each other their relative factor income shares remain relatively stable only marginally favoring equipment capital.

As regards skilled and unskilled labor inputs, corresponding trend developments look quite different. The bottom panel of Figure 1 shows that both relative input (NS/NU) and wage (WS/WU) developments favors skilled labor, i.e. both of them have an upward trend implying an even steeper trend in the skilled labor income to unskilled labor income ratio. This provides indication against a unit substitution elasticity between these two labor inputs, since in the Cobb-Douglas case factor shares are constant.

– Insert Figure 1 Here –

4 Estimation results

4.1 Overview

All the specifications were estimated using a nonlinear system estimator allowing for cross-equation parameter restrictions and correlated errors across equations as in the class of seemingly unrelated regression estimators (SURE). This estimator hence accounts for potential common shocks to the different factor markets and the production function. For estimation purposes, we used sample arithmetic averages for non-growing variables and geometric averages for growing variables as the point of normalization. Due to the nonlinearity of the CES function, the sample average of production need not exactly coincide with the level of production implied by the production function with sample averages of the right hand variables. Therefore, following Klump et al. (2007), we introduce an additional parameter ξ whose expected value is around unity (we call this the normalization constant).

Tables 1 to **4** show the estimation of equations (9) to (13) and their restricted counterparts. In terms of those equations, our results set $w_1 = cb$ (i.e., cost of building capital), $w_2 = cq$ (i.e., cost of equipment capital), $w_3 = ws$ (i.e., wage of skilled labor), $w_4 = wu$ (i.e., wage of un-skilled labor), and $V_1 = KB$, $V_2 = KQ, V_3 = NS$, and $V_4 = NU$. The tables also display residual ADF tests for each of the equations in the system.

The conventional single-step production function where capital and labor are single indices are shown in **Table 1**. This yields what might be considered standard results: the overall elasticity of substitution at around 0.5. This is consistent with the 0.4-0.6 range that Chirinko (2008) finds in typical of US estimates. Technical progress is strongly net labor saving ($\gamma_N >> \gamma_K$). Nevertheless, for the sample period considered, γ_K is still statistically significant at a rate of approximately 0.5% per year, which is not economically negligible.

– Insert Table 1 Here –

To assess the fit of this system, in **Figure 2** we graph the factor prices (in this case, merely the log real user cost and the log aggregate real wage) and that of potential output. The fit of all three series appears perfectly respectable, with that of the user cost noticeably good.[6] Wages are also fit reasonably well. It is important to stress that many production function specifications can yield a good

[6] León-Ledesma et al. (2010b) discuss the typical fit of the first-order equation for capital.

fit for output but, generally, they do a poor job at fitting factor prices, especially the user cost of capital.

– Insert Figure 2 Here –

The first two columns of **Table 2** show the case where the production-technology system disaggregates into two capital types, but a single labor type. Again, we see the elasticity between the capital aggregate and the single labor input is around 0.5. The elasticity of substitution between different capital types is around 0.9, but still statistically significantly different from 1. The value of the labor-augmenting growth rate (at 1.7%) is the same as before. In terms of capital augmentation, we see different signs between buildings and equipment. We shall see this pattern repeated in subsequent results and we will rationalize it accordingly. When you impose the two capital augmentation growth terms to be equal, as reported in the next two columns of the table, the elasticity between capital types increases, but it insignificantly different from zero. The model fit also tends to deteriorate when this equality (i.e. Hicks-neutrality for the capital component) is imposed.

– Insert Table 2 Here –

Table 3 represents the most general case whereby capital and labor types are disaggregated. The first two columns present the estimates with no restrictions imposed on technical progress. The elasticity of substitution between labor types is between 3 and 4 which, although marginally higher than some other estimates in the literature, nevertheless points to both gross substitutability between labor types.[7] The elasticity of substitution between capital types is, as before, around 0.8. Noticeably, the aggregate elasticity between capital and labor is again close to 0.5. As regards technical progress components, technical change augmenting equipment capital is high (3%) and higher than that of skilled labor (which in turn is six times as high as that associated with unskilled labor). As in Table 2, there is again negative technical change associated to equipment and a positive one to building/structures. The former can be rationalized as the natural outcome for a sustained decline in relative rental price (recall the middle panel of Figure 1), and an associated increase in usage. In short, equipment capital was not a scarce factor and – in the language of the "directed technical change" literature,

[7]Katz and Murphy (1992), found point estimates for the elasticity of substitution between skilled and unskilled labor of 1.4 (a value echoed by other studies such as Krussel et al. (2000) and Heckman et al. (1998)).

e.g., Acemoglu (2002a) – we would expect firms to bias technical improvements towards the scarce factor.

– Insert Table 3 Here –

As in the conventional system, we can assess the fit (**Figure 3**). Again, to a visual approximation, the fit of the system is good. The fit of output is very good except for the early 1980s period. The fit of the user costs components is very impressive. It is worth recalling that previous studies using nested production functions such as Greenwood et al. (1997) and Krussel et al. (2000) focus on the fit of relative wages, whereas our system's performance depends on the fit of *all* factor prices and output simultaneously. Models that are able to fit relative wages well do not necessarily match other relevant data and can thus be only partially judged. In our case, whilst we fit well the trend of relative skilled to unskilled wages (the wage premium), we cannot capture completely the dip observed in the late 1970s.

The next four columns of **Table 3** display the results restricting some technical progress parameters to zero. Although the elasticity between capital types increases, it becomes insignificant. However, the model fit deteriorates significantly judging by the log determinant and the ADF tests.

– Insert Figure 3 Here –

Finally, **Table 4** presents results with disaggregated labor types and a single aggregate capital stock measure. The results are well in line with what we had before. A common inter-class elasticity of substitution of around 0.5 and an intra-class labor elasticity of around 3. However, this appears not to be statistically significant. Regarding technical progress, that of skilled labor dominates unskilled labor (which is around the same rate as that of capital). We then constrained technical progress for both labor classes to be the same, i.e intra-class Hicks-neutral. The model, however, did not converge unless we imposed a large intra-class substitution elasticity (almost linear aggregation) as the value of 100 reported in the table. Even in that case, the model fit deteriorates dramatically, especially for both wages and output.

– Insert Table 4 Here –

4.2 Discussion

We can now extract some conclusions about the evolution of relative intra-class factor prices from the results above.

Recall that equations (9)-(10) imply,

$$\underbrace{\log \frac{cq_t}{cb_t} + \log \left(\frac{KQ_t}{KB_t} \right)}_{>0} = \left(\frac{\eta - 1}{\eta} \right) \left[\log \left(\frac{KQ_t}{KB_t} \right) + (\gamma_q - \gamma_b) \widetilde{t} \right] + C_1 \tag{15}$$

where $C_1 = \log \left[\frac{\beta}{(1-\beta)} \frac{KB_0}{KQ_0} \right]$.

Our previous data section made clear that the left hand price ratio $\frac{cq_t}{cb_t}$ has a negative slope and the relative equipment capital to structures capital ratio $\frac{KQ_t}{KB_t}$ has a positive one. In addition since we know that the slope of the latter is steeper upwards than the downward slope of the former, then the slope of the left-hand side of (15) is positive (as indicated).

Now, in the special case of Hicks neutral technical change ($\gamma_q = \gamma_b$) it is straight forward to see that $\eta > 1$. However, with technical progress more structures saving than equipment saving, i.e. $\gamma_q - \gamma_b < 0$, the right hand-side term in square brackets may turn negative if the trend in technical progress is stronger than that in $\frac{KQ_t}{KB_t}$. This would then be compatible with a below-unity substitution parameter $\eta < 1$. Our empirical results turn out to be fully in line with this case.

When we allow free factor augmenting technical progress we estimate negative equipment capital augmenting (i.e. equipment consuming) and positive structures capital augmenting (i.e. structures saving) technical progress and below unit substitution elasticity η. This result is in line with the fact the technical progress has continuously decreased the price of equipment capital increasing at the same time (especially the quality adjusted) volume of equipment capital. However, if we postulated a non-negativity constraint for technical change, our estimation results implied zero technical change for both capital components and above unit substitution elasticity η. However, this constraint decreased the fit of the system (see log determinant) and the stationarity properties of the residuals (ADF-test value).

Turning now to the skill-premium, equations (11)-(12) imply that,

$$\underbrace{\log \frac{ws_t}{wu_t} + \log \left(\frac{NS_t}{NU_t} \right)}_{>0} = \left(\frac{\zeta - 1}{\zeta} \right) \left[\log \left(\frac{NS_t}{NU_t} \right) + (\gamma_S - \gamma_U) \widetilde{t} \right] + C_2 \tag{16}$$

where $C_2 = \log\left[\frac{(1-\pi)}{\pi}\frac{NS_0}{NU_0}\right]$.

As both components of the left-hand side have a positive trend in our data it is evident that, at least with $\gamma_S - \gamma_U \geq 0$, it must be that skilled and unskilled labor are gross substitutes and the intra-class substitution $\zeta > 1$. Only in the counter-intuitive case of strongly net unskilled-saving technical change could gross complementarity be possible. Our estimation results are, in fact, in line with expectations. They suggest strong substitution between both types of labor inputs whilst technical progress is strongly skilled-labor augmenting. In fact, unskilled labor augmenting technical progress is only borderline statistically significant and around six times slower than skilled augmenting technical change.

It is also worth emphasizing that the inter class elasticity between capital and labor is close to 0.5 in most specifications, which is the value obtained also for the single-level CES production function. In the case in which a non-negativity constraint was imposed for all forms of factor augmenting technical progress the inter-class elasticity rose somewhat to around 0.65 but, simultaneously, the fit of the system decreased markedly.

5 Conclusions

We have empirically re-examined the two-level nested CES production function as in Sato (1967) but where we allow both capital and labor inputs to be disaggregated into two components: skilled and unskilled workers, and equipment and structures capital. We also paid particular attention to the role of factor augmenting technical progress. Several specifications of the two-level CES function were examined within a "normalized" system approach to estimate deep supply side parameters such as elasticities of substitution and factor augmenting technical progress coefficients. The results were examined against the backdrop of observed trends in the skill premium and the relative price of equipment capital in the US economy. This can have important consequences for analyzing the impact of factor accumulation and technical progress on inequality.

Several findings from our estimates stand out:

- Skilled and unskilled labor display a high elasticity of substitution (well above 1) and are hence gross substitutes in production.
- Equipment and structures capital appear to display mild complementarity properties, with an elasticity of substitution around 0.8.

- The forms of technical progress that dominate are net skilled labor-saving and net structures-saving. In the first case, because of substitutability with unskilled labor, this tends to increase its relative wage and share in labor income.

- Importantly, the supply side system formed by the production function and first order conditions fits *simultaneously* all factor prices and output very well. The system also allows to analyze whether common restrictions in the literature may help fitting one part of the system such as the skill-premium at the cost of a poor matching of other factor prices.

- Finally, the elasticity of substitution between aggregate capital and aggregate labor appears to be very robust to disaggregation. It takes a value of around 0.5, emphasizing the need to abandon the common usage of Cobb-Douglas functions in most growth and macroeconomic models.

This analysis, although exploratory, may open up interesting avenues for future research. One such avenue could be the use of the two-level CES system to test for different explanations of the rising relative wage of skilled workers observed in many countries for the last three decades such as the capital-skill complementarity or the skill-biased technical change hypotheses. The system provides natural a framework to test alternative production function nestings with different implications for intra- and inter-class factor substitution and their interaction with technical progress.

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A Normalization: A Primer

Let us start with the general definition of a linear homogenous production function:

$$Y_t = F\left(\Gamma_t^K K_t, \Gamma_t^H H_t\right) = \Gamma_t^H H_t f\left(\kappa_t\right) \tag{A.1}$$

where Y_t is output, K_t capital and H_t the labor input. The terms Γ_t^K and Γ_t^H capture capital and labor-augmenting technical progress, respectively. To circumvent problems related to the "Diamond-McFadden impossibility theorem",[8] researchers usually assume specific functional forms for these functions, e.g., $\Gamma_t^j = \Gamma_0^j e^{z_t^j}$ where z_t^j can be a stochastic or deterministic technical progress function associated to factor i. The case where $z_t^K = z_t^H > 0$ denotes Hicks-Neutral technology; $z_t^K > 0$, $z_t^H = 0$ yields Solow-Neutrality; $z_t^K = 0$, $z_t^H > 0$ represents Harrod-Neutrality; and $z_t^K > 0 \neq z_t^H > 0$ indicates general factor-augmentation. The term $\kappa_t = \left(\Gamma_t^K K_t\right) / \left(\Gamma_t^H H_t\right)$ is the capital-labor ratio in efficiency units. Likewise

[8] See Diamond and McFadden (1965).

define $\varphi_t = y_t/\left(\Gamma_t^H H_t\right)$ as per-capita production in efficiency units. The elasticity of substitution can then be expressed as:

$$\sigma = -\frac{f'(\kappa)\left[f(\kappa) - \kappa f'(\kappa)\right]}{\kappa f(\kappa) f''(\kappa)} \in [0, \infty]. \tag{A.2}$$

This definition can be viewed as a second-order differential equation in κ having the following general CES production function as its solution:

$$\varphi_t = a\left[\kappa_t^{\frac{\sigma-1}{\sigma}} + b\right]^{\frac{\sigma}{\sigma-1}} \Rightarrow Y_t = a\left[\left(\Gamma_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + b\left(\Gamma_t^H H_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{A.3}$$

where a and b are two arbitrary constants of integration with the following correspondence with the original Arrow et al. (1961) non-normalized form, which, after some rearrangements can be presented in conventional form:

$$Y_t = \mathbb{J}\left[\alpha\left(\Gamma_t^K K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(\Gamma_t^H H_t\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{A.4}$$

where efficiency parameter $\mathbb{J} = a(1+b)^{\frac{\sigma}{\sigma-1}}$ and distribution parameter $\alpha = 1/(1+b) < 1$. An economically meaningful identification of these integration constants a and b (and further $\mathbb{J}$ and α) is given by the fact that σ is a *point* elasticity relying on three baseline (or "normalized", $t = 0$) values: a given capital intensity $\kappa_0 = \Gamma_0^K K_0/\left(\Gamma_0^H H_0\right)$, a given marginal rate of substitution, $\frac{\partial(Y_0/H_0)}{\partial(Y_0/K_0)}$, and a given level of per-capita production $\varphi_0 = Y_0/\left(\Gamma_0^H H_0\right)$. For simplicity, and without loss of generality, we scale the components of technical progress such that $\Gamma_0^K = \Gamma_0^H = 1$. Accordingly, we can transform (A.4) into its key normalized form,

$$Y_t = Y_0\left[\alpha_0\left(\frac{K_t}{K_0}\Gamma_t^K\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha_0)\left(\frac{H_t}{H_0}\Gamma_t^H\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{A.5}$$

where distribution parameter $\alpha_0 = r_0 K_0/(r_0 K_0 + w_0 H_0)$ has a clear economic interpretation: the capital income share evaluated at the point of normalization. We see that all parameters of (A.5) are deep, demonstrated by the fact that at the point of normalization, the left-hand-side equals the right-hand side for all values of σ, α_0 and the parameterization of Γ_t^K and Γ_t^H. By contrast, as clearly shown in La Grandville (2009) (pp. 85-86), comparing (A.4) with (A.5) the parameters of the *non-normalized* function depend on the normalized value of the factors and factor returns as well as on σ itself:

$$\mathbb{J}(\sigma,\cdot) = Y_0 \left[\frac{r_0 K_0^{1/\sigma} + w_0 H_0^{1/\sigma}}{r_0 K_0 + w_0 H_0} \right]^{\frac{\sigma}{\sigma-1}} \tag{A.6}$$

$$\alpha(\sigma,\cdot) = \frac{r_0 K_0^{1/\sigma}}{r_0 K_0^{1/\sigma} + w_0 H_0^{1/\sigma}}. \tag{A.7}$$

Accordingly, in the non-normalized formulation, parameters $\mathbb{J}$ and α have no theoretical or empirical meaning. Hence, varying σ, whilst holding $\mathbb{J}$ and α constant, is inconsistent for comparative-static purposes.

Figure 1

Some key ratios related to capital and labor inputs

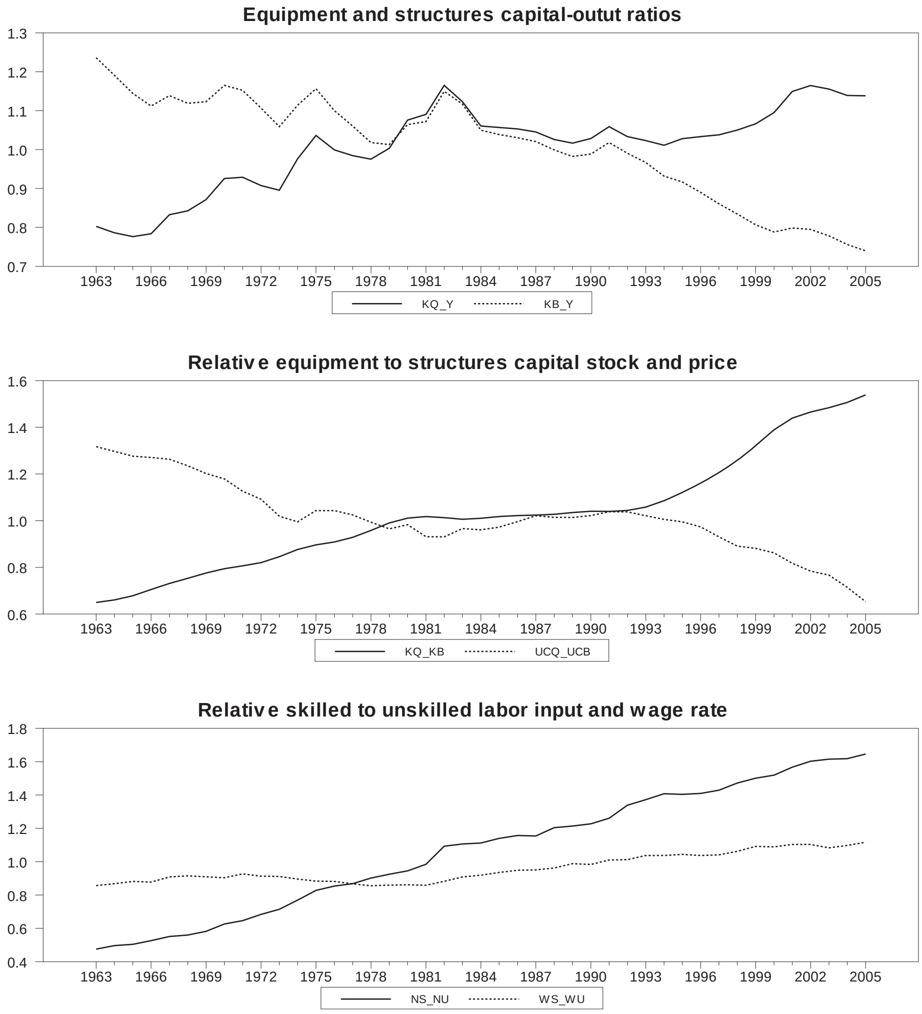
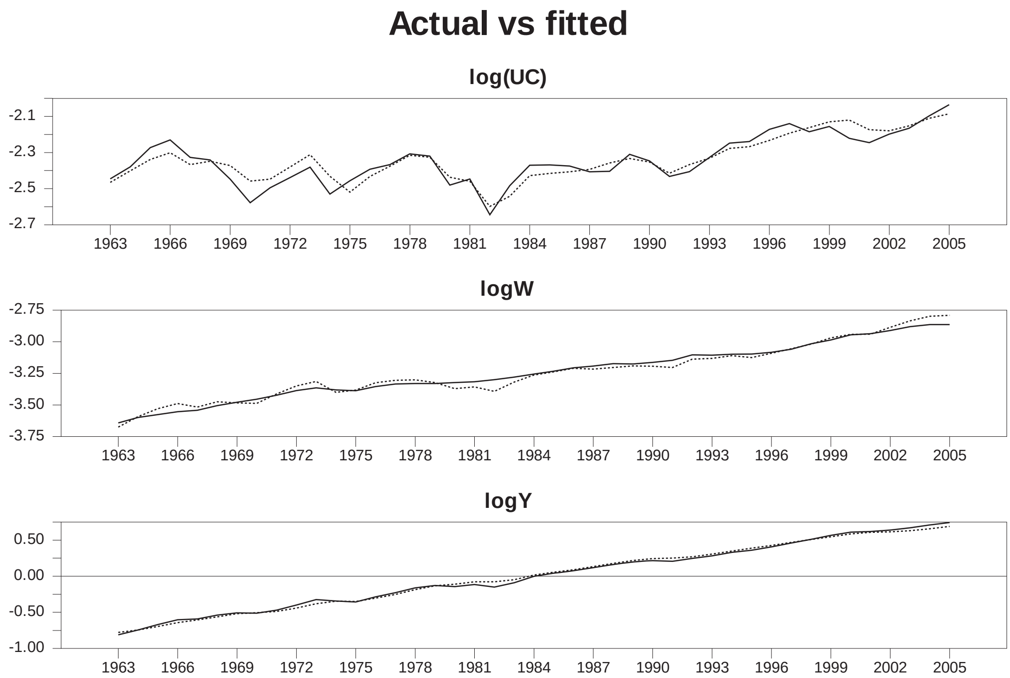
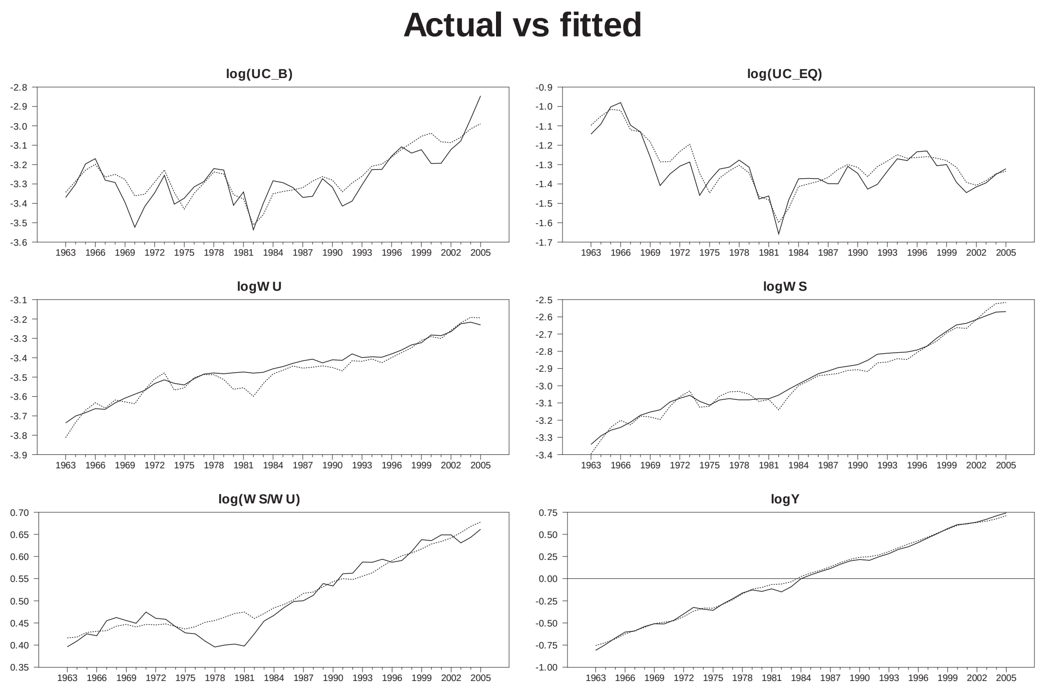


Figure 2: Single-Level Production-Technology System



Note: Solid line represents data; dashed line represents model-generated values.

Figure 3: Two-Level Production-Technology System



Note: Solid line represents data; dashed line represents model-generated values.

	Coef.	St. Err.
ξ	1.000	0.006
σ	0.512	0.002
γ^K	0.005	0.001
γ^N	0.017	0.001
Log Det.	-26.914	
ADF$_{FOC_K}$	-3.422	
ADF$_{FOC_N}$	-2.435	
ADF$_Y$	-2.255	

Table 1: Single-Level Production Technology System:
$Y = F[K, N, \sigma]$

	Coef.	St. Err.	Coef.	St. Err.
ξ	1.001	0.006	1.002	0.006
σ	0.516	0.002	0.531	0.002
η	0.871	0.043	1.748	0.136
γ^{KB}	0.045	0.015	-	
γ^{KQ}	-0.023	0.005	-	
γ^N	0.017	0.001	0.017	0.001
Log Det.	-34.008		-32.844	
ADF$_{FOC_{KB}}$	-2.695		-0.982	
ADF$_{FOC_{KQ}}$	-3.619		-2.773	
ADF$_{FOC_N}$	-2.424		-2.305	
ADF$_Y$	-2.316		-2.456	

Table 2: Two-Level Production Technology System:
$Y = F[(KB, KQ, \eta), N, \sigma]$, i.e., $V_1 = KB, V_2 = KQ, V_3 = N, \pi = 0$
Note: "-" indicates not applicable.

	Coef.	St. Err.	Coef.	St. Err.	Coef.	St. Err.
ξ	0.984	0.005	0.977	0.005	0.976	0.005
σ	0.535	0.002	0.665	0.003	0.652	0.004
η	0.819	0.077	1.631	0.116	1.647	0.110
ζ	3.640	0.680	3.156	0.475	2.841	0.067
γ^{KB}	0.032	0.013	–			
γ^{KQ}	-0.018	0.005	–			
γ^{NS}	0.024	0.002	0.025	0.002	0.026	0.001
γ^{NU}	0.004	0.002	0.002	0.002	–	–
Log Det.	-40.163		-39.634		-39.605	
$\text{ADF}_{FOC_{KB}}$	-3.012		-1.545		-1.480	
$\text{ADF}_{FOC_{KQ}}$	-3.667		-2.906		-2.910	
$\text{ADF}_{FOC_{NS}}$	-2.974		-2.658		-2.649	
$\text{ADF}_{FOC_{NU}}$	-2.967		-3.024		-3.231	
ADF_{Y}	-2.905		-3.171		-3.278	

Table 3: Two-Level Production Technology System:
$Y = F[(KB, KQ, \eta), (NS, NU, \zeta), \sigma]$, $V_1 = KB, V_2 = KQ, V_3 = NS, V_4 = NU$

	Coef	St. Err	Coef	St. Err
ξ	0.980	0.006	0.973	0.006
σ	0.457	0.001	0.464	0.010
ζ	3.525	0.641	100.0	-
γ^{K}	0.005	0.001	0.005	0.001
γ^{NS}	0.024	0.002	-	-
γ^{NU}	0.004	0.002	-	-
$\gamma^{NS} = \gamma^{NU} = \gamma$	-	-	0.011	0.001
Log Det.	-33.098		-27.1870	
$\text{ADF}_{FOC_{K}}$	-3.198		-3.2956	
$\text{ADF}_{FOC_{NU}}$	-3.067		-0.0222	
$\text{ADF}_{FOC_{NS}}$	-2.905		-2.0557	
ADF_{Y}	-2.829		-1.0459	

Table 4: Two-Level Production Technology System:
$Y = F[K, (NU, NS, \zeta), \sigma]$, $V_1 = K, \beta = 0, V_3 = NU, V_4 = NS$